THE STARS OF HIGH LUMINOSITY

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THE STARS OF HIGH LUMINOSITY

BY

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PREFACE

THE physical study of stars by means of their spectra has gone far in the last five years. In 1925 I attempted, in "Stellar Atmospheres" (Harvard Monograph No. 1), to survey and analyze the current knowledge of the subject. Ionization theory was then less than five years old, and its application was as yet general and empirical. Since 1925 there have been considerable advances in technique, theory has been extended, and, above all, there has been an accumulation of relevant observations. Since "Stellar Atmospheres" is out of print, and (to some extent) also out of date, I have been encouraged to replace it by the present volume.

"The Stars of High Luminosity" is in no sense a revision of the earlier book; the changing face of the subject, and the growth of my own outlook, have dictated the form of the present treatment. Moreover it is difficult to enumerate exactly the parts of the former monograph that are superseded in the present one. The whole picture is fuller, the detail more convincing, than they were five years ago. I believe that the chief advance (besides the improved technique that replaces qualitative by roughly quantitative results) resides in the wider correlation of data, and its effect on the background of astrophysical thought—matters entering the text implicitly rather than explicitly. In many ways the results are less clear-cut than they were five years ago, a consequence of the greater wealth of data; the picture of the stellar atmosphere sketched in Monograph No. 1 may well seem simpler and more convincing than the present one.

Although the analysis of the stellar atmosphere is still in its early phases, the present treatment marks a definite stage. It carries the work as far as I believe it can be carried with the

kind of material available to me—spectra of comparatively short dispersion, either unstandardized or standardized by simple and unrefined methods. It seems as though further work demands greater refinement of method, and probably also far larger dispersion, if the scope of the study is to be enlarged, or much greater accuracy attained.

But qualitative data have their uses. In the short history of ionization theory the majority of important advances have been made on the basis of simple, even qualitative observations. We do not yet know enough to interpret the more refined data, and assiduous sifting and classifying of facts will greatly help us to rough out our theories, and to decide which refined data are most worth pursuing. Especially is this thought stressed in regard to variable stars, which seem to be almost untouched from the broader astrophysical aspects; included for reasons of completeness, they have usurped a disproportionately large fraction of the book. But I am not even sure that it would be indefensible to make them the starting point of the whole attack on the star of high luminosity.

I should like to express my indebtedness to those who have helped me with material and the exchange of thought: Professor H. H. Plaskett, Professor B. P. Gerasimovič, and, above all, Dr. Shapley for his stimulating and generous discussion of most of the subjects of this monograph. Professor Rosseland, Dr. ten Bruggencate, Mr. Bok, and Miss Williams have each read part of the manuscript, and I am grateful for their helpful criticisms.

C. H. P.

CAMBRIDGE, MASS.
October, 1930.

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INTRODUCTORY

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CHAPTER I

SYNOPSIS OF THE PROBLEM

THE stars of high luminosity form a distinctive group, a group significant in itself and of fundamental importance in associated researches. Any one of these reasons should justify a general study of the high luminosity stars, and combined they make it a logical starting point for a survey of the spectroscopically accessible part of the universe. The present volume, planned with such a survey in mind, considers the high luminosity stars in their bearing on related stars and affiliated problems, instead of attempting to analyze them only as a restricted group.

This opening chapter aims to indicate the thread that connects—somewhat lightly in parts—the matters over which the survey has ranged. A brief synopsis of the later chapters is followed by a few general remarks on the high luminosity problem, on the value of the supergiant in related researches, and on the chief untouched or unsolved problems of the very luminous stars.

1. Synopsis of Later Chapters.—The technique of stellar spectrophotometry (largely developed in connection with the present and affiliated researches) is outlined in Chapter II, and the interpretation of the forms of spectrum lines is roughly discussed in Chapter III. In the remainder of the book, line contours are used as the basis of physical discussions, and the theoretical formulae that have been adopted are summarized, with references to original sources, in Chapter III.

The fourth and fifth chapters survey the material. In the former the 20,000 known high luminosity stars are summarized. Most of them are in distant systems, but presumably there is a comparable number of unrecognizable supergiants in the depths of our own Galaxy. The fifth chapter surveys the spectroscopic material and points out how unrepresentative it is, partly because spectra even of giant stars are available only to a distance of about one kiloparsec and partly because a considerable number of high luminosity stars of early type, such as Alcyone, are spectroscopically unrecognizable.

The spectral classes are then taken up individually; each is discussed from the aspects of distribution, luminosity, and the physical interpretation of the observed spectrum. Special topics discussed in these chapters (VI to XIII) are: the classification of the O stars; the temperatures of the B stars; problems of the bright-line B stars; the abnormally cool stars of early type (whose spectra are shown to require conditions other than the observed temperature, and any pressure at present regarded as probable); the general problem of the classification of second-type stars; and the interpretation of the "split" at the cool end of the spectral sequence. Throughout these chapters the aim is to evaluate physical conditions for the supergiant, and in every class the brighter stars are found to have a spectrum at lower pressure and temperature. Rational methods for absolute magnitude are thus foreshadowed.

The variable star forms one of the most important sections of the whole high luminosity problem, to which it is almost peculiar; the Cepheid variable, in fact, enables us to test our physical conclusions and our absolute-magnitude criteria. The principal sequence of variable stars—running from cluster-type Cepheids through classical Cepheids and RV Tauri stars to long-period variables—furnishes an invaluable parallel to the giant and supergiant branches and provides important evidence on the interrelationships of stars.

The observations contained in Chapters VI to XIV are largely presented without comment; they are summarized in

Chapter XV as far as the normal star is concerned. The new methods enable us to express the strength of the stellar absorption lines in terms of numbers of effective atoms over one square centimeter of stellar surface, and the summarized data constitute a real quantitative analysis of atmospheres along the spectral sequence.

The conditions in the atmospheres, deduced on the basis of these analyses, are found to be very near to those previously determined with cruder data and less satisfactory theory than those now available. The observed intensities of lines along the sequence can be shown to harmonize fairly well with the generalization of the Saha theory proposed by Milne.[1] Our present picture of the normal stellar atmosphere is explicit and in many ways hopeful.

What Chapter XV does for the normal star Chapter XVI does for the star of high luminosity. Pressures in the atmospheres of all such stars are very low—largely a result of low surface gravity—but for many, especially of early type, the temperatures and pressure are not alone competent to produce the observed spectrum.

Of especial interest is the comparison of the total amount of atmosphere in normal and supergiant stars; as Adams and Russell have already shown,[2] it is enormously greater for the latter.

The main part of the monograph is occupied in deducing from observations of the spectrum the physical conditions in the atmosphere of the star. But the analysis has a deeper significance than this. The correlation of certain surface conditions preferentially with certain surface temperatures is an observed fact—incompletely explained, it is true, but evident from the Russell diagram and the period-spectrum relation for variable stars, for example. The spectrum cannot fail to be a highly significant index to the star itself, and as such it must be classified on a physically significant basis.

[1] Milne, M. N. R. A. S., **89**, 17, 157, 1928.

[2] Mt. W. Contr. 359, 1928.

The problem of spectral and stellar classification is still to be solved. Effective temperature is a valuable parameter (though sometimes, perhaps usually, ambiguous), and it and the percentage ionization of a substance of suitable critical potentials are suggested as a basis for the classification of spectra. But it must be emphasized that to propose a system of classification is trivial; to apply it successfully to the arrangement of stellar spectra would be a contribution of real importance.

2. The Significance of High Luminosity.—Stars brighter[3] than absolute visual magnitude -2 are extreme in several respects: They are unusually massive, of very low density, of exceptionally high energy output (per unit mass), and exceedingly uncommon. The infrequency of the supergiant in space has an important bearing on its age and history, arguing for unlikely formation, or ready disruption, or brief duration—perhaps a combination of the three. An attempt to sort out the types of data that would detect and differentiate these effects may help us to approach later chapters with more perspective.

a. Probability of Formation.—Most spectroscopically and photometrically recognizable supergiants seem to be isolated stars; some are members of clusters (the supergiants in galactic clusters being preponderantly B stars,[4] and in globular clusters invariably red stars). Membership in a cluster is not always a concomitant of high luminosity; and there does not seem to be any systematic difference in brightness or spectrum between cluster supergiants and isolated supergiants. There is therefore no reason for supposing that supergiants are confined to condensed systems that might be guessed to have originated under conditions favorable to the formation of stars. The data for

[3] For the absolute magnitude of the high luminosity stars this limit has been arbitrarily adopted.

[4] In Chapter VII it is shown that the mean absolute magnitudes of Classes O, B1, and B2 place them within the high luminosity group. Such stars occur in galactic clusters of the Pleiades type but are rather uncommon. Although Trumpler classifies about half the spectroscopically accessible galactic clusters as of Pleiades type, the majority of these contain no stars earlier than B5, and therefore their members are not generally supergiants.

the galactic system are borne out by Shapley's observation that the most luminous stars in the Magellanic Clouds are not preponderantly members of clusters.

b. Stability of the Supergiant.—A second possible cause of uncommonness is ready disruption. Apparently the stars most subject to cataclysmic disruption—the novae—are originally of low luminosity. The supergiants, however, are more than commonly variable in brightness. The long-period variables are probably all supergiants;[5] so are the Cepheids of longer period than ten days, the RV Tauri stars, and most—perhaps all—other classes of variables. All N stars are variable, and also most M stars, including the presumably large supergiant group of which Betelgeuse and Antares are typical. Variability of radial velocity has also been detected for many (supposedly non-variable) supergiants of Class F, as well as for α Cygni and β Orionis; it seems that supergiants vary more commonly in brightness and in radial velocity than normal stars, from which it can be inferred that their surface conditions are unstable.[6] But stability in a star does not depend upon surface fluctuations, and the continuous pulsation of many variables shows that they have enough intrinsic stability to recover from the changes involved. No obvious reason exists for expecting massive (or luminous) stars to be especially unstable; the argument is more fruitful if reversed, postulating instability on the grounds of infrequency.

c. Duration of the Supergiant Stage.—The uncommonness of the supergiant might be explained if it were known that these stars have only a short life as such. The most definite evidence on the matter is very puzzling. The supergiants in globular clusters (presumably coeval groups) are all red stars (giants); they seem to occur in the same proportions in all the globular clusters. To account for this condition we may reject the idea

[5] See p. 224.

[6] It is of course possible that the sharpness of the lines of a supergiant makes such an effect in radial velocity easier to detect than for the normal star; but probably the effect is real.

of development, or the belief that the development is unidirectional. The most plausible resolution of the difficulty is that suggested by Shapley,[7] but it traces the dilemma to special conditions and does not suggest what inheritance might confer high luminosity on a star. There is in fact no direct observational evidence as to the permanence of the supergiant stage; indirectly we can make guesses based on uncommonness and on the unusually large energy output per unit mass; both these prepare us to believe that the supergiant does not live long.

It is tempting to associate the very luminous B stars with sources of mass and therefore of energy in the shape of diffuse galactic nebulosity (with which they are well known to be associated). But the approach is not very fruitful. The B stars make the nebulosity visible, and probably if it surrounded stars of other classes we should not detect it (for instance, only the brighter Pleiades, B5 to B8, are observed to be enmeshed in individual flecks of nebulosity), so we cannot assume that they alone are involved in it.[8] There are some striking galactic clusters, and some bright B stars, not now involved in visible nebulosity; and also it is hard to see why all the members of a cluster, and not merely a few, should have profited by passing through a rich district.

To summarize the observational data bearing on the uncommonness of supergiant stars: they do not seem to be formed more commonly in clusters than normal stars—rather the reverse; most variable stars (unstable as regards surface conditions) are of high luminosity; and the high luminosity stars have not obviously developed faster or slower than less luminous stars.

The output of energy per gram is greater for supergiants than normal stars of similar spectrum, and greater for hot supergiants than cool ones.[9] These are perhaps the most funda-

[7] H. B. 876, 1930.

[8] In the planetary nebula N. G. C. 246 there appears to be a star of Class B and one of Class G; Hubble (Ap. J., **56**, 182, 1922) enumerates six stars of late type that he considers definitely involved in nebulosity.

[9] Eddington, The Internal Constitution of the Stars, 311, 1926.

mentally important observations that have been made on these stars. I make no attempt to discuss them. Obviously they are closely linked with matters of evolution and stability.

The discussion of the infrequency of the supergiant has touched, as far as is desirable, on the problems of the stability and age of this type of star. It has sufficed to show how fundamental, and how far from solution, are the basic problems connected with the supergiant. The spectrum, it appears, is not the direct way to the solution of the supergiant problem, but for the direct approach the data are inadequate. The spectroscopic attack, though indirect, is one of the few possible; I return to it in Section 5.

3. Applications of the High Luminosity Star.—The highly luminous star, even though uncommon and perhaps abnormal, is our chief medium of communication with the more distant parts of the universe. Our power of measuring distances greater than about ten kiloparsecs depends almost entirely on our ability to recognize, and to establish the absolute magnitude of, high luminosity stars. The distances[10] of the Andromeda Nebula, Messier 33, and N. G. C. 6822, for instance, are deduced from the measurement of novae and long period Cepheids. Furthermore, the high luminosity stars in other systems (including the Magellanic Clouds) are our only medium for comparing physical conditions with our own system on the basis of spectra, for the spectra of fainter stars cannot be attained at present. For this reason alone the study of the high luminosity stars—especially the empirical study of the spectra of those in the galactic system—is an urgent matter. The very brightest stars in clusters and nebulae are certainly far from comparable with the *normal* nearby stars on which our notions of stellar atmospheres are largely based.

4. The Leading Problems.—It is important to determine the true frequency of supergiants in different parts of the galac-

[10] Hubble, Mt. W. Contr. 304, 1925; 310, 1926; 376, 1929.

tic system, and as far as possible in external systems; such data would throw light on origins and duration both of the stars and of the systems that include them.

Another problem embraces the stability of the supergiant, which demands an extensive theoretical attack. A study of the variability of the supergiant, with a view to determining if it is a physical or a statistical phenomenon, would further the same end. Has the region of vestigial variation in globular clusters a parallel among galactic stars, and are all stars of corresponding mass and density prone to variability? If so, with what other quantities is the amplitude related?

5. The Spectroscopic Approach.—The supergiants occur, with about the same brightness, and about in the same proportions, in all spectral classes, and hence with all surface temperatures. The similar intrinsic luminosities, and rather similar rates of evolution of energy per gram of material, in all spectral classes, show that no spectroscopic approach can contribute *directly* to the problems of origin, stability, and duration outlined above, though indirect contribution may be possible.

At first, however, we must proceed in another direction: the spectra of very luminous stars must be analyzed in comparison with normal spectra, and the differences used to deduce the physical conditions at the surface of the former. For stars of any one surface temperature we then have an opportunity of correlating ionization conditions at the surface with luminosity, surface gravity, mass, and energy output. This analysis will then be available for attacking the leading problems in high luminosity and for working out the connection with the normal giant, and with the star that constitutes effectively the whole stellar population—the main sequence dwarf. It is essential to recollect that, though the supergiant is exceedingly uncommon, the normal giant is hardly less so in comparison with the probable number of dwarf stars, at least in the later classes. Conclusions based on the spectra of either must be qualified until

the corresponding measures have been made for the dwarf. Examples will appear later[11] in which the mere qualitative data on the dwarf star are enough to rule out a theoretical interpretation that is satisfactory for giant and supergiant. The accessibility of intrinsically bright stars has its dangers.

[11] Cf., for instance, the hydrogen problem, Sections 81, 82, and 85.

CHAPTER II

ON THE PHOTOMETRY OF SPECTRA

THE accumulation and discussion of material is the first object of the present monograph. Descriptions of method are collected in this chapter and the one that follows it, but are not given in detail. Methods of observation have developed rapidly; for a more complete summary, reference should be made to Brill's discussion in the Handbuch der Astrophysik.[1]

6. Development of Spectrophotometry.—When the diversity of spectra had been recognized and used as a basis for classification, it is surprising how much attention was directed to the refinement of wave-length measurement and how little to the problem of measuring the strengths and forms of lines, or to the photometry of the continuous background. The principles were formulated early: the essentials for surface photometry were outlined by Hartmann,[2] and for the photometry of line contours by A. S. King and Koch, on the basis of Koch's earlier work.[3]

But although the existence of line structure was realized, described as "shading,"[4] or even illustrated,[5] it was not at once made the subject of precise measurement. Bottlinger first made accurate measures of the shapes of hydrogen absorption lines in A stars,[6] and Schwarzschild's measures for the H and K

[1] Handbuch der Astrophysik, **2**, 1929.

[2] Ap. J., **10**, 321, 1899.

[3] Ann. der Phys., **39**, 705, 1912; **40**, 797, 1912; **41**, 115, 1913; **42**, 1913; A. S. King and Koch, Mt. W. Contr. 77, 1914.

[4] Cf., for instance, Jewell, Ap. J., **8**, 51, 1898, though the observation here is not rightly interpreted; see Adams, Joy, and Merrill, P. A. S. P., **36**, 226, 1924.

[5] Cf. Jewell, Ap. J., **9**, 211, 1899.

[6] A. N., **195**, 117, 1913.

lines in the sun[7] stood for a long time as the only data on an important subject. Notwithstanding the urgency of the matter, the first systematic work executed with the precision now both possible and necessary was only begun in 1924, almost simultaneously, by Shapley[8] and Kohlschütter.[9] It has since been carried out chiefly at Harvard and at Amsterdam.[10]

The lack of data is the more surprising when it is noticed that some of the early material was well suited to measurement. Jewell's results, quoted above, show clearly the difference in shape between the solar hydrogen and metallic lines, so often noted since,[11] and first actually measured by Unsöld.[12] The wings of lines, as the "shading" came to be called, must have been obvious at a very early stage, but even as late as 1923, ten years after the pioneer measures of Schwarzschild and Bottlinger, they were discussed, even for the solar spectrum, only on a qualitative basis.[13] Russell, Dugan, and Stewart[14] pointed out that wings are a function of the strength of a line, and from this observation to the use of the measured contour in evaluating numbers of atoms is not a long step.

Photographic photometry of the continuous background was of even slower growth. E. C. Pickering[15] first seems to have attempted it, and his words are worth recalling: "The relative brightness of stars . . . of different colors cannot be correctly indicated by any single number or ratio. It is necessary to employ a . . . series of numbers which shall give a measure of the energy of rays of each different wave length . . . The intensities of rays of different wave lengths may be determined by comparing the densities of different portions of a photo-

[7] Sitz. d. Preuss. Ak. d. Wiss., **47**, 1183, 1914.
[8] H. B. 805, 1924.
[9] A. N., **220**, 326, 1924.
[10] Pannekoek, B. A. N., **4**, 1, 1927.
[11] Russell and Miss Moore, Ap. J., **63**, 1, 1926.
[12] Zs. f. Phys., **44**, 793, 1927; **46**, 765, 1928.
[13] Cf. Russell and Miss Moore, Ap. J., **63**, 1, 1926.
[14] Astronomy, 574, 1926.
[15] A. N., **128**, 377, 1891.

graphic spectrum." He referred to the solar spectrum as standard and published his results essentially in the form of color magnitudes, again adopted at Harvard nearly 30 years later.[16] His results were inaccurate and received little attention, but they were the pioneer attempt of photographic spectrophotometry.

The early history of background photometry is not dissimilar to that of line photometry; the photographic work dates chiefly from the present decade. As early as 1880, Vogel undertook absolute visual photometry of the continuous background, using a petroleum lamp as standard.[17] From this small beginning there was little visible advance until Wilsing and Scheiner's[18] absolute visual photometry. They used a carbon-filament lamp, calibrated in the laboratory, and successfully eliminated such sources of error as atmospheric extinction, which had remained uncorrected in Vogel's pioneer measures.

As with the contours of lines, there was no lack of material for background photometry. The spectra of stars of large and small proper motion (essentially stars of small and great luminosity), compared qualitatively by Adams,[19] would have yielded, on standardized plates, under the ideal observational conditions under which they were taken (on the same plate with the same zenith distance), a far better measure of the temperature difference between bright and faint stars than can be obtained from color indices. It was in 1914 that the first standardized photographic photometry was published by Brill.[20]

7. The Photometric Problem.—The spectroscopic problem in spectrophotometry is less complex than the photographic problem of deducing from the blackening of a photographic plate the intensity of the light that affected it. Hartmann's

[16] H. B. 848, 1927.

[17] Monatsber. d. Kgl. Preuss. Ak. d. Wiss., p. 801, 1880.

[18] Publ. Pots. Ap. Obs. No. 56, 1909; also Wilsing, Scheiner, and Münch, Publ. Pots. Ap. Obs. No. 74, 1919.

[19] Ap. J., **39**, 89, 1914.

[20] Publ. Pots. Ap. Obs. No. 70, 1914.

historic paper[21] states the essentials of surface photometry and outlines a way of complying with them. Mention should also be made here of Schwarzschild's solution of the problem by the Schraffierkasette, and its application in the Göttingen Aktinometrie.

The problem of the photometry of surfaces (including the conversion of other photometric problems, by out of focus methods, into surface problems) is discussed from many aspects in a series of papers published at Harvard during the last 12 years by E. S. King,[22] which are among the classics of standard photometry. These researches were directed primarily to the photometry of integrated starlight, or of the lunar surface, and did not encounter specifically, as a first order effect, the fundamental spectral problem of the sensitiveness of the film to different colors. However, they furnish the basis for spectral investigations, by establishing the principles that are "ideal for the comparison of two sources of light."[23]

I. The lights should be equalized by using apertures of different dimensions, or by setting the apparatus at different distances from the source of light, or by other similar devices.

II. The exposures should be of the same duration, and made either simultaneously, or as nearly consecutive as possible.

III. The images should be received on one plate, or parts of the same plate, and developed simultaneously in the same tray. The final work of measurement will then consist in determining the relation of images of nearly equal density.

IV. Every experiment should be repeated a sufficient number of times, under a variety of forms, to ensure the elimination of the subtle errors that may affect any one determination.

All of these principles are followed in the successful methods that have since been devised for the photometry of spectra.[24]

[21] Ap. J., **10**, 321, 1899.

[22] H. A., **59**, 1, 33, 223, 245, 1912; **76**, 83, 107, 1916.

[23] E. S. King, H. A., **59**, 34, 1912.

[24] For details of the fundamental physical work see Ornstein, Proc. Phys. Soc. London, **37**, 339, 1922; Phys. Zs. **28**, 688, 1927. On the detail of the analyzer, which follows the Moll pattern, see Moll, Proc. Phys. Soc. London, **33**, 207, 1921.

They do not specify any details of method for plate calibration, but such methods are enumerated in the papers quoted.

The table that follows summarizes the chief methods that have been used for standardizing spectrophotometric plates. The references are given so that the methods can be examined in detail; they are not intended to be complete, and they include, in general, only astrophysical applications.[25]

TABLE II, I.—SYNOPSIS OF METHODS OF PLATE CALIBRATION

A. Astronomical Calibrations

1. Prism crossed with grating	Hertzsprung[26] Greaves, Davidson, Martin[27] Hogg[28]
2. Various diaphragms on one star	Shapley[29] Payne and Shapley[30] Payne and Hogg[31]
3. Stars of known brightness, same spectral class	Payne and Hogg[31]
4. Stars of known brightness, different spectral class	Hogg[32]
5. Differences within same spectrum	Payne[33]
6. Exposure ratios, one star	Payne and Hogg[34]

B. Laboratory Calibrations

7. Wedge method	Merton and Nicholson[35] H. H. Plaskett[36]
8. Sensitometry	Brill[37] Yü[38]
9. Densitometry	King[39] Dunham[40] Miss Williams[41] Pannekoek and Minnaert[42]
10. Rotating sector	Elvey[43] Miss Anger[44]
11. Iron arc (comparison, slit spectra)	Hogg[45]
12. Standard curve	Sampson, E. A. Baker,[46] Hogg[47]

[25] Dorgelo, Phys. Zs. **26**, 756, 1925 gives a valuable discussion of some non-astrophysical applications of spectrophotometry.

[26] A. N., **207**, 75, 1918.

[27] M. N. R. A. S., **86**, 33, 1925; **87**, 352, 1927; **90**, 104, 1929.

[28] H. Repr. 48, 1927.

[29] H. B. 805, 1924.

[30] H. Repr. 28, 1926.

In spectroscopic work it is clearly not possible to comply with requirement I of King's list, but requirements II, III, and IV must be considered essential; and the importance of keeping as far as possible within a small range of density must never be forgotten. Methods 1 to 5, and 7, comply with requirements II and III, and in that it does not do so, method 6 must be considered very unsatisfactory.

The methods just mentioned calibrate the plate separately for each wave length, but often it has been considered satisfactory to standardize for all wave lengths together, as is done in methods 8 to 12. Differences of gradation at different wave lengths will introduce errors into measures thus calibrated; in order to eliminate these to some extent, and to confine the calibration to the region of the spectrum most represented in the spectrogram, Dunham and Miss Williams standardized through a blue filter, and Yü through an ultra-violet filter.

The method of Hogg, depending on multiplet intensities in the iron arc, is also of course a calibration of the integrated light (though it can be confined to certain spectral regions by suitable choices of multiplets). It is designed for use with otherwise uncalibrated slit spectra.

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[31] H. C. 301, 1927.
[32] H. C. 309, 1927.
[33] H. B. 855, 1928.
[34] H. C. 304, 1927.
[35] Phil. Trans., **317A**, 237, 1917.
[36] Publ. Dom. Ap. Obs., **2**, 213, 1923.
[37] Publ. Pots. Ap. Obs. No. 70, 1914.
[38] L. O. B., **12**, 104, 1926.
[39] H. A., **59**, 37, 1912.
[40] H. B. 853, 1927.
[41] H. C. 348, 1929.
[42] Verh. Kon. Akad. v. Wetensch. Afd. Natuurkunde, **13**, No. 5, 12, 1929.
[43] Ap. J., **58**, 145, 1928.
[44] Ap. J., **60**, 114, 1929.
[45] H. C. 337, 1929.
[46] M. N. R. A. S., **83**, 174, 1923; **85**, 212, 1925; Proc. Roy. Soc. Edin., **45**, 166, 1925; **47**, 34, 1927; **48**, 106, 1928.
[47] H. B. 856, 1927.

The use of a standard curve is the least accurate way to derive results from the measurement of spectrograms, but it enables us to use unstandardized spectra in large numbers and is of very wide application. It gives results that compare very well with those from methods 1 to 5.

8. The Standard Source.—A standard source is not required for line photometry; the line is compared with the neighboring continuous background—in practice, with the continuous background as the investigator supposes it would be if the line were absent. The inaccuracy of drawing the background is to my mind the most serious and unsuperable problem in line photometry; it must always depend on the judgment of the observer, and for instance to the violet end of the spectra of second-type stars, where wings become confluent, the task is insoluble.[48] To obtain spectra with very high resolving power removes the difficulty for faint lines, but the problem mentioned here does not really depend only on finite resolving power—the head of the Balmer series in A stars and the ultimate iron lines in cool ones would present insuperable problems at any resolving power, for the lines are actually confluent. Some method of tracing the background may possibly be devised for late-type stars of known temperature, but it is doubtful if the line intensities it would give would ever be entitled to accuracy; equally hard is the problem for such objects as the Wolf-Rayet stars and the late nova spectra, where it is impossible to distinguish where emission begins. We therefore confine ourselves to the approximate results that can be obtained for stars where the drawing of the background is not seriously subject to doubt; and keep in mind the uncertainty inherent in all measures of line depth, area, and contour from this cause alone.

In determining the energy distribution in the background, however, a standard source, either terrestrial or stellar, must be used. The use of either demands a knowledge of atmospheric

[48] Cf. also H. Repr. 44, 1928; further evidences on the difficulty of putting in the background are given in H. C. 352 by Miss Anger, and in Figure III, 8.

extinction coefficients, which enter absolutely for terrestrial standards, differentially for stellar ones. It might be thought that in the latter case the inherent errors should be smaller, but even when a comparison star is used for the determination of temperature, unless the comparison is made in exactly the same part of the sky, the extinction coefficient introduces a fruitful source of error, which is eliminated only when the comparison star is actually in the same field as the stars measured.

The correction for atmospheric extinction, which is different for each wave length, is taken to be proportional to the secant of the zenith distance of the star. For differential work, the difference of the secants of the zenith distances is involved, and the extinction correction should theoretically be zero if the stars are at the same altitude. But as the corrections differ from place to place,[49–51] evidently have a seasonal error,[52] probably change during one night,[53] and differ at one time in different parts of the sky, the corrections applied may well be in serious error. The best that can be done in the matter is to work only on good nights, to spread observations through the year as far as possible, and, if a comparison star is used, to have it as near to the measured star as possible, in azimuth as well

[49] Gerasimovič, H. C. 339, 1929. This paper contains the following illuminating comparison of the values of A in the formula $\Delta G = A \sec \zeta$, where G is gradient, ζ is zenith distance:

Cambridge, Mass.	0 54 ± 0 03
Greenwich	0 73 ± 0.05
Washington	0 38

A determination of atmospheric extinction separately for each night and position is probably feasible but has not so far as I know been carried out. As the Washington observers found, the atmospheric transmission is not a linear function of wave length (Ann. Astr. Obs., Smiths. Inst., **3**, 135, 1913); the value 0.38 just quoted was obtained by Gerasimovič by taking suitable means. For accurate spectrophotometry the form of the dependence on wave length cannot be ignored.

[50] Abbot and Fowle, Smithsonian Tables, 181, 1914.

[51] Greaves, Davidson, and Martin, M. N. R. A. S., **87**, 352, 1927.

[52] Miss Williams, H. C. 348, 1929; Greenstein, H. B. 876, 1930.

[53] Bottlinger, Veröff. Berlin-Babelsberg, **3**, No. 4, 1923.

as altitude. A network of standard spectra distributed over the sky will go far to meet this requirement, in the sense planned by Hogg;[54] the Greenwich observers and Kienle at Göttingen are laying the foundations of such a network.[55] A set of standard stellar sources requires not only careful inter-comparison but also a zero point, which must be determined by the use of terrestrial standards.

Various standard terrestrial sources have been used for stellar comparison; as the matter does not explicitly enter this monograph it will suffice to tabulate some determinations of stellar temperature that have been made by comparing the energy of the continuous background with that of standard terrestrial sources taken with the same optical system.

TABLE II, II.—ABSOLUTE DETERMINATIONS OF STELLAR TEMPERATURES

Standard	Reference
Petroleum lamp	Vogel[56]
Carbon filament	Wilsing and Scheiner[57]
Acetylene lamp	H. H. Plaskett[58]
Carbon arc	Baillaud[59]
Carbon arc	Greaves, Davidson, Martin[60] Storer[61]

For reference here also some determinations of temperature that have been made relative to stellar standards are summarized in Table II, III.

The assumption of 10000° as the temperature of the standard A0 star seems not to be far from the truth, as indicated by the determinations listed in Table II, II. The alternative of 13000° given by Greaves, Davidson, and Martin, or of 12000°

[54] H. Repr. 48, 1928.

[55] M. N. R. A. S., **90**, 104, 1929.

[56] Monatsber. d. Kgl. Preuss. Ak. d. Wiss., p. 801, 1880.

[57] Publ. Pots. Ap. Obs. No. 56, 1909; also Wilsing, Scheiner, and Münch, Publ. Pots. Ap. Obs. No. 74, 1919.

[58] Publ. Dom. Ap. Obs., **2**, 213, 1923.

[59] Bul. Astr., **4**, No. 3, 275, 1924.

[60] M. N. R. A. S., **86**, 33, 1925; **87**, 352, 1927; **90**, 104, 1929.

[61] L. O. B. 410, 1928.

TABLE II, III.—RELATIVE DETERMINATIONS OF STELLAR TEMPERATURES

Standard	Reference
Sun	Rosenberg[62]
Standard A star	Greaves, Davidson, Martin[60]
ζ Ophiuchi	Yü[63]
Polaris	Sampson[64]
Standard Bo stars	Payne[65]
Standard Ao stars	Hogg[66]
Standard Ao stars	Gerasimovič[67] Hufnagel[68]

by Gerasimovič, seems to be rather high. Miss Williams[69] has lately shown that the dispersion in temperature within one class near Ao is very small, so that any normal Ao star may probably be selected as a standard with some confidence, though any extensive program should of course rest upon several such standards, which should also be rigorously intercompared.

9. Outline of Procedure.—The investigations in the present monograph were standardized in the following ways.

TABLE II, IV.—METHODS USED FOR PLATE CALIBRATION

Material	Standard Method
Spectra of c-stars	2, 3, 4, 5, 12
Normal spectral sequence	3, 4, 5, 12
O stars	12
Cepheid variables	2, 3, 4, 12

All the spectra of stars of Class K (Chapter XI) were measured by method 12, and all that could so be measured (the great majority) by methods 3 and 4 also. The differences in the mean line intensities were inappreciable, so that method 12 (standard curve) may be satisfactorily used for the analysis of

[62] Abh. d. K. Leop.-Carol. Deutsch. Akad. d. Naturforscher, Nova Acta, **101**, No. 2, 1914.

[63] L. O. B., **12**, 104, 1926.

[64] M. N. R. A. S., **83**, 174, 1923.

[65] H. B. 848, 1927.

[66] H. C. 309, 1927.

[67] H. C. 339, 1929; Gerasimovič and Payne, H. B. 866, 1929.

[68] H. C. 343, 1929; H. B. 874, 1930.

[69] H. C. 348, 1929.

unstandardized spectra. It was accordingly used for the bright-line stars which could not otherwise have been studied. The more accurate methods of diaphragming, prism crossed with grating, or the wedge, will of course be applied in the future when such spectra are photographed. Of the other workers whose results are extensively quoted in the present monograph, Hogg used methods 2 and 12 primarily, and Dunham and Miss Williams used method 9.

In concluding this synopsis of method it is perhaps not out of place to urge that all spectrum plates taken in the future, for whatever purpose, should receive some type of standardization, preferably for all wave lengths separately, though a standardization by integrated light is better than none at all. The waste of material occasioned by the lack of such a procedure is almost incalculable, especially for the spectra of novae, comets, and the chromosphere.

CHAPTER III

ON THE INTERPRETATION OF THE FORMS OF SPECTRUM LINES

SINCE first spectra were photographed, it has been realized that stellar absorption lines are neither black nor monochromatic. The sun, our most accessible star, has unusually fine lines and probably caused a certain confusion about the true widths of absorption lines in stellar spectra. But the spectra of other stars taken, for instance, with the 100-inch Coudé arrangement at Mount Wilson, show lines far wider and less sharply defined than the sun does. This is even true for stars (such as α Cygni and α Persei) that had long been considered, as c-stars, to have very narrow lines. There is no doubt that stellar absorption lines have a very definite shape and that this shape is of great potential significance in the analysis of the stellar atmosphere.

10. Observations of Line Contour.—The universal presence of "wings" to absorption lines had been for some time a growing conviction when Russell and Miss Moore[1] showed that the winged lines in the solar spectrum were the strongest, and related the wings in very general terms to the amount of material present. This was the first step toward quantitative line photometry. At about the same time that Russell and Miss Moore discussed the winged lines, actual measurement of line contours was being systematically undertaken in several quarters.[2,3]

[1] Ap. J., **63**, 1, 1926.
[2] Shapley, H. B. 805, 1924.
[3] Kohlschütter, A. N., **220**, 326, 1924.

Many of the results in the present monograph depend on measures of contour, and it is proposed to devote a chapter to summarizing the data bearing on contour as such, and to comparing observed values with the predictions of theory.

11. Sources of the Contours of Lines.—The measured intensity curve of an absorption line, relative to the continuous background of the star, is produced jointly by several effects, of which the chief are:

1. The actual distribution of energy in the light that leaves the star.
2. Atmospheric (and possibly interstellar) effects.
3. Resolution of the spectrograph.
4. Resolution of the analyzer.
5. Photographic effects.
6. Shallowing effects of the diffraction pattern.
7. Relative motion of different parts of the source (rotating and expanding stars).

The theories of line contour apply, of course, to (1) only, and until the other factors have been convincingly eliminated or shown to be negligible, it is useless to apply theory in any detail to measured contours. We may also recall that the difficulty of tracing the continuous background, alluded to in Chapter II, will in most cases introduce an error at least comparable to those arising from the other sources enumerated. Unlike the other errors the error involved in putting in the background depends on judgment, and the hope of eliminating it is remote.

The contributors to the final contour may be classed as intrinsic line shape; stellar disturbances; optical disturbances (including atmospheric effects); and photographic effects.

Optical Disturbances.—The resolving power of the one-prism objective-prism camera used for most of the work discussed below is about 2,000, so that, even apart from any imperfections of performance, the observed contours of all stellar absorption lines, except those of hydrogen, calcium, and at times helium,

are purely instrumental. Investigation of contours is therefore here confined to the lines named, though a few other lines were examined with greater dispersion. The derivation of total absorptions from a single-parameter measure for lines that are entirely instrumental is discussed in a later section.

There is no point in using a much greater resolving power with the analyzer than is possessed by the spectrograph. The analyzer used throughout the writer's work is the Moll microphotometer of the Harvard Observatory. For the most part the resolving power used with the microphotometer was about twice that of the spectrograph, so that there is no danger of reducing the effective resolving power with which the analyses were made.

Atmospheric unsteadiness has a direct effect on the objective-prism spectrum, shallowing and blurring the lines by increasing the size of the effective stellar image.[4] Although it does not operate precisely as a reduction of the resolving power,[5] the effects cannot and need not be separated in practice. The adopted procedure was to decide from experiment the effective limits of the resolving power (compounded of spectrographic, analyzer, and atmospheric effects) and to undertake no work that demanded greater resolution.

Carroll[6] has discussed the formation of an absorption line, initially black, by a diffraction grating and shown that owing to the effects of the diffraction pattern it cannot appear more than 96 per cent black at the center. The effect is calculated for a narrow, totally black line and would of course be much smaller for lines such as are studied here, 30 or 40 Angstroms wide at the wings. Though the effect is probably just sensible, even for such lines, we can be sure, from the real differences, of quite another order, that are seen on comparing measured central intensities,[7] that the neglected diffraction effect is not

[4] Payne and Hogg, H. C. 302, 1927.
[5] Because of the change of dispersion with wave length in prismatic spectra.
[6] M. N. R. A. S., **88**, 154, 1928.
[7] See Section 82, p. 274.

leading us astray by simulating incomplete blackness at the center of lines that are really black. Observation points more and more conclusively to light at the centers of all lines. Carroll's effect will help to evaluate its amount but will not dispose of it altogether.

Stellar Disturbances: Rotating Stars.—The Doppler effect imposed on the spectrum by rotation of a star will of course widen the absorption lines in a definite manner. A very complete discussion of the effect of rotation on the stellar spectrum has been made by Shajn and Struve[8] and by Carroll;[9] the former give a long bibliography, and therefore I shall not make complete references to the subject.

Shajn and Struve discuss the effects of rotation on the contour of a line of arbitrarily chosen shape (percentage light loss 50 at the center) and evaluate the resulting contours for various initial widths, in one case also allowing for darkening at the limb. The results can scarcely be regarded as more than empirical, since probably true line contours do not follow the formula assumed by Shajn and Struve:

$$I = I_0 e^{-k^2(\lambda-\lambda_0)^2}$$

One of the most promising stars for the measurement of line contour is V Puppis, in whose spectrum Miss Maury[10] noted the shallowed lines as the effects of rapid rotation. Measures of contour made from plates of V Puppis confirm the type of contour illustrated by Shajn and Struve; they are reproduced in Figure III, 1, in a form suitable for comparison with the later Figures III, 3 and III, 4. The shallowness and flatness are most striking: in a normal star of similar spectral class the lines of hydrogen are between two and three times as deep. This same shallowness makes accurate measures of the contours very difficult, so that the result cannot be considered as more than an empirical confirmation of Shajn and Struve's empirical prediction. There is the further important point,

[8] M. N. R. A. S., **89**, 222, 1929.
[9] M. N. R. A. S., **88**, 548, 1928.
[10] H. A., **84**, 157, 1920.

which will emerge on comparison of Figure III, 1 with Figures III, 3 and III, 4, that a logarithmic intensity plot should provide a means of detecting Doppler disturbances affecting the contours of lines.

The effect of stellar pulsation falls also in the present category. That the lines would be somewhat broadened, and shifted, at maximum and minimum light, was shown by

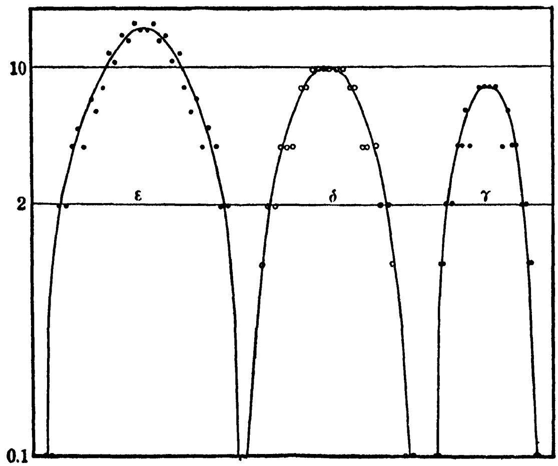

FIGURE III, 1.
Contours of lines in the spectrum of V Puppis. Ordinates are percentage light losses, plotted logarithmically; abscissae are wave lengths, on arbitrary but comparable scales. Note that the scale of ordinates differs in this diagram from those of Figure III, 3 and Figure III, 4.

Shapley and Nicholson,[11] who, however, pointed out that "the change observed in line width [for Cepheid variables] . . . appears to have twice the period and about three times the amplitude of the variation which would be produced in a pulsating star by the distribution of velocity over the stellar disk . . . " The observations on the changing line contours for Cepheid variables, given in Chapter XIV, confirm this view and show that though the effects of pulsation on the contour may be present, there is also a far larger effect that has not yet

[11] Mt. W. Comm. 63, 1919.

been interpreted, which obscures any other. The immense shallowing and broadening shown by the H and K lines at minimum is one of the most important features of the spectral variation.

12. Observed Intensity Distribution.—We conclude that for lines that are wide and strong (hydrogen and ionized calcium in particular) the effects of finite resolution are not serious (with the Harvard one-prism dispersion); and the shallowing effect of the diffraction pattern is approximately negligible. Furthermore, except for some special stars that can probably be spectroscopically selected by inspection, or put aside because of known binary character and short period, rotation effects are not important. We shall assume that effects of "pulsation" are not associated with stars not otherwise known to be variable. The discussion is therefore made, without further qualification, of the contours of the hydrogen and calcium lines in normal stars. Unless abnormalities appear in the course of the measures, we shall not consider them to be present.

The measures of contour (in the form of widths of lines at various percentages of the continuous background) are given at intervals throughout the book and summarized in Chapters XV and XVI. They will not be repeated here; but the meaning of contours and the formulae used in describing them are summarized in the coming sections with the aid of tables and diagrams.

The method of measuring contours was alluded to at the end of the preceding chapter, where the procedure of drawing the tangent background across the line, and smoothing the wings, was mentioned. After the contour has been obtained it is necessary to determine the analytical formula that best describes it. Without this, the only assertion that can safely be made is that most contours are symmetrical about the fundamental wave length—the helium lines in low luminosity B stars are probably an exception.[12]

[12] Struve, Ap. J., 70, 85, 1929.

13. Theories of Line Contour.—The pioneer investigations of Sir Arthur Schuster[13] remained, as did the early work on line photometry, long without a successor. Not until 1924 was a workable advance made, when the paper of Stewart[14] was aimed at astrophysical application, and at the same time Slater and Harrison[15] measured and discussed laboratory absorption contours. Unsöld[16] followed up these researches with a contour formula of the same form as Stewart's, applied it directly to the stellar conditions, and analyzed with its aid his measures of contours in the solar spectrum. Unsöld's formula is primarily designed for detailed or topographic use on the sun and must be simplified for the stellar applications, according to the darkening at the limb of the star concerned. In my applications, when using the Unsöld formula, I have used the same law of darkening for all spectral classes: our knowledge of darkening is very inadequate, being restricted to eclipsing binaries, which do not altogether support the contention of Stebbins[17] that darkening increases from a small value at Class B to a large one at Class M.

Unsöld's formula, simplified for stellar use, is as follows:

$$\frac{b}{B} = \frac{1}{1 + \sigma H} = 1 \Big/ \left(1 + \frac{2\pi e^4 \lambda_0^2 f}{3mc^4(\lambda - \lambda_0)^2} \cdot NH\right)$$

where:

e = electronic charge.
m = electronic mass.
c = velocity of light.
λ_0 = wave length of resonance line.
λ = wave length considered.
N = number of atoms per cubic centimeter.
f = oscillatory strength (= 2/3 and 1/3, respectively, for H and K).

If this formula is computed for the K line of calcium, we obtain the following working table which relates half breadths ($\lambda - \lambda_0$) to logarithms of numbers of atoms per square centimeter surface.

[13] Ap. J., **21**, 1, 1905.
[14] Ap. J., **59**, 30, 1924.
[15] Phys. Rev., **26**, 176, 1925.
[16] Zs. f. Phys., **46**, 765, 1928.
[17] Ap., J., **54**, 91, 1921.

TABLE III, I.—WORKING TABLE OF UNSÖLD CONTOURS

Half Breadth in Angstroms at Which Percentage Light Losses Are Computed

Log *NH*	1	2	5	10	15	20	25	30	35	40
17.0	85	96	99	100	100	100	100	100	100	100
17.1	82	95	99	100	100	100	100	100	100	100
17.2	79	94	99	100	100	100	100	100	100	100
17.3	74	92	99	100	100	100	100	100	100	100
17.4	70	90	98	100	100	100	100	100	100	100
17.5	65	88	98	99	100	100	100	100	100	100
17.6	59	85	97	99	100	100	100	100	100	100
17.7	54	82	97	99	100	100	100	100	100	100
17.8	47	79	96	99	99	100	100	100	100	100
17.9	42	74	95	97	99	100	100	100	100	100
18.0	37	70	94	98	99	100	100	100	100	100
18.1	32	65	92	98	99	99	100	100	100	100
18.2	27	59	90	97	99	99	99	100	100	100
18.3	22	54	88	97	98	99	99	100	100	100
18.4	19	48	85	96	98	99	99	99	100	100
18.5	16	42	82	94	97	98	99	99	100	100
18.6	13	37	78	94	97	98	99	99	99	100
18.7	10	32	77	93	96	98	99	99	99	99
18.8	8	27	70	90	95	97	98	99	99	99
18.9	7	23	65	87	94	97	98	99	99	99
19.0	6	19	60	85	93	96	97	98	99	99
19.1	4	16	53	82	91	95	97	98	98	99
19.2	4	13	48	79	89	93	96	97	98	98
19.3	3	10	42	75	87	92	95	96	97	98
19.4	3	8	37	70	84	90	93	95	97	97
19.5	2	7	32	66	81	88	92	94	96	97
19.6	1	6	27	60	77	85	91	93	94	96
19.7	1	4	22	54	72	82	88	91	93	95
19.8	0.9	4	19	48	68	79	85	89	92	93
19.9	0.7	3	16	42	62	75	82	87	90	92
20.0	0.6	2	13	37	56	70	78	84	88	90

In Figure III, 2 observations of a number of stars are compared with the theoretical contour contained in the above table. For the K line in all spectral classes, the fit is not too good, especially at the line center. The hydrogen lines deviate more from the theoretical curves than the calcium lines do, both at the center and in the wings: and they differ more widely from

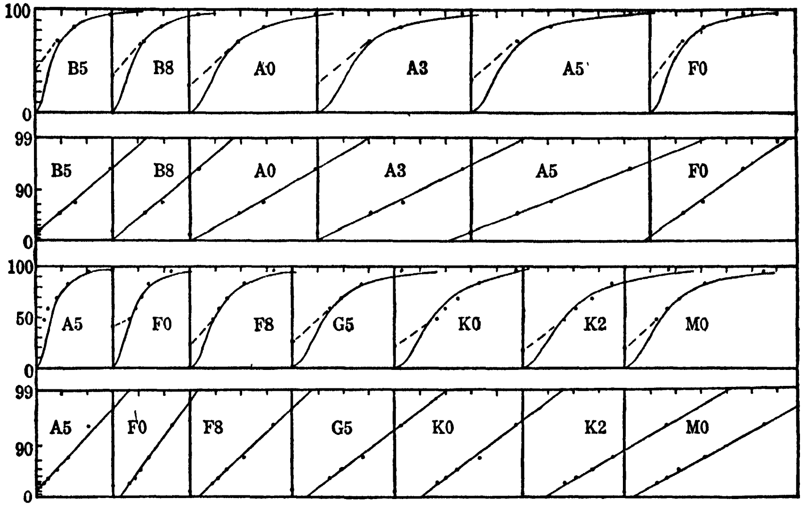

FIGURE III, 2.

Observed contours compared with the Unsold formula and the exponential formula. The two upper strips represent the mean reflected contours of $H\gamma$ in various spectral classes, full lines representing the Unsold contour (above) and the exponential formula (below). Broken lines represent the observed central depth, which (for instrumental reasons) is certainly too shallow. Ordinates are percentage light losses; abscissae are Angstrom units. Short horizontal lines are drawn on the left, at intervals of 10 per cent in percentage light loss, and short vertical lines are drawn at intervals of 5 Angstroms in wave length. The two lower strips represent similar data for the K line in exactly the same form.

class to class. Both of these remarks are exactly illustrated by Unsöld's early attempt to fit contours to the lines in the solar spectrum.

The deviations are always in the direction that the wings are too deep and the center of the line shallower than the formulae require. All theoretical formulae for stellar line contour hitherto published, indeed, call for lines black, or very nearly black, at the center. The interpretation of centrally

intense lines has to be made especially, with additional assumptions.[18] We leave the matter of central intensities to a later section.

Though the fit of the lines to the contour tabulated in Table III, I is only fairly good, we consider that probably the widening of lines is primarily an effect of numbers of atoms and use the computed contour to derive numbers of atoms from our measures. We measure the half breadth at a number of specified percentage light losses *dl* (actually values of *dl* equal to 4, 17, 31, 51, 60, 68) and then determine graphically the number of effective atoms given by the formula for these half breadths at their respective percentage light losses. As the fit is not perfect, there will be a systematic difference between the numbers of atoms given by different levels of the line. In the work now presented the mean log NH for all the measured half breadths is adopted and tabulated. The procedure is rough, semiempirical indeed; but neither data nor theory justify greater refinement.

The Unsöld formula is not necessarily the one most suited to practical application; Shajn and Struve[19] suggest one of the form

$$I = I_0 e^{-K^2(\lambda-\lambda_0)^2}$$

$$\text{or } \log I = \text{const.} \times (\lambda - \lambda_0)^2$$

If this contour were plotted with log I as one coordinate, and distance from the center of the line as the other, a curve would result, but clearly from Figure III, 3 and III, 4 the observed relation is linear, so that the formula is ruled out by observation—at least for the lines now discussed.

Milne[20] has discussed theoretically the contours of stellar absorption lines and shown (using the same scattering coefficient as Unsöld) that the form given by Unsöld differs slightly from the rigorous value; but the deviation is smaller than can be distinguished by observation.

[18] Unsöld, Festschrift für Sommerfeld, 72, 1928.
[19] M. N. R. A. S., 89, 222, 1929.
[20] M. N. R. A. S., 89, 3, 1928.

14. The Empirical Evaluation of Line Intensities.—There is another method of expressing the contours of absorption lines that has great practical advantages. As pointed out by the writer,[21] absorption line contours are pretty well represented by an exponential formula, first used in this connection by Merton and Nicholson.[22] To test the representation of the facts by such a formula, Elvey[23] plotted (reduced) intensities

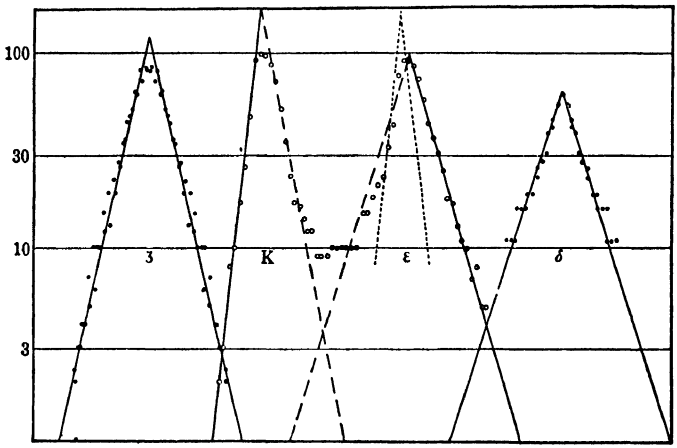

FIGURE III, 3.

Contours of four lines in the spectrum of Canopus, as an illustration of the exponential formula. Abscissae are wave lengths (on arbitrary but comparable scales); ordinates are percentage light losses, plotted logarithmically. The points for Hδ and Hζ are reflected. Broken lines for K and Hε represent reflected contours; dotted lines at Hε represent the position of H (Ca+) and its contour, deduced from K.

along a line contour on semilog paper against distance from the line center in Angstroms; if the line contour follows an exponential law the points will then lie on a straight line. His data showed that the relation was indeed linear.

The exponential formula can be made to fit the observations so well because it has essentially two disposable parameters—the slope of the line and its depth. In Elvey's treatment the

[21] H. Repr. 46, 1928.

[22] Phil. Trans., **216A**, 459, 1916.

[23] Ap. J., **68**, 145, 1928.

absorptions at all points were expressed in terms of the observed central absorption I_0; the relation is of course linear if we plot instead the uncorrected depth. Such a plot eliminates a serious source of ambiguity in Elvey's method—the use of the measured line depth, a quantity affected, probably for all available spectrograms, by finite resolving power. The data for two stars, one of Class B9, the other of Class F0, are shown, plotted in this manner, in Figures III, 3 and III, 4. The fit is excellent, as was shown also by Elvey's observations.

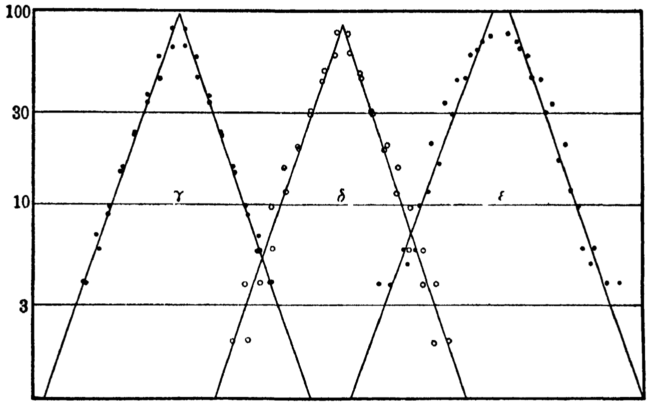

FIGURE III, 4.

Contours of hydrogen lines of υ Pavonis (Class B9). Ordinates are percentage light losses, plotted logarithmically; abscissae are wave lengths, on arbitrary but comparable scales. The observed points are reflected for all three lines.

The two straight lines drawn through the points observed meet the zero line, of course, only at infinity; the place where they pass the point where the percentage light loss is 1 per cent is about the observed edge of the line. They intersect at a point which might be regarded as the true effective depth at the center; but sometimes it is above 100 per cent, so the interpretation is less simple. We cannot at present assume that the exponential contour represents the normal contour of the line (though the correspondence with observation looks promising), thus the difference between the observed and "limiting" central intensities is best regarded, at present, in an empirical light.

Figure III, 5 compares the contour required by the Unsöld theory with the one given by the exponential formula, as explained in the legend. The curves cover all the observed cases of measurable contour; contours wider than the one corresponding to log NH = 20 are apparently never observed. It is evident that if observations fit the Unsöld curve closely they will not fit the exponential formula; and the data of Figures III, 3 and III, 4 seem to pronounce in favor of the latter. The fit of the Unsöld formula is best for the H and K lines.

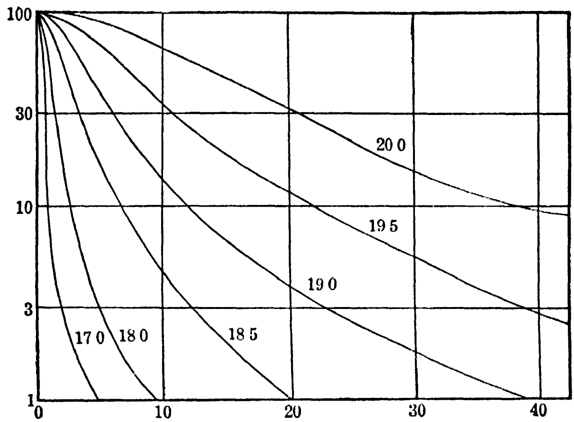

FIGURE III, 5.

Logarithmic plot of curves corresponding to the Unsold contour. Ordinates are percentage light losses (plotted logarithmically); abscissae are Angstrom units. Evidently a precise linear fit of points thus plotted is incompatible with an Unsold contour.

The total absorptions of the means for the spectral classes, derived as described by Miss Williams[24] from the data of Table XV, I and II, are given in Table III, II. The numerical value of the total absorption is first given, followed by the logarithm of the total absorption to two places, enclosed, according to the usual custom, in brackets. The total absorptions are compared, in Figure III, 6 with the mean numbers of atoms, per square centimeter surface, derived from the Unsöld formula for the same spectral classes. We treat NH as representing a

[24] H. C. 348, 1929.

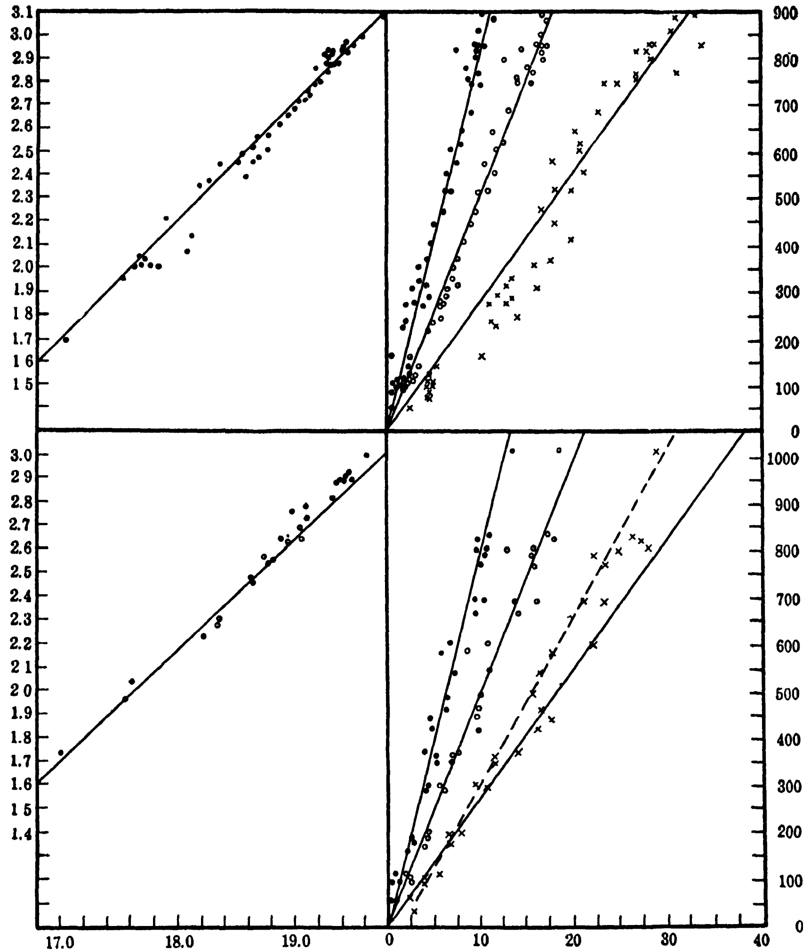

FIGURE III, 6.

Contour and total absorption for hydrogen (above) and calcium lines. Left side: relation of total absorption (ordinate, plotted logarithmically) to log NH (abscissa). The straight line represents the expected relation log total abs. = 2 log NH − const. The value of the constant, deduced from the present data, is 32.02. Right side: relation of total absorption (ordinate) to line half width, at r = 0.96 (crosses), 0.83 (circles), and 0.69 (dots). It is possible that the writer has been too generous in drawing the extreme wings of moderately broad hydrogen lines and too sparing in drawing them for very wide lines; the close linear relation shown by other percentage light losses is probably typical for the extreme wing also. The full lines connecting the points for hydrogen and calcium are the same; the broken line for calcium represents the observations better than the line corresponding to the observations for hydrogen.

TABLE III, II.—TOTAL ABSORPTIONS

Class	Hγ	Hδ	Hε + H	K	4227
O8.5	90 [1.95]	103 [2.01]	99 [2.00]	... ...	... .
B2	240 [2.38]	161 [2.21]	302 [2.48]	... .	... .
B3	285 [2.46]	233 [2.36]	275 [2.44]	... .	... .
B5	403 [2.61]	365 [2.56]	442 [2.64]	... ...	... .
B8	509 [2.71]	468 [2.67]	... .	... ...	... .
B9	608 [2.79]	596 [2.78]	745 [2.87]	... .	... .
A0	821 [2.91]	739 [2.87]	676 [2.83]	54 [1.73]	... .
A2	889 [2.95]	762 [2.88]	749 [2.87]	... .	... .
A3	967 [2.99]	818 [2.91]	... .	171 [2.23]	... .
A5	825 [2.91]	769 [2.89]	898 [2.95]	289 [2 46]	... .
A7	... .	... .	... .	298 [2 47]	... .
A8	... .	... .	... .	348 [2.54]	... .
F0	547 [2.74]	511 [2.71]	570 [2.76]	358 [2.55]	26 [1.42]
F2	354 [2.55]	320 [2.51]	808 [2 91]	580 [2.76]	... .
F5	221 [2.34]	268 [2.43]	852 [2 92]	830 [2.92]	92 [1.96]
F8	311 [2.49]	273 [2.44]	644 [2.84]	536 [2.73]	107 [2.03]
G5	109 [2.04]	100 [2.00]	... .	690 [2.84]	100 [2.00]
K0	102 [2.00]	... .	795 [2 90]	798 [2.90]	190 [2.28]
K2	... .	... .	813 [2 91]	1011 [3.00]	201 [2.30]
K5	120 [2.08]	135 [2.13]	... .	661 [2.82]	368 [2 57]
dK5	49 [1.69]	107 [2.03]	... .	... .	... .
M0	... .	... .	... .	798 [2 90]	490 [2 69]
M1	... .	... .	... .	821 [2 91]	422 [2 63]
M2	... .	... .	... .	787 [2 90]	444 [2.65]
M3	... .	... .	... .	603 [2 78]	460 [2.66]
M5	... .	... .	... .	770 [2.89]	... .

number of atoms, but always bear in mind that it is only an empirical quantity; if number of atoms *is* proportional to a function of *NH* the constant of proportionality differs for every atom and for every line of any one atom. But in spite of the empiricism of the result, or perhaps because of it, the data derived from contour fitting and line integration can be widely applied.[25]

[25] The most complete discussion of the relation of total absorption to line width is that made by Miss Anger (H. C. 252, 1930, completed after this chapter was written). Her results, derived before mine, are based on more, better, and more homogeneous material, and supersede the present ones quantitatively. In the main our conclusions are the same.

15. Photometry of Faint Lines.—It has been mentioned that most lines in stellar spectra are beyond the resolving power used in the present investigation. Their contours are entirely

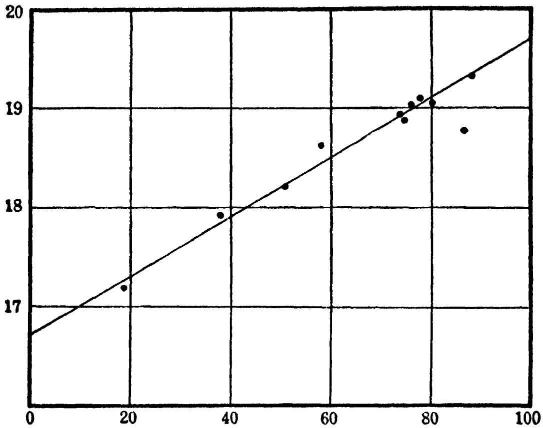

FIGURE III, 7.
Relation of log *NH* (ordinate) to measured percentage light loss for one-prism dispersion.

instrumental, and therefore all similar. Thus a measure of any single parameter of the "contour" should give the total absorption of the line. The quantity that suggests itself at

TABLE III, III.—RELATION OF LOG *NH*, PERCENTAGE LIGHT LOSS, AND TOTAL ABSORPTION

Class	Log *NH*	Mean *dl*	Log Total Absorption
A0	17 20	19	1.73
A2	17 91	38	. . .
A3	18 22	51	2 23
F0	18 61	58	2 55
F2	18 78	87	2 76
F8	19 04	76	2.73
G0	18 89	75	. .
K0	19.10	78	2 90
K2	19.32	88	3 00
K5	18 94	74	2 82
M	19 05	80	2.90

once for measurement is the depth of a line. In order to calibrate the line depths for the dispersion used, the H and K lines were analyzed by contour from two-prism plates and measured by depths from plates of smaller dispersion. The results of the measures are summarized in Table III, III, which

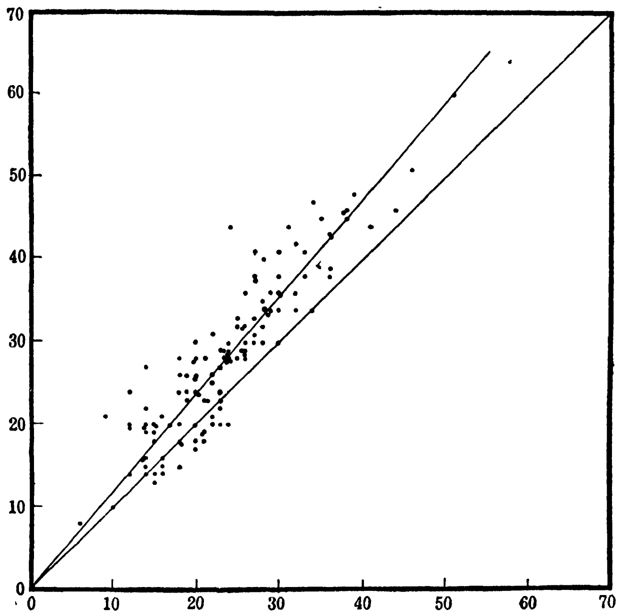

FIGURE III, 8.

Reduction of line depth from two-prism to one-prism dispersion. Ordinates are percentage light losses measured from a two-prism spectrogram; abscissae are percentage light losses from a one-prism spectrogram for the same lines. The diagonal line would represent equality; the mean line represents the adopted relation. The data of this diagram are from the spectrum of γ Cygni; similar comparisons for other stars lead to the same relation.

gives for each spectral class the mean log *NH* for ionized calcium, the mean *dl* (percentage light loss at center), and finally the mean total absorption as described in Section 14.

This table supplies us with an empirical curve connecting percentage light loss with log *NH*. Extensive use will be made in later chapters of this curve, reproduced in Figure III, 7.

If we could relate measured line depth to resolving power we could use Figure III, 7 for various spectral dispersions. With the aid of such a curve, line depths can be converted to the one-prism scale, and numbers of atoms deduced.

16. Outline of Procedure.—The various methods mentioned in the present chapter may be summarized in the following form:

Measured	Method of Analysis	Derived Quantity	Section
Contour	Unsöld formula	Log NH	13
Contour	Shajn and Struve*	Number of atoms	..
Contour	Exponential formula	Total absorption	14
Depth (faint lines)	Empirical curve	Log NH	15

* Not adopted.

Further, the deviations from the exponential curve may be used to detect rotation, using the curves given for V Puppis to get an idea of the form that rotation will impart to a line.

No mention of the Stark effect has appeared in the present chapter, which has treated the contours quite empirically. The matter is discussed in Chapter IX. The question of central intensities also seems to fall more naturally in a later discussion (Chapter XV), where the contrast in surface conditions with different values of surface gravity is considered.

II

THE MATERIAL

CHAPTER IV

THE STARS OF HIGH LUMINOSITY

SEVERAL hundred thousand stars can confidently be regarded as members of the high luminosity group.[1] They are collected from three sources: individual stars of known brightness; classes of stars known to be very luminous; and stellar systems of known distance.

17. Individual Stars of Known Brightness.—Comparatively few high luminosity stars are individually known. The eclipsing and spectroscopic binaries in Table IV, I have been selected on the assumption that the mass-luminosity law can be applied, the mass $8.5 \times \odot$ corresponding[2] to absolute *bolometric* magnitude -2. The members of galactic clusters are placed here rather than in the later section because their brightness depends on a spectral parallax, both the membership of the star in the cluster and the relation to other stars being a matter of individual study. The c-stars are discussed in more detail in the later chapters of the book; to anticipate conclusions we may mention that stars with the spectroscopic c-character are almost all very luminous but that they are not necessarily representative of the very luminous stars— very bright stars may also have "normal" spectra.

The spectra of all the stars in Table IV, I are known, and it is of the utmost importance to make them the central point of the study of high luminosity. The stars of this section and the one that follows are the only ones for which we have indi-

[1] The survey of the available material on stars of high luminosity is arbitrarily limited in the present study at absolute visual magnitude -2, irrespective of color. It might be better to draw the line at higher luminosities for red stars than for blue ones, but the colors of most of the stars enumerated in the present chapter are unknown.

[2] Eddington, The Internal Constitution of the Stars, 153, 1926.

TABLE IV, I.—INDIVIDUAL BRIGHT STARS OF KNOWN LUMINOSITY

Criterion	Number	Source
Double stars, known M........	15	Beer, Veröff. Berlin-Babelsberg, **5**, No. 6, 1927 Struve and Pogo, Ap. J., **68**, 335, 1928
Double stars, unknown M, but very luminous.......	6	*Ibid.*
Individual parallaxes... ...	27	Russell, Dugan, and Stewart, Astronomy, **2**, 724, 1927
Members of galactic clusters....	55	See Chapter V, p. 53
c-stars	300:	See Chapter V, p. 55

vidual knowledge of spectrum and brightness; our other bases of selection are numerous and appear secure, but they are indirect.

18. Classes of Stars Known to Be Very Luminous.—Without exception the N stars are of high luminosity,[3] and the O stars (excepting the nuclei of planetary nebulae) also fall in the list.[4] Most variable stars are very luminous, the long-period variables,[5] the Cepheid variables with periods greater than ten days,[6] and the novae[7] have been definitely shown to have absolute visual magnitudes brighter than -2. The case for the irregular variables of Class M is less certain, but possibly they, too, fall in the class of high luminosity stars as here defined; Hubble finds irregular M stars absolutely as bright as -4 in Messier 31. Certainly the brightest known M stars, Betelgeuse and Antares, belong to the highly luminous class, but spectroscopically they are not typical, and in luminosity also they are probably exceptional.

About 90 per cent of the high luminosity stars thus defined are red variables, probably an indication of really large numbers.

[3] Chapter XIII, p. 185.

[4] Chapter VI, p. 63.

[5] Shapley, H. B. 804, 1925; Shapley, H. Repr. 53, 1928; Gerasimovič, H. Repr. 54, 1928; cf. R. E. Wilson, A. J., **35**, 125, 1925.

[6] Chapter XIV, p. 198.

[7] Stratton, Handbuch der Astrophysik, **6**, 254, 1928.

Long period variable stars have smaller absolute photographic magnitudes than novae and long period Cepheids, and thus a complete survey down to a given apparent magnitude embraces a far larger volume of space for the two latter.[8] On the other hand, because of their greater range long period variables have a greater discovery chance than Cepheids. But that the scarcity of long period Cepheids is probably not a matter of discovery chance is illustrated by Miss Swope's findings in

TABLE IV, II.—CLASSES OF VERY LUMINOUS GALACTIC STARS

Class	Number of Stars	Reference
Long period variables	1,927	Harvard Catalogue, 1928; H. B. notes from 1928 to 1929
Long period Cepheids	67	See Appendix B
Novae	70	Stratton, Handbuch der Astrophysik, **6**, 254, 1928
Class N	146	Henry Draper Catalogue
Class O	166	Chapter VI
Total	2,376	

MWF 185—the cluster type Cepheids, which are as unfavorably placed, as regards discovery chance, as the Cepheids of longer period, are about as freely found as the long period variables.[9]

Even the long period Cepheids may be regarded as red variables, for their spectra are preponderantly of Class K,[10] so

[8] If we regard as typical for galactic novae Hubble's mean absolute photographic magnitude −5.7 at maximum for 86 novae in the Andromeda nebula (Pop. Astr., **37**, 85, 1929) and adopt for long period variables a value of −0.5, the novae down to a given apparent magnitude are visible throughout a volume of space a thousand times greater than for the long period variables. A survey of all novae as bright as the fourteenth photographic magnitude at maximum should include the whole nova population to a distance of about a hundred kiloparsecs (of the order of size of the galactic system). A complete survey of galactic O stars ($M = -4$) would go down to the sixteenth photographic magnitude; of the N stars, down to magnitude 17.5; and of the long period variables and cluster type stars, down to the twentieth magnitude.

[9] Miss Swope, H. B. 857, 862, 1928; 863, 867, 1929.

[10] Shapley and Miss Walton, H. C. 313, 1927; Shapley, H. B. 861, 1928.

that all the stars in Table IV, II, with the exception of the inconstant nova and the exceptional O,[11] are of this type. Any attempt to evaluate the importance and trace the history of the high luminosity stars must therefore be made with serious reference to the long period variable and other red variables.

19. Members of Systems of Known Distance. *a. Globular Clusters.*—The mean absolute photographic magnitude of the 25 brightest stars in a globular cluster[12] is -1.5 ± 0.28, and the dispersion about this quantity is about a magnitude.[13] But all the brightest stars in a globular cluster are red, with color indices of a magnitude or more, so that their absolute *visual* magnitudes place a considerable number of the members of each globular cluster within the limits which I have adopted for high luminosity stars. In Messier 22, for instance, there are 46 stars brighter than -2.0 visually.[14]

Combined data for the three clusters M 22, M 3, and M 13 are given in Table IV, III, which is taken directly from Shapley's data.[15] The tabulated magnitudes are photovisual.

TABLE IV, III.—LUMINOSITY CURVE FOR THREE GLOBULAR CLUSTERS

Magnitude Interval	k5	k5-ko	ko-g5	g5-go	go-f5	f5-fo	All
−4.0 to −3.5	1	1	1	0	0	0	3
−3.5 to −3.0	11	0	1	0	0	1	13
−3.0 to −2.5	18	13	5	0	2	0	38
−2.5 to −2.0	5	11	16	5	1	1	39
−2.0 to −1.5	0	6	32	34	7	1	80
−1.5 to −1.0	2	4	32	28	24	8	98
−1.0 to −0.5	0	1	20	71	85	18	195
−0.5 to 0.0	0	1	7	73	136	103	320
0.0 to +0.5	0	1	5	44	132	91	273
+0.5 to +1.0	0	0	1	8	31	39	79
+1.0 to +1.5	0	0	0	1	31	85	117

[11] Cf. Chapter VI, p. 63.

[12] Shapley, M. W. Contr. 151, 1918.

[13] Shapley, H. Mon. No. 2, 1930.

[14] Shapley, H. B. 874, 1930.

[15] H. Mon. No. 2, 1930.

Out of a possible 200,000 stars down to absolute photovisual magnitude +4, about 40 therefore are definitely supergiants. Incidentally the relatively few long period Cepheid variables in the globular clusters are always among the brightest cluster stars; the cluster type variables are of course below our adopted limit for high luminosity, with absolute magnitude about zero.

b. The Magellanic Clouds.—The two Magellanic Clouds are unusually rich in high luminosity stars. The general features of the photographic luminosity curves of the Small Magellanic Cloud, and of N. G. C. 6822, a similar system, are given in Tables IV, IV and IV, V, together with the data for other systems that show similarities and contrasts. The preponderance of very bright stars in the Cloud and of stars of intermediate brightness in the three clusters of Table IV, III is of great interest.

c. Spiral Nebulae.—The spiral nebulae that have been resolved display a large number of very luminous stars, as shown in Table IV, IV. The data[16] for Messier 33 and for N. G. C. 6822[17] have been adjusted so that they represent roughly the total number of stars in the system on the basis of Hubble's statement that the counts covered respectively one fifth and one half of the total numbers of stars. The Andromeda nebula[18] also contains very large numbers of luminous stars, but fewer in proportion to population, as it is considerably larger and somewhat nearer than Messier 33, though less resolved (the ratio in the radii given by Hubble is 6,400/2,300, rather less than three to one).

To discuss Tables IV, IV and IV, V is outside the scope of this monograph. I note that dots (. . .) indicate the absence of a record as a result of inaccessibility of the object, not of its nonexistence. Zeros, on the other hand, indicate that the object is certainly not present or is present in negligibly small numbers; they probably are the most significant entries in the table. The rôle of very luminous stars is evidently bound up

[16] Hubble, Mt. W. Contr. 304, 1925.
[17] Hubble, Mt. W. Contr. 310, 1926.
[18] Hubble, Mt. W. Contr. 376, 1929.

TABLE IV, IV.—HIGHLY LUMINOUS STARS IN STELLAR SYSTEMS (PHOTOGRAPHIC DATA)

System	Stars Brighter than		Cepheid Variables	Period $<1^d$	Period $>10^d$	Novae	Note
	−3.5	−2					
47 Tucanae	0	4	4:	4:	[3]*	0	1
Messier 68	0	3	27	27	0	0	2
Messier 3	0	0	150:	150:	0	0	3
Small Magellanic Cloud	2,350	..	170 (2,000:)	14	23	0	4
N. G. C. 6822	136	..	11	..	11	0	5
Messier 33 ..	1,070	..	35	...	35	2	6
Messier 31	.....	..	40	...	40	85	7
(Galactic system)	.....	3×10^5:	288	116	69	70	8

* Long period.

REFERENCES

[1] Shapley, H. B. 783, 1923.
[2] Shapley, Mt. W. Contr. 175, 1919.
[3] Shapley, P. A. S. P., 29, 247, 1917.
[4] Shapley, H. C. 260, 1924; H. B. 765, 1922; H. C. 280, 1925.
[5] Hubble, Mt. W. Contr. 304, 1925.
[6] Hubble, Mt. W. Contr. 310, 1926.
[7] Hubble, Mt. W. Contr. 376, 1929.
[8] Variables are taken from the collected material of Appendix B. Novae are from the compilation by Stratton, Handbuch der Astrophysik, 6, 254, 1928.

TABLE IV, V.—APPROXIMATE PHOTOGRAPHIC LUMINOSITY CURVES

System	Number of Stars Brighter than Magnitude							
	−6.5	−6.0	−5.5	−5.0	−4.5	−4.0	−3.5	−3.0
Messier 3... .	0	0	0	0	0	0	0	0
Small Cloud*......	340	480	600	740	900	1,100	2,350	3,500
Messier 33... .. .	15	30	105	290	630	2,055	4,350	5,350
N. G. C. 6822....	0	0	6	18	42	78	136	330

* Unpublished data recently obtained at Harvard by Dr. Shapley show that the Large Magellanic Cloud contains over 5,300 stars brighter than absolute photographic magnitude −3.

with the occurrence of long period Cepheids but not necessarily with novae or with cluster type variables, as they are neither universally coexistent nor mutually exclusive.

There is some apparent similarity between the galactic system and the Andromeda Nebula,[19] where the novae are

[19] Hubble, Mt. W. Contr. 376, 1929.

commoner at the center, and the (long period) Cepheids at the edges. Lack of data prevents the establishment of a positive analogy, but in our own system there are two very significant absences: there are few long period Cepheids toward the galactic center, though the search there has been thorough, and there are comparatively few novae in other directions, though it would be expected that they would have been found if they occurred, for instance, in the Cygnus regions.[20] Irregular and dissimilar distribution of distinctive objects receives, however, a ready explanation in terms of the recent "supergalaxy" hypothesis of Shapley.[21]

20. Summary.—For the study of highly luminous stars we can locate about 3,000 galactic objects visually brighter than -2.0, but spectra are attainable for only about half of them, and for most of those can be studied only with very small dispersion. In distant systems, on the other hand, more than 24,000 such stars can be observed, but the spectra of only a few exceptional objects are attainable.

For purposes of physical study the individual contributions of the different spectral classes to the general luminosity curve are the obviously important feature, and the next chapter attempts to indicate the distribution of the high-luminosity stars in spectral class. Table II, III contains data for globular clusters—based on colors and not necessarily representative of the stars in general. Evidently even an approximate idea of the spectral distribution is possible only for the galactic system, since the spectra of the large preponderance of very luminous stars are inaccessible, and colors, as will be shown in Chapter VIII, are not always a reliable substitute. The vital question as to whether the distribution is similar in the Galaxy and distant systems, or even in different parts of the galactic system, can at present be discussed but not answered.[22]

[20] Cf. Baade, A. N., **232**, 65, 1928.

[21] H. C. 350, 1930.

[22] Cf. the discussion by Krieger, L. O. B. 416, 1929.

CHAPTER V

THE SPECTRA OF THE HIGH LUMINOSITY STARS

ACCORDING to the tabulations of the previous chapter, only a small fraction of the known stars of high luminosity are spectroscopically accessible. Before making a spectroscopic study it is well to examine how far the accessible stars are typical of the group. The spectroscopic data for the three groups of high luminosity stars enumerated in the last chapter are summarized; they are then examined to decide how far they are representative, and accordingly how far they can be used as a key to the problem of high luminosity.

21. Spectroscopic Binaries.—The observations of the spectra of the 22 stars mentioned in the last chapter are summarized in Table V, I. They are taken from Beer's compilation[1] and from the papers of Struve and Pogo.[2]

Almost all the stars in Table V, I are of early type. Only three are c-stars, and the letter n (nebulous) is assigned to one or both components for nine of the others; the lines of 27 Canis Majoris also are hazy with emission edges.[3] Making allowance for the effect of rotation with the revolution period to account for the haziness of the lines of the three n-lined stars with periods less than three days (cf. Shajn and Struve, M. N. R. A. S., **89**, 235, 1929, where this period is adopted to separate "n" and "s" stars), there are still six stars, certainly of high luminosity, whose spectra do not show the c-character. The case of β Lyrae is especially noteworthy; the B8 star—much the fainter of the pair—is a sharp-line star, while the B2 star is not. For

[1] Berlin Veröff., **5**, No. 6, 1927.
[2] Ap. J., **68**, 335, 1928.
[3] Struve, Ap. J., **68**, 112, 1928.

TABLE V, I.—SPECTROSCOPIC DATA FOR MASSIVE STARS

Star	$m_1 \sin^3 i$	$m_2 \sin^3 i$	Spectra		Period d
27 CMa	....	....	B5p		4,000:
H. D. 47129	75.6	63.3	O8nke	O8nke	14.41
ι Ori	21.8	13.3	B1k	B1k	29.14
30 CMa	15.6	11.0	O		154.80
Boss 46	17.6	16.4	O8.5nk	O8.5nk	3.523
V Pup	33.0		B1n	B3n	1.454
Y Cyg	16 4	15 2	B2nk	B2nk	2.996
Boss 6142	18.5	12.7	B0nk	B0nk	13.44
H. D. 19820	18.9	9.2	O8k	O8k	3.369
H. D. 191201	13.8	12.9	O9.5nk	O9.5nk	8.334
H. D. 216014	14.2	12 4	B0nk	B0nk	2.288
ζ_2 CrB	13 3	13.1	B8n	B8n	12.58
β Lyr	16.6	6 8	B2ep	cB8	12.92
β Sco	13.0	8 3	B1nk	B1nk	6.828
ζ Cen	16.4		B2	B3	8.024
μ_1 Sco	16.5		B3	B3	1.446
29 CMa	..	...	O7e		4.393
υ Gem	....	...	B5k		3,506:
ε Aur	..	....	cF5		29 years
μ Sgr	...	...	cB8		....
(υ Sgr)	>20		(cA5)		....

the early B stars, then, sharp lines do not always accompany high luminosity. It is unlikely that they all have high rotation speeds, and probably the haziness of their lines has a cause in the conditions of the atmosphere, thus being real rather than optical.

The lines of V Puppis, discussed in Chapter III,[4] are an excellent example of rotational widening. But those of 27 Canis Majoris are not certainly widened in that way, and it is well to remember that a B star with a hazy spectrum may be exceptionally bright, which is not necessarily true of later types.

Measures of the total intensity of the lines of the luminous B stars in Table V, I have been made for only a few such as V Puppis, and the lines are definitely found to be weaker than those of normal stars. When the spectrum of Y Cygni is

[4] Chapter III, p. 27.

examined with small dispersion, the hydrogen lines appear narrow,[5] though J. S. Plaskett reports them wide and hazy with larger resolving power. The weakness of the hydrogen and helium absorption in the spectra of luminous B stars is discussed in Chapters VI and VIII.

22. Individual Parallaxes.—Fifteen of the stars from Table V, I are of earlier class than B8; their spectra do not display any special peculiarities, and further reference to them is deferred to Chapters VI and X. Typical high luminosity stars from the other spectral classes are contained in Table V, II.

TABLE V, II.—TYPICAL SUPERGIANTS

Star	Class	Absolute Magnitude	Note
β Ori	cB8	−5.8:	1, 2
α Cyg	cA2	−5.2:	1
α Car	F0	−7 4	1, 3
ρ Pup	cF5	−2.1	4
γ Cyg	cF8	−3 0	4
β Dra	cG0	−3.5	4
α Ori	cM0	−2 9	1
α Sco	cM0	−4 0	1

REMARKS

1 Parallax from Russell, Dugan and Stewart, Astronomy, 2, 637.
2 Companion, Class B8, six magnitudes fainter.
3 The lines in the spectrum are well defined but not sharp.
4 Spectroscopic parallax, Mount Wilson.

The stars in this list are apparently, as well as absolutely, bright, and upon them our detailed knowledge of the supergiant star is apt to be centered. The spectroscopic remark on Canopus is to be noted; this star is the only one in the list that does not show the c-character. Before determining to what extent these spectra are typical, the data for the other luminous galactic stars must be summarized.

23. Members of Galactic Clusters.—Several galactic clusters for which the spectra are available, the distances have

[5] Harvard plates examined by the writer.

been derived, and high luminosity stars have been recognized, are tabulated below. The sources for the adopted parallax are given in the remarks.

TABLE V, III.—SPECTROSCOPIC MATERIAL FROM GALACTIC CLUSTERS

Cluster	Modulus	High Luminosity Stars	Remarks
Pleiades*..	5.77	2	$\pi = 0''.007$
Messier 35...	9.5	0	Wallenquist[6]
Messier 11.. ...	10.5	4	Trumpler[7]
	(12.0)		Lindblad[8]
h and χ Persei	10.0	33	Balanowsky[9]
N. G. C. 6231....	9.0	16	Shapley and Miss Sawyer[10]
η Carinae*....	.....	?	. ...

* Associated with nebulosity.

These clusters are all of the "Pleiades" type; in the "Hyades" type only isolated stars fall within the limits here adopted.

The material represented by Table V, III is without doubt the most important that we have for determining the spectroscopic features that accompany high luminosity. Even though the actual distances are very uncertain (depending, except for

TABLE V, IV.—QUALITY OF SPECTRA IN GALACTIC CLUSTERS

Class	Pleiades	Messier 35	Messier 11	Perseus Clusters	N. G. C. 6231	Carina
B5	Hazy; emission	....	....	c; low color	Normal	(c and normal)
B8, 9	Hazy	(Normal)	.. ....	c; low color	Normal	(c and normal)
A0,5[11]	Hazy	(Normal)	cA0,cA5 (low color)	c; low color	Normal	(c and normal)

[6] Bosscha Annals, **3**, 19B, 1929.
[7] L. O. B., **12**, 10, 1924.
[8] Mt. W. Contr. 228, 1925.
[9] Poulkova Bul. 92, 1924.
[10] H. B. 846, 1927.
[11] Lindblad, Mt. W. Contr. 228, 1924; the stars referred to are Stratonoff 437 545.

the Pleiades, on spectral parallax), the relative brightnesses of stars of different spectral classes are most instructive.

For stars of Class B 5, then, having about the same brightness, we may compare Alcyone, with hazy lines and hydrogen emission, and sharp-line stars, also showing emission spectra, in the Perseus clusters. Luminous stars have not necessarily sharp-lined spectra. Leaving out the Carina group, to be mentioned later, it seems that spectra with the c-character are associated with low color temperature; though both the Pleiades[12] and N. G. C. 6231[13] have rather low temperatures in comparison to the normal stars of the corresponding B classes,[14] they do not compare with h and χ Persei and Messier 11 in color excess.[15] There still seems to be a residual color excess for the brighter stars of Messier 11 even if Küstner's photographic magnitudes are adopted. The presence or absence of sharp lines is apparently unconnected with nebulosity, for the Pleiades are nebulous, and Messier 35 and the Perseus cluster are free. The question is more fully discussed in Chapter X.

The stars in the Carina region include one of the three conspicuous groups of c-stars,[16] but evidently the depth through which the apparent clustering extends is so great that relative brightness of different spectral classes has little meaning, and the assignment of an individual star to a particular group, none at all. Most of the c-stars near the group are of about the ninth magnitude and are probably further away than the group of O-stars that appears to surround η Carinae. At present we cannot decide with which part of the group (the O stars, or the c-stars, or the brighter B stars) the nebulosity is connected, nor where η Carinae itself is placed with respect to the other stars. Hence this group of c-stars, though undoubtedly real,

[12] Hogg, H. C. 309, 1927; Greaves, Davidson, and Martin, M. N. R. A. S., **87**, 360, 1927; Gerasimovič, H. C. 339, 1929.

[13] Payne, H. B. 848, 1927.

[14] See Chapter VI.

[15] Balanowsky, Poulkova Bul. 92, 1924; Trumpler, L. O. B., **12**, 10, 1924.

[16] Schilt, B. A. N., **2**, 47, 1922.

cannot be discussed from the point of view of association with nebulosity; unfortunately the colors in this region are also unknown.

From the available data on the spectra of galactic clusters we conclude that sharp lines are always an index of high luminosity for the spectral classes encountered (B5 to A2) but that the converse is not necessarily true. A group of high-luminosity stars made up on the criterion of sharp lines will therefore be homogeneous but not representative. The sharp lines tend to be associated with low temperature, an observation discussed more fully in Chapters VI and X.

24. The c-stars.—Miss Maury's group[17] of stars showing the c-character was at once recognized by Hertzsprung[18] as a collection of highly luminous objects. A list of c-stars has been made up from the Remarks to the Henry Draper Catalogue, the two Mount Wilson lists of "stars with Cepheid characteristics,[19] the additional narrow-line stars recorded in the Perseus clusters by Trumpler,[20] Hertzsprung's lists of abnormally red A and B stars,[21] and a few additional spectra examined at Harvard by the writer.[22] About 500 stars are included in the list, not including Class N, Class O, or the long period variables.

Practically all stars in this list (which is given in detail in the Appendix) are of high luminosity.[23] The collection is of course not homogeneous; abnormally bright B and O stars are as yet spectroscopically indistinguishable from normal stars of the same classes; the distinction becomes clear only at Class B5. At the other end of the scale, the very bright K and M stars have not especially narrow lines and are included only

[17] H. A., **28**, 1897.

[18] A. N., **179**, 374, 1908; **192**, 262, 1912.

[19] Mt. W. Contr. 199, 1919; P. A. S. P., **31**, 184, 1919; **37**, 160, 1925.

[20] P. A. S. P., **38**, 350, 1926.

[21] B. A. N., 35; 37, 1923.

[22] Chiefly those of Cepheid variables; see Chapter XIV.

[23] Cf. Harvard Monograph No. 1, 176, 1925, for a star wrongly assigned on the basis of narrow lines. Probably the sun falls in the category of narrow-lined stars of low luminosity.

if attention is paid to the greater strength of hydrogen in their spectra.

The magnitude limit of completeness for the c-stars is brighter than for normal stars, because, with small dispersion, narrow lines become imperceptible more readily than wide ones. Furthermore, K and M stars, when very faint, may be classified by noticing the distribution of energy in their spectra, when lines are practically invisible, so that the earmarks of high luminosity, even if present, might not be noted. The commonness of very luminous K stars in the Magellanic Cloud[24] suggests that we underestimate their numbers near the Sun or that the spectroscopic composition of the two districts may differ.

TABLE V, V.—NARROW-LINE STARS BRIGHTER THAN MAGNITUDE 8.25

Class	c-stars	All Stars	Percentage c-stars	
B0–B5	53	2,061	2 56	
B8	17	1,604	1.06	0.61
B9	9	2,752	0 34	
A0	23	6,320	0.36	0.58
A2–A3	39	5,208	0.75	
A5	4	1,352	0 31	
F0	19	3,208	0.59	
F2	10	1,976	0 53	
F5	19	3,504	0.54	
F8	17	2,488	0.68	
G0	17	2,784	0 61	
G5	8	5,248	0.15	
K0–K5	11	18,340	0.06	
M	12	1,200	1.00	

The c-stars form about the same fraction of each spectral class from B8 to G0. Earlier than B8 other factors besides luminosity affect the line sharpness. Beyond G0 there is so large an admixture of dwarfs that the ratio would not be expected to remain steady; also the line criteria for the c-character are no longer effective. But for the intermediate spectral classes we may guess that the fraction of high luminosity stars

[24] Shapley and Miss Walton, H. C. 288, 192.

is about the same for all.[25] The counts both of c-stars and normal stars are made to, and are supposed to be complete to, apparent magnitude 8.25. If, then, the average c-star is regarded as three magnitudes brighter than the average giant, about one tenth of one per cent of the stars brighter than 0^M are c-stars; if the difference is two magnitudes, about one half of one per cent are c-stars. The corresponding number derived from the luminosity curve is very much larger,[26] an indication that the c-stars do not represent all the high luminosity stars. Although the Kapteyn luminosity law appears to lead us to expect too many supergiants,[27] the excess of prediction over observation is not so great as our count of c-stars would require.

In the matter of the distribution and motions of the c-stars, little need be added to Schilt's discussion,[28] which was based on similar, though slightly less, material. The difference in mean parallax between the two groups B0 — A3 and A5 — G5 being negligible, the mean absolute magnitude -3.2 was derived for all by Schilt, so that the estimate of frequency, one-tenth of one per cent, made in the preceding paragraph, would seem to be near to the truth.

Schilt gives diagrams of the distribution of the c-stars in the galactic plane (to which they are closely confined); his conclusion that the early c-stars, and to a great extent also the late ones, practically cease at a distance of about a thousand parsecs is in my opinion probably the result of the impossibility of recognizing the c-character for faint stars. It is well known that very luminous stars extend to much greater distances than this, but only in special circumstances—such, for instance, as Cepheid variability—can the c-character be inferred for a very remote star. There can be no doubt that CG Sagittarii[29]

[25] It seems probable that the large percentage of cB8 and cA2 stars in the table, accompanied by a small percentage of cA0 and cB9, is the result of classifying by mentally comparing with α Cygni and β Orionis.

[26] Cf. Chapter IV, p. 48.

[27] Luyten, Ap. J., **62**, 8, 1925.

[28] B. A. N., **2**, 47, 1922.

[29] Gerasimovič, H. B., 846, 1927.

and H. V. 4585[30] would both have spectra showing the c-character, and there seems no reason to consider that other equally bright stars should be less common in other districts than in our own. It should be remembered that the distance within which we can recognize the spectra of c-stars is less than a tenth of the distance from here to the galactic center.

25. Spectra of Stars in Remote Systems.—Besides the spectra of a few exceptionally bright stars in globular clusters, the data are drawn entirely from the Magellanic Clouds. In view of their scarcity in our lists of nearer stars, the large number of K stars there is of great interest.

Lindblad[31] records the spectra of four bright K stars:

Cluster	Star	Spectrum	M	Remarks
M 13	Scheiner 47	Ko	−3.19	Cyanogen weak; supergiant
	Scheiner 127	Ko	−3.07	Cyanogen weak; supergiant
	Scheiner 63	Ko	−2.69	Cyanogen weak; supergiant
M 3	von Zeipel 752	late	−3.18	Pseudocepheid spectrum

Shapley and Wilson's list[32] of spectra and magnitudes in the Large Magellanic Cloud is condensed in the next table.

TABLE V, VI.—SPECTRA IN THE LARGE MAGELLANIC CLOUD

Object	Number	Mean Pg. M
P Cygni stars	9	−5.8
O stars	32	−4.8
Variable stars (K and M)	11	−3.3
S Doradus	..	−8.7
Planetary nebulae	8	−6 5
Diffuse nebulae	13	−6.4 to −13.8

From their counts of stars of different spectral classes near the Large Cloud, Shapley and Miss Walton[33] concluded that there is an excess of Class A and Class K over the normal back-

[30] Miss Swope, H. B., 866, 1929.
[31] Mt. W. Contr. 228, 1929.
[32] H. C. 271, 1924.
[33] H. C. 288, 1925.

ground stars in about equal numbers at M = −5. This observation is the most positive one available concerning the absolute and relative numbers of supergiants of a given spectral class, and in that it differs so much from our own neighborhood, we may conclude that the distribution of supergiants within the various spectral classes differs in the two places.

That the composition of the Magellanic Clouds differs from that inferred by us from our (seriously selected) study of the galactic regions is suggested, also, by the richness of the Clouds in variable stars, their poverty in novae, their large numbers of Wolf-Rayet stars, their apparent poverty in bright blue stars, and their rich population of supergiants—400 stars brighter than −4.5. The data of Table IV, IV emphasize the diversity of other systems also. The larger aspects of high luminosity obviously cannot be adequately treated with reference only to the galactic specimens.

26. The Luminosity Curves for the Spectral Classes.—The general luminosity curve, regarded as superposed curves for each spectral class separately, is naturally affected by a change in the relative contributions of the various spectral classes. This of course becomes most evident when a small number of stars in an isolated system are considered, humps on the smooth curve arising from a preponderance of one spectral class. Thus Trumpler[34] points out the effect of the F stars on the general curve for Messier 11, and Shapley's "critical luminosity"[35] is connected with a large number of stars of a particular color and brightness.

It is worth enquiring whether there is evidence for such a "heaping up" of high luminosity stars of any particular color; evidences of local excesses among the A stars have been given by Malmquist,[36] Wallenquist[37] for Messier 36, Schalén for

[34] L. O. B., 12, 10, 1924.
[35] Mt. W. Contr. 155, 1919.
[36] Lund Medd., Series 2, 46, 1927.
[37] Ups. Medd. 36, 1927.

galactic stars,[38] and Wallenquist[39] for Messier 35, but they correspond to absolute magnitudes no brighter than zero. Malmquist[40] considers that "the [galactic] Cepheids and pseudocepheids seem to form a special group of supergiants," but that appears to result from the fact that the determinations of their parallaxes discussed by him were spectroscopic, and if a star had been assigned to the supergiant reduction curve, its absolute magnitude was automatically assumed to be very bright. The luminosity curves given by van Rhijn for the various spectral classes[41] were derived without reference to local irregularities, and their form tells nothing of the distribution of luminosities among the c-stars of a given class. Considering the available number of stars the question cannot be settled statistically. Physically it will be shown in later chapters that the relationship between supergiants and normal stars is close and continuous, and there is no evidence of a heaping up in any class of high luminosity stars.

[38] Ups. Medd. 37, 1928.
[39] Bosscha Ann., 3, 19B, 1929.
[40] Lund Medd. 32, 1924.
[41] Groningen Pub. 38, p. 71, 1925.

III

RESULTS OF OBSERVATION

CHAPTER VI

THE STARS OF CLASS O

THE O stars are in many ways unique. Although they constitute one of the smallest spectral classes, they embrace a greater diversity than any other. In spectrum, in brightness, and in effective temperature they have as large a dispersion as any class, and they display many features, such as wide, structureless emission, that are not found elsewhere.

27. Census of the O Stars.—All the stars now regarded as of Class O, 238 in number, are tabulated in the present chapter.[1] Of these, 166 are galactic O stars, 39 are the nuclei of planetary nebulae, and 33 are in the Magellanic Clouds. The O stars are roughly divided into two groups—Wolf-Rayet stars and absorption O stars. Extreme members of the groups differ widely, but the line between them is hard to define; intermediate stars may be alternatively regarded as Wolf-Rayet stars with absorption tendencies or as absorption O stars with Wolf-Rayet tendencies.

Tables VI, I, VI, II, and VI, III contain the H. D. numbers, the spectral classifications, and the apparent photographic magnitudes of the three collections—galactic O stars, nuclei of planetary nebulae, and Magellanic O stars. The data on the nuclei of planetary nebulae, and their spectral classifications, were very kindly given to me, from an unpublished investigation, by Prof. H. H. Plaskett, who classified the spectra partly from Victoria spectrograms and partly from published Lick material.

[1] Many additions to Class O will doubtless be made when faint B stars are examined with greater detail. See J. S. Plaskett, Pub. Dom. Ap. Obs., **2**, 316, 1924, and the report on a brief search for O stars by the writer, in Harvard Bulletin 846.

Table VI, I.—Classification of Galactic O Stars

H. D.	Pg. Mag.	Spectrum	Remarks
108	7.12	O6ew	1
4004	10.2	Wb*	
5005	7.5	O6	2
5980	.....	Wa	
6327	10.1	Wa	
9974	10.5	Wc	
14605	9.7	O	
14633	7.5	O8	3
14947	8.04	O5w WI	4
16523	9.98	Wa	
16691	8.4	O6w WI	5
17638	10.2	Wa	
24431	6.7	O9	6
24912	4.0	O7c	7
25638	7.04	O9	8
30614	4.14	O9	9
34078	5.57	O9	10
34656	6.7	O6(w) (WIII)	11
36861	3.7	O9	12
36862	5.6	O8	
37022	5.36	O7	13
37043	2.9	O8	13*a*
42088	7.4	O6	14
45166	9.6	O	
45314	7.1	O8e	15
46056	7.72	O8	16
46149	7.47	O8	17
46150	6.61	O6	18
R	6.80	O6	19
46223	6.95	O5	20
46966	7.1	O8	21
47129	5.82	O8e	22
47839	4.7	O7	23
48099	6.01	O6	24
49798	7.6	O5	
50891	9.2	O9	
50896	6.9	WIbn!	25
53179	8.9	O9e	26
53667	7.8	O8	
54662	6.2	O7s	27

* The letters a, b, c, d are inserted to indicate the previous Harvard classes Oa, Ob, Oc, and Od.

TABLE VI, I.—(*continued*)

H. D.	Pg. Mag.	Spectrum		Remarks
56925	12.1		Wb	27*a*
57060	4.9	O7e		28
57061	4.4	O9		29
60848	7.7	O8e		30
62150	8.2	O6		
62910	9.7		Wa	
63099	10.8		Wa	
63150	8.7	O9		
65865	10.1		Wc	
66811	2.27	O5	WIIIdsk	31
68273	2.22	O6w!	WIIIan!k	32
69106	6.9	O9		
73882	7.9	O9		
−45° 4482	10.		WI	33
76536	8.8		WIIIan!	
79573	11.0		Wa	
86161	8.3		W	
88500	10.1		Wa	
89358	11.1		Wa	
90657	9.8		Wa	
91824	8.2	O		
91969	6.9	O9	WI	34
92554	9.1	O		
92740	6.3		WIIIcsk	35
92809	9.1		WIIIan!	36
93128			WIII	37
93131	6.3		WIcs!k	38
93162	8.8		WIcn	39
93250	7.6	O6	WI?	40
93843	7.1	O		
94305	12.0		Wa	
94546	10.6		Wa	
94663	9.4	O		
95435	11.5		Wa	
96548	8.2		WIcn	41
97152	7.9		WIIIan!	41a
97253	7.3	O5w		42
97434	8.3	O5w		43
97950	9.2	Ow	WI	
104994	10.1		W?c	
105056	6.9	O9		44

TABLE VI, I.—(*continued*)

H. D.	Pg. Mag.	Spectrum		Remarks
112244	5.6	O8		45
113904	5.4	O9w	WIIIan!	46
115473	9.0		WIIIan!	47
117297	10.9		Wa	
117688	10.9		W?c	
117797	10.1		WI	
119078	9.4		WIIIan!	48
120521	8.5	O		
121194	11.0		Wa	
124314	7.6	O9?		49
134877	.....	....	....	50
135240	5.24	O7w		51
135591	5.5	O7		
136488	9.1		WIIIbn!	52
137603	10.4	Ow	WI	53
143414	9.2		WIbn!	
147419	10.5	O	Wb	
149038	5.6	O9e		54
150135	7.12	O6		
150136	5.59	O7?		
150958	7.8	O9w	WIIIs	55
151804	5.8		WIIIs!	56
151932	6.4		WIcsk	57
152147	7.2	O9w		58
152233	7.0			59
152270	7.2		WIIIan!k	60
152386	8.8	O6w	WIsk	61
152408	5.6	O8w	WIIIcsk	62
152424	6.9	O9		63
153919	6.7	O6w	WIIIdsk	64
156327	10.0		Wa	
156385	7.5		WIIIan!k	65
157451	10.2		Wa	
157504	(11.8)		Wa	
158860	11.1		Wb	
159176	6.1	O7		66
160529	7.6	Oe		67
163181	7.5	Oe		68
163454	8.3	Ow	WI	
163758	7.1	O5w	WIIIsk	
164270	8.6		WIIIbn!p	69

TABLE VI, I.—(*continued*)

H. D.	Pg. Mag.	Spectrum		Remarks
164492	6.8	O7		70
164794	6.2	O5		71
165052	7.0	O6		72
165688	9.7		WIbn!	
165763	7.7		WIIIan!	72a
166813	var.		WIc	73
167264	5.18	O9		74
167633	8.7	O		
167771	6.37	O8		75
168206	8.87		WIIIan!	
169010	10.77		Wa	
175876	7.1	O8e		76
177230	11.12		Wc	
184738	10.0		WIIIcs!p	77
186943	9.98		Wa	
187282	11.0		Wa	
188001	6.29	O7w	WIII	78
190002	11.09		Wa	
190429	6.69	O5w		79
+35° 3930S		O9		80
190864	8.0	O6w	WIIIs	81
190918	7.01	O9	WIn!k	82
191765	7.80		WIbn!	
191899	11.6		Wa	83
192103	7.94		WIIIan!	
192163	7.44		WIbn!	83*a*
192639	7.02	O7		84
192641	7.94		WIan!	
193077	8.0	O9w	WIcn	85
193576	8.04	O5w	WIbn	86
193793	6.83	O5	WIIan!k	87
193928	9.43		Wa	
195177	13.		Wa	
199579	6.01	O6		88
203064	5.06	O8		89
206267	5.64	O6		90
208220	9.0	O9		91
209975	5.2	O9		92
210839	5.19	O6w	WIIId	93
211564	11.07		Wc	
211853	9.0	O7w	Wb	
213049	11.0		Wa	
214419	8.92		Wb	
214680	4.91	O8		94
219460	9 2		Wa	

Remarks

References to the Henry Draper Catalogue are prefixed with H. D. C.; those to J. S. Plaskett, Pub. Dom. Ap. Obs., **2**, 287ff., 1924, are prefixed with J. S. P., followed by the page number. Other references are made in full.

[1] H. D. 108. H. D. C., Class B; spectrum nearly continuous, dark lines very faint. J. S. P., 302, notes hydrogen lines of P Cygni type, and narrow Wolf-Rayet emission; radial velocity constant.

[2] H. D. 5005. H. D. C., B2. J. S. P., 303, no emission. A nebulous star (Hubble, Mt. W. Contr. 241, 1922).

[3] H. D. 14633. H. D. C., Class B, another spectrum superposed. J. S. P., lines of fair quality; 4686 v. str., 4634, 4640 ft., Si IV rather pronounced; velocity variable; spectroscopic binary.

[4] H. D. 14947. J. S. P. reproduces spectrum; velocity variable; spectroscopic binary.

[5] H. D. 16691. J. S. P. reproduces spectrum; velocity variable; spectroscopic binary.

[6] H. D. 24431. J. S. P., 304. Temperature 7500°, Gerasimovič, H. C. 340.

[7] H. D. 24912, ξ Per. H. D. C., emission wing to 4649; spectroscopic binary with stationary H and K. Temperature 9000°, Hertzsprung, Leid. An., **14**, 1922; 8500°, Gerasimovič, H. C. 340. Possibly associated with N. G. C. 1499 (Hubble, Mt. W. Contr. 214, 1922).

[8] H. D. 25638. H. D. C., B0. J. S. P., between O9 and B0; lines poor; spectroscopic binary.

[9] H. D. 30614, 9 Cam. H. D. C., B0; H. A., **56**, 105, intermediate between Oe5 and B0. Spectroscopic binary. Temperature 9500°, Gerasimovič, H. C. 304.

[10] H. D. 34078. H. D. C., B0p; H. A., **56**, 105, intermediate between Oe5 and B0. J. S. P., 304, beautifully sharp lines; large difference in velocity between star and Ca+.

[11] H. D. 34656. J. S. P., 304, 4686 absent; 4634, 4640 hovering between absorption and emission; spectroscopic binary.

[12] H. D. 36861, **2**, λ Ori. J. S. P., 305, lines better than average O8 (ι Ori). Temperature 11000°, Hertzsprung, Leid. An., **14**, 1922; 12000°, Gerasimovič, H. C. 340.

[13] H. D. 37022, θ^2 Ori, the Trapezium. H. D. C., the spectra of the four stars are probably similar. J. S. P., 305, however, classified A, B, C, D, as B2, B2, O7, B1, the O star being the brightest of the four. These classifications are undoubtedly good. H and K weaker than usual for O stars; spectroscopic binary.

[13a] H. D. 37043, ι Ori. Spectroscopic binary.

[14] H. D. 42088, N. G. C. 2175. J. S. P., 305, lines of fair quality; spectroscopic binary. Nebulous star; color excess +1.0 (Seares and Hubble, Mt. W. Contr. 187, 1920).

[15] H. D. 45314. J. S. P., 306, spectrum practically continuous, except for doubly emissive H lines; velocity variable.

[16] H. D. 46056. H. D. C., B0. J. S. P., lines rather poor with indications of doubling; spectroscopic binary.

[17] H. D. 46149. H. D. C., B2. J. S. P., 307, lines fair.

[18] H. D. 46150. H. D. C., B2. J. S. P., 307, lines fair.

[19] R. 6^h $26^m.6$ + 5° 00′. J. S. P., 307, lines fair.

[20] H. D. 46223, H. D. C., B2. J. S. P., 307, lines fair.

[21] H. D. 46966. H. D. C., B2. J. S. P., 307, lines good.

[22] H. D. 47129. H. D. C., B0p; H slightly bright, lines narrow, 4200.3 strong. Spectroscopic binary; orbit discussed by J. S. Plaskett, Pub. Dom. Ap. Obs., **2**, 147, 1922.

[23] H. D. 47839, S Mon. H. D. C., the lines are wide. J. S. P., 308, lines rather weak and broad; spectroscopic binary. Temperature 11000°, Hertzsprung, Leiden An., **14**, 1922.

[24] H. D. 48099. H. D. C., B2. J. S. P , lines rather broad but strong; on some plates appears earlier than O7; spectroscopic binary.

[25] H. D. 50896. o_1 CMa. Spectrum described in H. B. 844.

[26] H. D. 53179. H. D. C., Bp; the K line is very strong for Class B.

[27] H. D. 54662. H. D. C., the lines are very narrow and sharply defined. J. S. P., 309, fair quality lines; constant velocity.

[27a] H. D. 56925. Associated with nebulosity (Hubble, Mt. W. Contr. 241, 1922).

[28] H. D. 57060, 29 CMa. Spectroscopic binary.

[29] H. D. 57061, τ (30) CMa. Spectroscopic binary.

REMARKS.—(*continued*)

[30] H. D. 60848. J. S. P., spectrum nearly continuous; spectroscopic binary.
[31] H. D. 66811, ζ Pup.
[32] H. D. 68273, γ Vel. Color index $-0^m.39$, King, H. A., **85**, 186, 1928.
[33] $-45°$ 4482. Miss Cannon, H. B. 789.
[34] H. D. 91969. H. D. C., Bo.
[35] H. D. 92740. Spectrum, H. B. 843; temperature 10500°, Gerasimovič, H. C. 340.
[36] H. D. 92809. Spectrum, H. B. 844.
[37] H. D. 93128. Spectrum, H. B. 846.
[38] H. D. 93131. Spectrum, H. B. 843; temperature 16000°, Gerasimovič, H. C. 340.
[39] H. D. 93162. Spectrum, H. B. 843.
[40] H. D. 93250. Previously unpublished O star.
[41] H. D. 96548. Spectrum, H. B. 843; temperature 10000°, Gerasimovič, H. C. 340.
[41a] H. D. 97152. Temperature 18000°, Gerasimovič, H. C. 340.
[42] H. D. 97253. Spectrum, H. B. 846. H. D. C., B8.
[43] H. D. 97434. Spectrum, H. B. 846. H. D. C., B2.
[44] H. D. 105056. Spectrum, H. B. 846. H. D. C., Bop; H. A., **56**, 153 notes peculiarities.
[45] H. D. 112244. H. D. C., faint 4686; strong 4514.
[46] H. D. 113904. θ Mus. Lines hazy. Possibly associated with nebulosity (Shapley, H. B. 843).
[47] H. D. 115473. Spectrum, H. B. 844.
[48] H. D. 119078. Spectrum, H. B. 844.
[49] H. D. 124314. H. D. C., spectrum of Class B or Oe5.
[50] H. D. 134877. H. D. C., not an O star, though it appears as such in several lists.
[51] H. D. 135240. δ Cir.
[52] H. D. 136488. Bands at 4686, 4633, sharp edge to violet.
[53] H. D. 137603. H. D. C., spectrum faint, 4686 bright.
[54] H. D. 149038. μ Nor. H. D. C , near Hδ the spectrum resembles Class Oe5.
[55] H. D. 150958. H. D. C., the spectrum is peculiar in that 4638 is stronger than 4686. Spectrum, H. B. 842. Temperature 8500°, Gerasimovič, H. C. 340.
[56] H. D. 151804. Spectrum, H. A., 28, 175, rem. 12; H. B. 842.
[57] H. D. 151932. Spectrum, H. B. 843.
[58] H. D. 152147. Spectrum, H. B. 846. H. D. C., Bo.
[59] H. D. 152233. Spectrum, H. D. 846. H. D. C., B.
[60] H. D. 152270. Spectrum, H. B. 844.
[61] H. D. 152386. Spectrum, H. B. 843. Temperature very low, Gerasimovič, H. C. 340.
[62] H. D. 152408. Spectrum, H. B. 842.
[63] H. D. 142424. Spectrum, H. B. 846. H. D. C., Bo.
[64] H. D. 153919. Spectrum, H. B. 842. H. D. C., spectrum more like ζ Pup than any yet studied. Temperature 7000°, Gerasimovič, H. C. 340.
[65] H. D. 156385. Spectrum, H. B. 844. Temperature high, Gerasimovič, H. C. 340.
[66] H. D. 159176. H. D. C., the lines are very wide.
[67] H. D. 160529. H. D. C., the K line is strong; bright lines suspected.
[68] H. D. 163181. H. D. C., the lines are so indistinct that the spectrum appears at first nearly continuous.
[69] H. D. 164270. Spectrum, H. B. 843.
[70] H. D. 164492. In the region of the trifid nebula. Somewhat hazy. Spectroscopic binary (J. S. P.; Hubble, Mt. W. Contr. 214, 1922).
[71] H. D. 164794. H. D. C., no lines distinctly seen except H and He +. J. S. P., 309, lines rather diffuse. Spectroscopic binary. The two brightest stars in Messier 8.
[72] H. D. 165052. Spectroscopic binary.
[72a] H. D. 165763. Temperature 11500°, Gerasimovič, H. C. 340. Spectrum, H. B. 844.
[73] H. D. 166813. Y Coronae Austrinae. Irregular variable, 12.0–12.9.
[74] H. D. 167264. H. D. C., Bo; the lines are somewhat narrow. Spectrum, H. B. 846.
[75] H. D. 167771. J. S. P., no trace of 4686. Spectroscopic binary.
[76] H. D. 175876. Double-lined spectroscopic binary.
[77] H. D. 184738. The hydrogen-envelope star.

REMARKS.—(*continued*)

[78] H. D. 188001. J. S. P., 310, N++ in emission, 4686 barely seen.

[79] H. D. 190429. J. S. P., 310, +35°3930N, Wolf-Rayet emission; variable velocity. Spectroscopic binary. Temperature 7000°, Gerasimovič, H. C. 340.

[80] +35°3930S, spectrum with diffuse lines, spectroscopic binary.

[81] H. D. 190864. J. S. P., 311, fair quality lines; 4634, 4640 in faint emission; 4686, when present, absorption. Spectroscopic binary.

[82] H. D. 190918. H. D. C., lines very indistinct. J. S. P., Class B1.

[83] H. D. 191899. Suspected by me to be a B star with bright Hβ.

[83a] H. D. 192163, N. G. C. 6888. Hubble, Mt. W. Contr. 214, 1922.

[84] H. D. 192639. J. S. P., lines of fair quality.

[85] H. D. 193077. Reproduced by J. S. P.

[86] H. D. 193576. Reproduced by J. S. P.

[87] H. D. 193793. Reproduced by J. S. P. Spectroscopic binary.

[88] H. D. 199579. J. S. P., 312, good spectrum but lines diffuse; spectroscopic binary.

[89] H. D. 203064. A Cyg. J. S. P., broad diffuse lines. Spectroscopic binary. Temperature 8500°, Gerasimovič, H. C. 340.

[90] H. D. 206267. J. S. P., diffuse lines; spectroscopic binary. Temperature 6500°, Gerasimovič, H. C. 340.

[91] H. D. 208220. J. S. P. classifies as B0.

[92] H. D. 209975. J. S. P., lines not so sharp and numerous as in 10 Lac; spectroscopic binary. Temperature 7500°, Gerasimovič, H. C. 340.

[93] H. D. 210839, λ Cep. J. S. P., absorption diffuse and faint; emission weak. Temperature 7000°, Gerasimovič, H. C. 340.

[94] H. D. 214680, 10 Lac. J. S. P., numerous sharp lines; constant velocity. Temperature 11000°, Gerasimovič, H. C. 340; 13000°, Hertzsprung, Leid. An., 14, 1922.

The absorption O stars in the tables have been classified according to the scheme devised by H. H. Plaskett[2] in the form adopted by the International Astronomical Union;[3] the small w indicates Wolf-Rayet lines in addition to the continuous spectrum. The Wolf-Rayet stars are classified according to a provisional system suggested to me by H. H. Plaskett, which forms a new Class W, WI having the line at 4686 stronger than the combined lines near 4340, and WIII having this line the weaker. Roman numerals are used in recognition of the tentative nature of the classification; its physical meaning will be discussed later.[4] Stars for which both absorption and Wolf-Rayet spectra are conspicuous are classified in both systems.

The absorption and Wolf-Rayet stars, clearly very intimately connected, will be discussed side by side throughout the chapter.

[2] Pub. Dom. Ap. Obs., 1, 336, 1922.

[3] Pub. I. A. U., 3, 167, 1928.

[4] A recent discussion by Beals of the classification of the Wolf-Rayet stars, still unpublished, finds valuable criteria in the yellow and red portions of the spectrum. The significance of bands in the yellow suggested the survey work carried out at Harvard in 1925 and briefly reported in Harvard Bulletin 836.

TABLE VI, II.—NUCLEI OF PLANETARY NEBULAE

Nebula	Magnitude		Spectrum
	Nebula	Star	
N. G. C. 40	10	12	WIII
N. G. C. 246	8	9	O6
I. C. 1747	13	15	W
I. C. 351	12	15	W
I. C. 2003	10	.	Cont.?
N. G. C. 1514	10	9	O8
N. G. C. 1535	9	12	Cont., H. Obs.
I. C. 418	9	11	O7w
N. G. C. 2022	12	14	Cont.
I. C. 2149	10	13	O7
N. G. C. 2392	11	11	O8w
N. G. C. 3242	8	11	Cont.
N. G. C. 4361	10	12	Cont.
I. C. 3568	9	12	Cont.
N. G. C. 6058	13	13	Cont.
I. C. 4593	11	12	Cont., O7
N. G. C. 6210	8	12	W
I. C. 4634	9	..	Cont.
N. G. C. 6543	9	11	WI
N. G. C. 6572	8	11	WI
N. G. C. 6629	12	14	Cont.
I. C. 4776	11	..	WIII
N. G. C. 6720	9	14	Cont.
N. G. C. 6751	13	13	WIII
N. G. C. 6790	9	..	Cont.?
N. G. C. 6803	10	14	Cont.?
+30° 3639	..	.	WIII
N. G. C. 6826	9	11	O6w
N. G. C. 6833	10	.	Cont.?
N. G. C. 6879	14	.	Cont.
N. G. C. 6891	12	12	Cont.
I. C. 4997	12	..	Cont.?
N. G. C. 6905	11	14	WIII
N. G. C. 7009	7	12	Cont.
N. G. C. 7026	11	15	WIII
I. C. 5217	10	..	WIII
I. C. 1470	..	12	O
N. G. C. 7635	..	8	O7
N. G. C. 7662	8	13	Cont.

TABLE VI, III.—THE MAGELLANIC O STARS

H. D.	Pg. Mag.	H. D.	Pg. Mag.	H. D.	Pg. Mag.
5980	...	34632	12.2	38029	11.3
32109	12 9	34783	13.8	38030	12.6
32125	13.2	35517	11.5	*24*	14.4
32228	9.9	36063	12.3	*25*	11.4
32257	13.7	36156	12.2	*26*	13.1
32402	12 3	36402	10.8	38282	10.6
6	13.2	36521	11.4	*28*	11.8
33133	11.5	37026	12.9	38344	12.6
8	[15	37248	12 2	38448	13.1
34169	13.2	37680	12.6	38472	13.4
34187	14.0	*21*	13.6	*32*	12.1

Italicized numbers refer to Miss Cannon's list in Harvard Bulletin 801, from which the magnitudes for all the stars are also taken.

Table VI, IV shows the number of absorption and Wolf-Rayet stars (galactic) in different magnitude intervals; the tabulation will be of interest in several connections later.

TABLE VI, IV.—FREQUENCY OF APPARENT MAGNITUDE FOR O STARS

Apparent Magnitude	Wolf-Rayet	Absorption	Total
]2.7	1	1	2
2.8– 4.2	0	4	4
4.3– 5.7	1	18	19
5.8– 7.2	6	33	39
7.3– 8.7	14	25	39
8.8–10.2	28	9	37
10.3–11.7	20	1	21
11.8–13.2	5	0	5
Total	75	91	166

In Table VI, IV the stars have been assigned to *one* of the two groups on the basis of the preponderant feature of the spectrum; thus all stars originally called Od and Oe in the Harvard system are classed as absorption O stars. For this reason the table, and accordingly the conclusions to be drawn from it, differ from those of Wilson and Luyten,[5] who used a

[5] H. Repr. 18, 1925.

rather smaller list of stars than is given in Table VI, I (140 in all) and counted stars of Classes Od and Oe with the emission stars, thus placing them with the Wolf-Rayet stars.

28. Distribution and Luminosity of the O Stars.—The familiar distribution of the O stars, with their extreme galactic concentration, and their tendency to form groups, is illustrated in Figure VI, 1. The chief galactic groups are those in Orion, Monoceros, Carina, Cygnus, and Scorpio. The groups in Orion and in Carina are associated with nebulosity (cf. page 88), but those in Monoceros and Scorpio seem to be clear.

The luminosities of the absorption O stars, the Wolf-Rayet stars, and the nuclei of planetary nebulae must be considered separately. J. S. Plaskett[6] has shown that the galactic absorption O stars have an absolute visual magnitude of −4, a value incidentally corroborated by Gerasimovič.[7] Struve[8] finds −3.1, −0.3, −0.4 and −6.0 for Oa, Ob, Oc and Od, respectively, from his analysis of stationary calcium lines. The absolute magnitudes of stars with broad strong emission must be taken

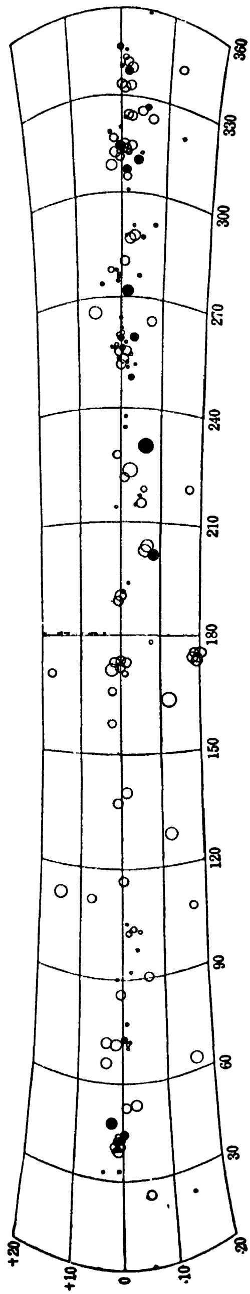

FIGURE VI, 1.

Galactic distribution of O stars (excluding the planetary nebulae and the Magellanic specimens). Circles represent absorption O stars; dots, Wolf-Rayet stars. The sizes of the circles indicate approximately the brightness of the stars.

[6] Pub. Dom. Ap. Obs., **2**, 329, 1924.
[7] A. N., **226**, 327, 1926.
[8] Ap. J., **67**, 388, 1928.

for what they are worth; their meaning is less obvious than for normal stars.

The luminosities of the Wolf-Rayet stars may at present be deduced only from their membership in stellar systems of known distance or from considering their distribution and dip. Data on radial velocity are inaccessible for most of them, because of spectral peculiarities. The mean of 31 stars in the Large Magellanic Cloud[9] gives, with Shapley's revised distance of 26 kiloparsecs[10] a value of -4.7. An estimate of $-2^m.9$ may be similarly derived, from Shapley and Miss Sawyer's provisional parallax,[11] for the mean of the four Wolf-Rayet stars in the cluster N. G. C. 6231.

The Wolf-Rayet stars are probably complete down to magnitude 10.5, as accepted by Wilson and Luyten,[12] but the assumption that the same is true of the absorption O stars has less justification. A faint absorption O is almost certain to be classified as a B star on a short-dispersion spectrogram, and probably the group is not complete even to the sixth magnitude; I have added 10 bright stars to the list,[13] and J. S. Plaskett has increased their number by 13.[14]

The maximum in frequency for the Wolf-Rayet stars at median apparent magnitude 9.4 (total of 80 stars) is probably real. If the corresponding maximum at 6.4 for the absorption O stars were also real, we should infer that they were at least three magnitudes brighter absolutely than the Wolf-Rayet stars, but there is no doubt that selection operates to produce a spurious maximum for the absorption stars, and these data can furnish us with no information as to the relative luminosities of the two groups.

The obvious grouping of the O stars prompts us to consider separately the various groups in comparing the brightness of

[9] Miss Cannon, H. B. 801, 1924.
[10] Pop. Astr., in press.
[11] H. B. 846, 1927.
[12] H. Repr. 18, 1925.
[13] H. B. 846, 1927.
[14] Pub. Dom. Ap. Obs., 2, 316, 1924.

the two classes of stars, instead of taking a general mean over the whole sky, although, as will appear shortly, a general mean would give approximately the same result.

The known groups of O stars that can be considered sufficiently marked to be regarded as definite systems are those in Cygnus, Carina, Scorpio (N. G. C. 6231), Orion, and Monoceros. There are also a number of O stars in the Scorpio-Ophiuchus region, toward the galactic center, and these may be combined with the Scorpio group in N. G. C. 6231, or considered separately, or omitted as not properly belonging to a group. All these proceedings lead to approximately the same result, and the combined results for the two groups in Scorpio will be used in the tabulation that is appended. The groups in Monoceros and Orion contain no Wolf-Rayet stars.

As the Wolf-Rayet stars are probably complete down to the tenth magnitude—perhaps to the eleventh—and the absorption O stars are probably very incomplete beyond the ninth magnitude, in comparing the mean magnitudes of the two classes in a given region it is evidently not fair to take the mean for all the Wolf-Rayet stars. Table VI, V, which contains means for all O stars brighter than the *ninth* magnitude and, also, for comparison, the means for all known O stars, is self-explanatory.

The difference in intrinsic brightness between the absorption O and Wolf-Rayet stars is evidently small, but whether the mean value $+0^m.7$ is accurate, or even necessarily indicates that the Wolf-Rayet stars are the fainter, cannot be said until the completeness of the Henry Draper Catalogue for absorption O stars is better known.[15]

The apparent magnitudes have a mean range of 5.1, most of which must be attributed to real differences of brightness, unless the groups are far more extended in the line of sight than across it, which is improbable. The apparent magnitudes in the Large Magellanic Cloud have a range of $4^m.7$, and probably some fainter O stars there are missed, which would make the

[15] The two brightest O stars in the sky, ζ Puppis (absorption) and γ Velorum (Wolf-Rayet), are close together and about equally bright.

TABLE VI, V.—MEAN APPARENT VISUAL MAGNITUDES OF WOLF-RAYET STARS AND ABSORPTION O STARS

Group	Brighter than $9^m.0$				All Stars				Range in Brightness		Difference W. R. − Abs.	
	W. R.	No.	Abs.	No.	W. R.	No.	Abs.	No.	W. R.	Abs.	$9^m.0$	All
Scorpio I	7.4	4	6.8	11	9.3	7	6.8	11	5.3	2.9	+0.5	+2.4
Scorpio II*	8.8	4	6.8	11	9.7	7	6.8	11	2.3	3.3	+2.0	+2.9
All Sco	8.1	8	6.8	22	9.5	14	6.8	22	4.6	3.5	+1.3	+2.7
Cygnus	7.7	7	6.6	6	8.9	12	6.6	6	5.6	3.1	+1.1	+2.3
Carina	8.0	8	8.2	10	9.3	14	8.2	10	5.2	2.3	−0.2	+1.3
Means	...	.	...	.			..		5.1	3.0	+0.7	+2.1
Magellanic Cloud	...	.	..	.	..	..	.	..	4.7	...	.....	.....
Monoceros	...	.	..	..	..	..	...	..	...	4.5	.....	.....
Orion	...	.	..	..	...	.	..	..	.	5.0	....	.....

* Scorpio II is probably a collection of stars toward the galactic center. Note the large differences in W. R. dispersion and W. R. − Abs. between this region and all the others.

accordance better still.[16] Shapley and Miss Wilson[17] noted that "a normal error curve . . . can be fitted . . . to the distribution curve for the brighter half of the O stars. The number and distribution of fainter O stars is . . . unknown because of the magnitude restrictions on spectral classification." The large spread in brightness for the groups in Orion and Monoceros (the apparently brightest groups considered) suggests definitely that some absorption O stars are being missed at the faint end of the other groups (such as the scattered one in Scorpio) and thus justifies the exclusion from the means for Wolf-Rayet stars those of magnitude greater than 9. The spread also suggests that the dispersion in real brightness is about as great for the absorption O stars as for the Wolf-Rayet stars, although we have no guarantee that a number of faint

[16] The Magellanic Cloud contains no recorded absorption O stars, and few B stars, so that here either the Wolf-Rayet stars are considerably brighter or else are unaccompanied by absorption O stars. In no other group are Wolf-Rayet stars found without absorption O stars, though the reverse arrangement occurs in Monoceros and Orion, for instance.

[17] H. C. 271, 1925.

emission O stars have not also been missed, so that their range may be yet larger.

The dips of the two classes of stars with respect to the galactic circle furnish another way of estimating their relative absolute magnitudes. Reference to Figure VI, I will show that the two classes of stars do not greatly differ in galactic concentration or dip—if anything the Wolf-Rayet stars appear on these criteria to be the brighter.

To summarize our conclusions on the luminosities of the Wolf-Rayet stars; they appear to be not more than a magnitude fainter, on the average, than the absorption O stars, and their dispersion is at least two magnitudes and a half and may be far greater.

Of the celebrated problem of the luminosities of the nuclei of planetary nebulae I can discuss but one aspect. The photographic absolute magnitudes of the central stars have been shown by Russell, Dugan, and Stewart[18] to be +4 or +7, according as the parallaxes are derived from proper motions or directly measured. A similar result of +3.4 has been derived by Gerasimovič[19] from various kinds of data.

It is possible that the total luminosity of the nebula gives a better idea of the *bolometric* magnitude of the central star than the photographic magnitude of the star itself. Data for over 90 planetary nebulae have been given to me by Prof. H. H. Plaskett; for 54 of them the magnitudes were determined for both nebula and included star. The mean difference of apparent magnitude between star and nebula[20] is +3.3. The

[18] Astronomy, **2**, 834, 1927.

[19] H. B. 864, 1929.

[20] For those planetaries in which both have been observed, which are obviously those with the smallest difference. For over thirty no star is included, and for these the mean difference, star − nebula, may be as great as five magnitudes. Gerasimovič (H. B. 864, 1929) has attempted to explain the anomalous absolute magnitudes as a systematic effect applying to nebulae with bright nuclei and little disturbing nebulosity. But this suggestion, implying that the absolutely faint nebulae are those for which (star − nebula) is small, appears to be at variance with the facts.

mean apparent photographic magnitude of the integrated nebulae is 11.0, from which, on various assumptions as to the brightness of the nuclei, we derive the absolute magnitudes of the integrated nebulae at the bottom of the table below. We may compare with the very different value of −6.3 obtained by Shapley[21] for eight integrated nebulae, classed as planetaries, in the Large Magellanic Cloud.[22] Dr. Shapley points out to me, however, that the latter were selected for gigantism.

The luminosities of the O stars are collected in a final tabulation.

TABLE VI, VI.—ABSOLUTE MAGNITUDES OF O STARS

Class	Magnitude	Reference
Absorption	− 4.0	J. S. Plaskett
Wolf-Rayet	− 3.3	See p. 76
Nuclei of planetaries	(*a*) + 4	Proper motion
	(*b*) + 7	Parallax
	(*c*) + 3.4	Gerasimovič
Total planetaries	(*a*)] + 0.8	
	(*b*)] + 3.7	
	(*c*)] + 0.2	

The many attempts to account theoretically for the low luminosities of the nuclei of planetaries are of great interest but have not led as yet to crucial observations.

29. The Temperatures of the O Stars.—The available evidence, although incomplete and not homogeneous, clearly shows that the O stars have not the high energy temperatures that might be expected from the high ionization temperatures deduced by the Saha theory and its modifications.[23,24] The approximate temperatures of 24 stars are tabulated below.

[21] Using the revised parallax.

[22] Shapley and Miss Wilson, H. C. 271, 1924.

[23] Cf. Fowler and Milne, M. N. R. A. S., 83, 403, 1923; Stellar Atmospheres, 1925.

[24] Gerasimovič, H. C. 340, 1928.

TABLE VI, VII.—TEMPERATURES OF O STARS

H. D.	Spectrum	Approximate Temperature	Reference
		°	
24431	O9	7500:	Gerasimovič, H. C. 340
24912	O7e	8500; 9000	Gerasimovič; Hertzsprung, Leiden Ann., 14
30614	O9	9500	Gerasimovič
36861, 2	O9	11000; 12000	Hertzsprung; Gerasimovič
42088	O6	[color excess + $1^m.0$]	Seares and Hubble, Mt. W. Contr. 187
47839	O7	11000	Hertzsprung
68273	WIIIn!k	Very high	King, H. A., 85, 186
92740	WIIIsk	10500:	Gerasimovič
93131	WIs!k	16000:	Gerasimovič
96548	WIn	10000:	Gerasimovič
97152	WIIIn!	18000:	Gerasimovič
150958	O9w	8500	Gerasimovič
152386	O6w	Very low	Gerasimovič
153919	O6w	7000	Gerasimovič
153685	WIIIn!k	High	Gerasimovič
165763	WIIIn!	11500	Gerasimovič
190429	O5w	7000	Gerasimovič
203064	O8	8500	Gerasimovič
206267	O6	6500	Gerasimovič
209975	O9	7500	Gerasimovič
210839	O6w	7000	Gerasimovič
214680	O8	13000; 11000	Hertzsprung, Gerasimovič

The scattering of the temperatures and the great inaccuracy inherent in such measures (cf. H. B. 846 for a discussion of the difficulties) deprive the individual temperatures of much significance. The means for absorption and Wolf-Rayet stars are 8600° and 13200°, and the difference is undoubtedly real: the Wolf-Rayet stars, rather less luminous and accordingly less massive, have higher spectrophotometric temperatures than the absorption O stars. We note that the nuclei of the planetary nebulae, still less luminous, appear to have still higher temperatures[25] (if the distribution of energy in their spectra

[25] Zanstra (Nature, May 19, 1928) obtains theoretically (cf. Ap. J., **65**, 50, 1927; Bowen, Ap. J., **67**, 1, 1928) temperatures of the order of 50,000° to 100,000°.

conforms to the black-body law) according to the observations of their energy distribution made by Wright[26] and by Hubble.[27]

The difference of temperature is not alone adequate to account for the differences between the three classes of stars, nor can their spectra be accounted for by ionization theory at the lower temperatures.

30. Relation of O Stars to Nebulosity.—In addition to the nuclei of the planetaries, a number of O stars are apparently associated with diffuse nebulosity. Attention is drawn to all published data on the matter in the notes to Table VI, I; in addition, all the O stars in the immediate region of η Carinae are clearly involved in nebulosity. There appear to be all stages of association, from the planetaries, through such stars[28] as those in N. G. C. 1514 and 7635 which recall the nuclei of planetaries, and θ Muscae,[29] to those in N. G. C. 6231, which are apparently not directly connected with surrounding diffuse nebulosity.

There does not appear to be any positive indication that O stars are necessarily situated in nebulosity, although Hubble has shown that when they are, the spectra of nebula and star are correlated[30]; the close concentration of the O stars and the diffuse nebulae in the Milky Way is probably enough to lead to the observed coincidence.

31. Spectroscopic Binaries of Class O.—Of the 160 stars enumerated in Table VI, I, 31 are spectroscopic binaries, but periods are known for only 10. We note that the relation pointed out by Shajn and Struve[31] between rotation and line quality is borne out by them; not only have those of shorter period the most diffuse lines, but also three stars of definitely

[26] Lick Obs. Pub., 13, 251, 1918.
[27] Ap. J., 56, 436, 1922.
[28] Hubble, Ap. J., 56, 184, 1922.
[29] Shapley, H. B. 843, 1927.
[30] Mt. W. Contr. 241, 1922.
[31] M. N. R. A. S., 89, 222, 1929.

stationary velocity[32] (H. D. 34078, 54662, 214680) have especially sharp lines. The southern O stars have not been measured for radial velocity, but the occurrence of diffuse lines for several suggests that they would be worth examining in this connection.

32. The Interpretation of the Spectra of Wolf-Rayet Stars.—The spectra of both absorption and emission O stars contain the lines of H, He+, C++, N++, and Si+++, and are obviously in a high state of excitation.[33] The absorption O can be interpreted on simple ionization theory at a temperature of 25000° to 35000° and a pressure of about 10^{-4} atmospheres. But the spectrophotometric temperatures of these stars, as shown in an earlier section, are below 10000°, corresponding, with an absolute magnitude of −4 to the low value of 1,000 centimeters per second per second for the surface gravity. As I shall show in greater detail in the chapter dealing with the cool B stars, the observed degree of ionization in the spectrum could be attained only, at a temperature of 10000°, with pressures of the order of 10^{-15} atmospheres; and at such pressures,[34] the numbers of effective atoms of hydrogen and helium would be well below the lower limit for visibility (about 10^{16} per square centimeter surface on Harvard one-prism plates; see Chapter III) instead of about $10^{17.5}$, as observed. So far as the absorption O stars are concerned, the observed spectrum cannot be produced by any mechanism that considers only the normal theory of ionization as at present accepted, if the measured energy temperatures really represent the temperatures of the reversing layers.

[32] J. S. Plaskett, Publ. Dom. Ap. Obs., 2, 329, 1924.

[33] Spectrophotometric measures of the lines of absorption O stars are included in the tables of the following chapter. References to descriptions of spectra are given in the notes to Table VI, I.

[34] Assuming that κ = const. (see Chapter XV). If $\kappa \propto P_e$ the quandary is equally serious, for with these low temperatures the hydrogen lines should be as strong as for B8 stars, because of Milne's "null-effect" (H. B. 871, 1929); and actually they are much weaker.

A similar quandary holds for the Wolf-Rayet stars. The spectrophotometric temperature is somewhat higher, but the pressure required to produce the observed degree of ionization is still very low. Moreover, all the Wolf-Rayet stars display one of the two varieties of emission lines. Figure VI, 2 represents the measurement of the bright bands in the spectra of six representative Wolf-Rayet stars. The most extreme cases are not shown because for them the continuous background is too weak to be measured. Remarks on the individual spectra are given in the legend of the figure.

An interpretation of the Wolf-Rayet spectrum must not only account for the observed degree of ionization but also find the way in which the energy is thrown into the bright bands. Apparently for these stars, and also for the absorption O stars, we are driven to postulate either large departures from black-body distribution or some mechanism of superexcitation. We have little basis for predicting the form that these would take, and so it seems best to examine the data empirically.

There seems to be no progression in energy temperature from O9 to O5. The relative numbers of stars in the various absorption O classes are: O9, 24; O8, 16; O7, 13; O6, 19; O5, 11. The later O classes tend to contain more stars; they presumably represent conditions that occur more commonly and that are more similar to those for the B stars. Hence we infer that the excitation is greatest for O5, least for O9. There are equal numbers of stars in Classes WI and WIII, which does not suggest that the conditions producing stronger 4686 are any less common than those producing stronger 4633. On the other hand, the old Harvard classes Oa, Ob, Oc, Od occur in the numbers 39, 15, 15, 3—suggesting that the excitation is least for Oa, greatest for Od, a conclusion in harmony with the fact that all Od stars are now placed in the most highly excited classes O5 and O6, and reversing the old scheme . Furthermore, WIII and Oa are correlated, and WI is related more closely to Ob, Oc, and Od, suggesting that the most highly excited Wolf-Rayet stars (to judge by their numbers) are those with 4686 stronger than 4633,

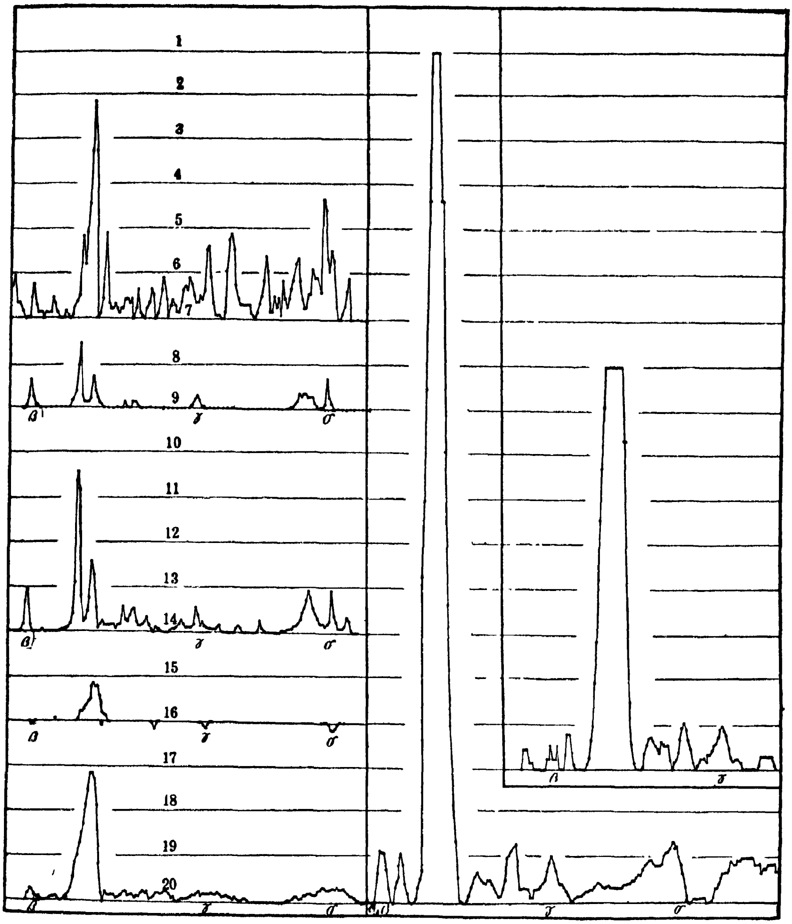

FIGURE VI, 2.

Distribution of energy in Wolf-Rayet emission bands.

H. D. 164270
H. D. 92740
H. D. 151932 — H. D. 115473
θ Muscae
H. D. 152270 — H. D. 165763

Ordinates are expressed in percentages of the corresponding continuous background, all on the same scale; horizontal lines (numbered serially for convenience) are drawn at intervals of 100 per cent. The first three stars are examples of peaked emission, the next two, of rounded emission lines. The stars on the right show flat-topped lines of ionized helium, but the other lines are more rounded.

though the relative numbers of stars of Classes WI and WIII do not substantiate the suggestion. In general terms this is in harmony with the conclusion of J. S. Plaskett[35] that Wolf-Rayet emission is not related to the stage of ionization of the associated absorption O spectrum.

The proposal has recently been made[36] by Beals[37] that the spectra are the result of continuous ejection of matter from the stars. He reaches his conclusion from a consideration of the Doppler widening of the lines, and the analogy with the nova spectrum. The general idea is of considerable interest, and it is important to assemble all the facts of which a completed theory on these lines must take account.

1. Various forms of emission lines occur: parabolic, both intense and weak; flat topped, both intense and weak; and peaked, both intense and weak. The weak peaked type is represented by stars like ζ Puppis, which are not illustrated. They connect evidently with the remaining types, though by some not regarded as Wolf-Rayet stars at all.

2. Line width is unrelated to line intensity.

3. The lines at 4686 (He+) and 4640 (C++, N++) are by far the most intense and vary greatly in relative strength; the hydrogen lines, though next in intensity, are weak, even in comparison with the Pickering series.

4. Violet absorption is strong for the lines of 4640 when they are intense; undisplaced central absorptions are simultaneously shown, however, by some of the hydrogen lines, and

[35] Pub. Dom. Ap. Obs., **2**, 329, 1924.

[36] The continuous ejection of matter has already been discussed in connection with the Wolf-Rayet stars by Dingle (Modern Astrophysics, p. 288, 1924), who saw reasons for rejecting the explanation. Miss Clerke foreshadowed a similar thought in her discussion of novae (The System of the Stars, p. 96, 1905): "Novae undergo changes of a prescribed kind in a settled order. Nor are any of their phases necessarily and essentially unstable, since each is exhibited permanently by the members of other sidereal families. The conditions to which novae are temporarily subjected cannot . . . be adequately explained without reference to the fact that they are durable elsewhere." This passage follows a specific reference to the Wolf-Rayet peculiarities in the nova spectrum. Cf. also p. 246.

[37] Pop. Astr., **37**, 577, 1929; M. N. R. A. S., **90**, 202, 1930.

many of the strong emission bands are unaccompanied by strong violet absorption edges. Occasional very strong violet absorption is also shown by emission lines not especially strong, e.g., the lines of neutral helium[38] in H. D. 96548.

The violet absorption edges are used by Beals to support the hypothesis of continuous ejection; the central absorptions of the hydrogen lines have, however, been regarded by Gerasimovič as confirming the idea that the bright lines are formed at low levels in the atmosphere and are broadened by pressure. A satisfactory theory should cover both types of absorption. Professor Rosseland suggests that the central reversals might be produced by reabsorption at the edges of an expanding shell. Assuming the Wolf-Rayet star to have a definite size and surface as determined by its absolute brightness and effective temperature, we obtain the accompanying series of surface gravities for the temperatures indicated at the heads of the columns. The bolometric reduction is taken from Hertzsprung's formula.[39]

We note that Beals considers that "the possibility of the ejection of atoms from a star by radiation pressure depends

TABLE VI, VIII.—LOGARITHMS OF SURFACE GRAVITIES (Cm/Sec²)

Type of Star	Absolute Visual Magnitude		Temperatures				
			8600°	10000°	13500°	15000°	20000°
Wolf-Rayet	Low	−0.8	3.4	3.6	4.0	4.2	4.5
	Mean	−3.3	2.9	3.1	3.5	3.7	4.2
	High	−5.8	2.7	2.9	3.5	3.5	...
Absorption	Low	−1.5	3.3	3.5	3.9	4.1	4.4
	Mean	−4.0	2.8	3.0	3.4	3.6	4.1
	High	−6.5	2.5	2.8	3.2	.	...
Planetary	Low	+7	5.7	.	.	.	6.7
Nuclei*	High	0	3.6	..	..	...	4.7

* Low estimate is from parallax (cf. Russell, Dugan, and Stewart, 2, 834, 1926); high estimate is for integrated light of the nebula.

[38] H. B. 843, 1927.

[39] Eddington, the Internal Constitution of the Stars, 137, 1926.

on . . . (*a*) the temperature of the photosphere, and (*b*) the value of gravity at the stellar surface." From Table VI, VIII we see that the higher the temperature the higher the surface gravity and the less the tendency for atoms to escape. At 20000° the surface gravity is of the same order as on the Sun; even at 10000° it is considerable. The two causes of ejection considered thus tend to work one against the other. We note incidentally that the value of gravity at the surface of the nucleus of a planetary nebula is probably greater than 10^5 centimeters per second per second, as the evidence for their high energy temperatures seems fairly good, and their luminosities are almost certainly less than zero. That such a star can and often does show a Wolf-Rayet spectrum, very similar to that shown by a star at whose surface gravity is probably a hundred times less, demands a versatile method of continuous ejection.

The mechanism of ejection, if it is indeed the cause of the spectral peculiarities, must be peculiar to the Wolf-Rayet stars and vary among them. For there are numerous stars of similar temperatures, and even greater mass and dimensions, that do not show emission of the same kind: α Cygni and Rigel are two of a large class.

In discussing the general tendency of atoms to leave the surface of a Wolf-Rayet star one matter is worthy of comment. Low surface gravity might be produced by low temperature and large size—but it might also result from low mass, and it is just possible that the masses of the Wolf-Rayet stars are low. Consider, for instance, a nova just after maximum, when the bright bands are beginning to appear. It is ten magnitudes brighter than it was before the outburst, and there is a possibility that if any mass can be assigned to it on the basis of luminosity, it should be on the basis of the luminosity before outburst—say absolute magnitude +5. A Wolf-Rayet star has similar characteristics; may it not be similarly distended so that its absolute brightness is no direct index of its mass?

CHAPTER VII

THE NORMAL CLASS B STAR

It is of interest that the stars that now stand at the head of the normal stellar sequence, Class B, were first placed between Class A and Class F, though they were soon seen to be better suited to a position preceding Class A. This corrected position harmonizes with what is known of the degree of ionization in their atmospheres, but in energy distribution not a few B stars apparently fall in the place originally assigned to the whole class. The position and relationships of Class B are by no means finally fixed, and attention can merely be focused on several open problems.

33. Distribution of the B Stars.—The present chapter deals with Classes B0, B1, B2, and B3, which constitute "group B" as defined by Shapley and Miss Cannon.[1] The high galactic concentration (more than 90 per cent of the B stars fainter than $7^m.26$ are within ten degrees of the galactic equator) is only one of many indices of their great brightness. Several estimates of their absolute visual magnitude are summarized in Table VII, I.

Of the 220,000 stars in the Henry Draper Catalogue, less than 4,000, or about two per cent, belong to group B, and this uncommonness is real and significant. We encountered it, in yet more pronounced terms, for the O stars of Chapter VI. Because of their great brightness the B stars are of course even less numerous than appears from their numbers in the Catalogue, contributing but one half of one per cent to the giant population of the solar neighborhood. Their uncommonness,

[1] H. Repr. 6, 1924.

TABLE VII, I.—ABSOLUTE VISUAL MAGNITUDES OF B STARS

Class	Kapteyn[2]		Charlier[3]	Plummer[4]	Robb[5]	Struve[6]	Pannekoek[7]	Adopted
B0	(0.34 ± 0.23)		−2.99		−1.72	−2.8	−3.1	−2.65
B1	−2.00	16	−2.78	−3.7	−2.86	−1.3	−2.5	−2 29
B2	−1.21	16	−2.18		−2.03	−2.5	−1.8	−1.94
B3	−0.38	06	−1.01	−1.2	−1.28	−1.0	−1.2	−0.97
B5	−0.21	09	−0.80		−0.84	.....	−0 8	−0.66

brightness, and consequent high mass are undoubtedly closely connected; we recall the similar case of the O stars, the long-period Cepheids, and the c-stars as a group; the rarity of large masses is an important aspect of the high-luminosity problem.

The B stars are the class most peculiar to the local system, about 1,200 of the 2,000 known B stars brighter than $8^m.25$ being local system stars, including 90 per cent of those brighter than visual magnitude 5.25[8]; nearly all B stars spectroscopically studied in any detail belong to the local system.

The concentration of the B stars in the local system is linked with their apparent connection with the bright galactic nebulae. The coincidences of distribution are obvious, and the spectral correlations[9] well known. The stars are effective in stimulating the nebular spectra, and it is possible that the nebular substance also affects the stars. The influence might take several forms: reddening of the starlight by molecular scattering and accretion of mass by absorption of the nebular material occur to the mind at once.

Stars of Class B fainter than $8^m.25$ are for the most part not members of the local system.[8] They are strongly concentrated

[2] Mt. W. Contr. 147, 1918.
[3] Lund Medd. 34, 1926.
[4] M. N. R. A. S., 73, 186, 1913.
[5] Lund Medd., Series 2, No. 44, 10, 1926.
[6] Ap. J., 67, 388, 1928.
[7] Publ. Astr. Inst. Amsterdam, 2, No. 2, 1929.
[8] Shapley and Miss Cannon, H. C. 239, 1922.
[9] Russell, Dugan, and Stewart, Astronomy, 2, 838, 1926.

to the Milky Way,[10] but not markedly to the galactic center (in contrast to the intrinsically fainter planetary nebulae, and to the far brighter novae). In low galactic latitudes very faint stars of color class b are found; in high galactic latitudes there are few or no faint B stars.[11]

The motions of the fainter B stars appear to be very continuous with those of the bright ones, as is shown by J. S. Plaskett's recent demonstration that their radial velocities fall in with the general galactic rotation.[12] This observation does not suggest discontinuity between the local system and the more distant B stars.

Groupings of the B stars, and possible discontinuities, are suggested by the analyses of Pannekoek[13] (but the conclusions are subject to the adopted dispersions within the various spectral classes, and the outlines of the individual groups could be blurred into imperceptibility if the adopted dispersion were large enough—not perhaps much larger than the evidence of undoubted galactic clusters suggests that it is). The discontinuity noted by Krieger[14] in the direction of the Scutum cloud would, if substantiated, strengthen the case for separate groups materially. Indeed, if Shapley's picture of the galactic system[15] describes the arrangement better than the idea of a mixed galaxy in uniform rotation, such discontinuities are almost a necessary requirement. The stars of spectral class B and color class b[16] will undoubtedly furnish the evidence most

[10] If the galaxy has a diameter of seventy kiloparsecs, and if the B stars are coextensive with it, early B stars of similar brightness to the local specimens should cease at about $16^m.5$ in the direction of the galactic center, and at about $14^m.5$ in the opposite direction. Measures of the galaxy in all dimensions are thus attainable.

[11] Shapley, Mt. W. Comm. 30, 1917.

[12] Science, **71**, 225, 1930.

[13] Publ. Astr. Inst. Amsterdam, **2**, 1929.

[14] L. O. B. 416, 1929.

[15] H. C. 350, 1930.

[16] It is particularly unfortunate for the purposes of such studies that the energy temperatures of all the B stars are rather low, and those of many are so low that on the basis of color index the stars would be assigned to Classes A and F. Moreover the most luminous B stars, which would be of the greatest service in exploring

essential in resolving the picture, and studies of their brightness, distribution, and motions are of paramount importance.

34. Synopsis of the Spectral Criteria.—Class B0 is recognized by the strength of the hydrogen lines, the presence of a weak Pickering series, and especially by the strength of the lines of Si++ and O+. The latter lines are the criteria chiefly used in the classification of faint B0 stars.[17]

Class B0 is rather inhomogeneous, and abnormalities, when they occur, are in the direction of weakened O+ and Si++ and of strengthened He+ (both Pickering and 4686). There is a suggestion that the stars with these features are less luminous than ϵ Orionis, the typical star, which is undoubtedly something of a supergiant; its sharp lines and its relative brightness suggest high luminosity on comparison with δ and ζ Orionis, which have hazy lines and the abnormalities mentioned above.[18] These changes in the relative line strengths could be produced by increased ionization, through an increase of temperature, or a lowering of pressure.

Class B1 is recognized by stronger helium than Class B0 and weaker O+ and Si++. The line of C+ is also noted. Sharp-line stars that are undoubtedly abnormally bright for the class (θ Arae and ϵ Canis Majoris) seem to have abnormally strong N+, O+, and C+, and strengthened Si++ (except for γ Arae, which is possibly an ac star). Ionized oxygen is as strong here as anywhere. The abnormalities in the spectra of the brighter stars are the results of increased ionization and make the spectrum resemble more nearly that of Class B0.

Class B2 is recognized by the helium lines at their maximum intensity, and the lines of O+, N+, Si++, C+ are prominent,

---

the distances of the galaxy, tend to be the "coolest" and are thus colorimetrically unrecognizable. The best that can be done, when color indices alone are available, is to feel assured that very small color indices belong to B stars. Stars of larger color index *may* be B stars. Any satisfactory analysis requires spectra as well as colors. See Section 44 for a discussion of the coolness of certain B stars.

[17] Miss Cannon, H. A., **28**, 151, 1897.

[18] *Ibid.*, 176, 1897.

though weaker than in Class B1. Abnormal intensities are not recorded in this class, peculiarities referring to sharp lines (9 Cephei) or bright hydrogen lines. Apparently the dispersion in degree of ionization within Class B2 is not great.

Class B3 is defined by the greater weakness of the C+, N+, Si++ and O+ lines that are striking in B2 and by stronger Si+ and Mg+ than in that class. Its abnormal members are chiefly bright-line stars. Few abnormal line intensities are recorded. As in Class B2, the dispersion in degree of ionization appears not to be great.[19]

Class B5, defined by He and Si+, has interesting abnormal members. Stars with unusually broad lines (such as Achernar) have a faint K line, and those that have very sharp lines, such as o_2 and η Canis Majoris, have abnormally strong Ca+, N+, He, and O+ (the latter, which is very weak in Class B3 and absent from normal B5 stars, recalls the appearance of helium at Class A2).[20] The bright stars represent higher ionization than the fainter ones, and their spectra tend to resemble those of stars of earlier spectral class. The lines strengthened in the two supergiants o_2 and η Canis Majoris are in fact nearly as strong as in Class B1.[21]

35. Temperatures of Normal B Stars.—The ionization temperature of the B stars, derived from the method of maxima[22] on the assumption of a uniform partial electron pressure of 10^{-4} atmospheres, proved to be very high—far higher than the temperatures based on energy distribution in their spectra. A comparison of the temperatures from several sources is contained in the next table. In commenting on the color temperatures it should be noted that Hertzsprung's are based on numerous inhomogeneous color equivalents, King's on color indices measured with great accuracy, and Gerasimovič's on the

[19] The difference between Class B2 and B3, and its effect on the classification of bright-line stars, is discussed in Section 38.

[20] See Chapter X, p. 143.

[21] Miss Cannon, H. A., **28**, 183, 1897.

[22] See H. Mon. No. 1,116, 1925.

photometry of short dispersion spectra. Hertzsprung's tend to be the lowest and King's the highest, but all agree in not being much greater for early B stars than for later ones.

Table VII, II.—Temperatures of Stars of Group B

Class	Ionization Temperature[23]	Color Equivalent Temperature		Spectrophotometric Temperature Gerasimovič[26]	Mean Energy Temperature
		Hertzsprung[24]	King[25]		
	°	°	°	°	°
B0	20000	10200	15300	14200	13200
B1	18000	8900	14700	9500	11000
B2	18000	9000	16400	10700	12000
B3	17000	117000	14100	11200	12300
B5	15000	10700	14400	11800	12300

We note that Hertzsprung says: "Among the stars here considered there is none the c_2/T value of which is certainly below 1 or in other words the effective temperature of which is above 14600° on the scale of Wilsing," and that actually there are only four such stars in his table—moreover two of these are of Class B3, and one is of Class B5. His table assigns the highest mean temperature of all to Class B3—a result of some weight, for it depends on 47 stars.

The ionization temperatures in Table VII, II were derived under the assumption of a constant partial electron pressure—the same that had been shown to fulfil the requirements of observation earlier in the spectral sequence. But evidence accumulates that the same value of P_e is not present all along the spectral sequence,[27] and it is worth examining whether at the energy temperatures derived for the various classes of B stars, the observed ionization conditions could be produced with plausible values of the surface gravity.

[23] See H. Mon. No. 1, 139, 1925.

[24] Ann. Leiden Obs., **14**, 1922.

[25] H. A., **85**, No. 10, 1928.

[26] H. C. 339, 1929.

[27] Payne and Hogg, H. C. 334, 1928.

Table VII, III contains the mean absolute visual magnitudes of the spectral classes of group B; the mean temperatures taken from Table VII, II; the logarithms of surface gravities, in centimeters per second per second, computed by Eddington's equation.[28]

TABLE VII, III.—LUMINOSITIES, TEMPERATURES, AND SURFACE GRAVITIES OF B STARS

Class	Absolute Visual Magnitude	Temperature	Logarithm of Surface Gravity
O (absorp.)	−4.	11600°	3.0
B0	−2.6	13200	3.2
B1	−2.3	11000	3.6
B2	−1.9	12000	3.8
B3	−1.0	12300	4.1
B5	−0.7	12300	4.2

36. Ionization in the Atmospheres of B Stars.—In examining the ionization conditions for the classes that make up Group B, I shall use hydrogen, for which the data are most complete, and shall for the moment neglect the Stark effect. The data to be used in making the test are contained in the following brief tabulation:[29]

Class	Temperature °	Log g	Log NH (Mean β, γ, δ)
O	11600	3.0	17.50
B0	13200	3.2	17.95
B3	11000	3.6	18.08
B5	12000	3.8	18.25
(A0)	(10000)	(4.1)	(19.00)

As a first approximation we take a uniform apparent effective temperature of 12200° for all classes earlier than B5—only for Class O is this temperature demonstrably outside the probable limits.[30]

[28] The Internal Constitution of the Stars, 120, 1926.

[29] See Table VIII, VIII, p. 113.

[30] For a second approximation to the temperatures see Chapter VIII, p. 113; cf. also Chapter XV, p. 272.

In Chapter XV the generalized Saha equations[31] are compared with observation. It develops that for hydrogen in the earlier spectral classes observation is best represented by the assumption of an absorption coefficient that varies with the pressure (to a power rather less than the first). This assumption is adopted in the following numerical estimates.

The ionization is expressed by Milne's Problem III: atoms just becoming ionized in the presence of excess of atoms already once ionized. The number of neutral atoms is given by the expression

$$N_0 = \frac{2\epsilon}{mg}\left[P - K_1 \log\left(1 + \frac{P}{K_1}\right)\right]$$

where

$$K_1 = \frac{q_1}{q_0}\frac{(2\pi m_e)^{3/2}}{h^3}(kT)^{5/2}e^{-\chi_1/kT}$$

The quantity K_1 is a function of the temperature T, the ionization potential χ, and the constants q_1, q_0, π, m_e, h and k. For hydrogen at 12200° it has the value $10^{4.16}$, and so $N_0 mg = 2\epsilon P$, since we know on general grounds that P is never greater than 10^2 dynes per square centimeter. Thus we have:

Class	Log N_0	Log g	Log ϵP	Log P_e
O	17.50	3.0	−3.6	−6.6
Bo	18.0	3.2	−2.9	−5.9
B3	18.1	4.1	−1.9	−4.9
B5	18.3	4.2	−1.6	−4.6
(Ao)	(19.0)	(4.7)	−0.4	−3.4

The last row and the last column are computed on the basis of the value of log P ($\bar{4}.6$) already derived[32] for Class Ao from measures of neutral and ionized calcium. This value for the pressure serves to provide a zero point for the partial pressures of the earlier classes.

These final pressures are the test of the plausibility of our assumption that the measured energy temperatures of normal B stars are adequate to produce the observed spectra under

[31] Milne, M. N. R. A. S., **89**, 17, 157, 1928.

[32] Payne and Hogg, H. C. 334, 1928.

normal conditions of ionization. In order to examine this they may be compared with the pressures derived by the Ca/Ca+ ratio for later classes.[32] The agreement in scale is sufficiently satisfactory, and the zero point depends only on the value of ϵ used. The chief uncertainty in the above calculations is the assumed value for ϵ, and the final pressures may well be five times too large or too small. The quantity here inserted as ϵ is empirically derived, refers to *effective atoms*, and may well contain unspecified terms, so that its physical significance is not obvious.

It remains to examine the ionization of other elements. Helium has a maximum at Class B1.5, which occurs at a pressure of $10^{-6.8}$ atmospheres,[33] a value very similar to the one that would be obtained by interpolation from the previous table (10^{-5}). The case for this element is rather different from that of hydrogen; the latter is changed very little in ionization by changing temperature in the B stars, for its maximum is past (and P/K_1 therefore negligible) but for helium the number of atoms in the neutral state is not simply proportional to p/g. The hydrogen lines are insensitive to small differences of temperature occurring among the B stars, but the helium lines will react to them, as will such lines as those of O+, and N+, present in the early B stars. All these lines will tend to be strongest in the brighter stars—and a bright B star that appears to be normal for the spectral class in which its hydrogen places it will have to be cooler than the average star of that luminosity. This is no doubt the reason for the general tendency of the absolutely brighter B stars of *one Draper class* to be cool. The abnormalities shown by B stars that appear from their line quality to be unusually bright (see Section 39) are all in the direction of placing them in a class earlier than that called for by the strength of the hydrogen lines. A general study of the bright B stars from the point of view of luminosity and line intensity is very much to be desired.

[33] Assuming a temperature of 12000° and the new Milne formulae.

It appears that as far as hydrogen is concerned, the observed energy temperatures, luminosities, and spectra of the normal B stars can be reconciled in terms of the new Milne theory. The differences shown by lines of other elements are not in disharmony with the picture. Whether the abnormally cool B stars can be explained on similar lines is more doubtful; the question is examined in the next chapter. It is also well to add that the existence of such effects as "superexcitation" on the ionization in the atmospheres of normal B stars is not to be specifically denied; they may be present, even appreciable, but they are not necessary to explain the observed sequence of types. We need not feel hesitant about accepting a temperature as low as 14000° for early B stars.

CHAPTER VIII

THE HIGH LUMINOSITY STARS OF CLASS B

THE stars of Group B include Classes B0, B1, B2, B3, and B5. The present chapter takes up briefly, in order, the demarcation of supergiants within the group; the title of the bright-line B stars of high luminosity; the spectra of supergiant B stars; the temperature of supergiant B stars; and, finally, their probable physical condition. Several of the fundamental problems of stellar spectroscopy are prominent in the chapter: What is a supergiant? What is meant by spectral classification in general and the Draper classes in particular? What do the low temperatures of the brighter B stars signify physically? And this last question is linked with the general problem of the meaning and expression of stellar temperatures.

37. Supergiants within Class B.—The absolute magnitudes of the normal B stars are reproduced in[1] Table VII, I. Classes B0 (−2.6) to B2 (−1.9) are included wholesale in the high luminosity category, defined as being composed of stars brighter than −2 visually. But the dispersion in absolute magnitude in these classes is very large,[2] so that fainter members of the early B classes are not in the "high luminosity" group, and the brighter members of Classes B5 and B3 fall within it.

The absolutely bright B stars will be found, in later spectral classes, to be spectroscopically distinguishable from their fainter relations; but the line between supergiant and normal must be drawn at a higher luminosity in Class B than in Class G if the distinction is to be spectroscopically obvious, or, indeed, physically significant.

[1] See p. 88.

[2] Cf. for instance, Lundmark, L. O. B. 338, 1922.

The B stars that are known to be spectroscopic binaries brighter than $-2^m.0$ in absolute visual magnitude are contained in the first table of Chapter V. The general comment on the table may be recalled—six out of the ten stars there enumerated have lines described as nebulous, and certainly cannot be classed as c-stars.

The Henry Draper Catalogue contains seventy B stars noted as having narrow-lined spectra and to be regarded, therefore, as possible supergiants. Fifty-three of them are brighter than $8^m.25$—a percentage of 2.56 of the whole number of B stars included down to the same magnitude, more than twice as large as the percentage of narrow-lined stars in any other class, and four times as large as the average.

The difference in spectrum between c-stars and normal stars is so intangible as to be almost meaningless in the early B classes, and for B0, B1, and B2 the narrow-line stars grade continuously into normal stars. There are very few cB3 stars—six can be asserted with more or less confidence, and one other has a somewhat peculiar spectrum. Only in Class B5 do c-stars become distinctive and numerous.

Data on the physical condition of the obvious cB stars of earlier type are scanty, though we may safely consider them to be among the brightest of their class. But a glance at the list of very massive stars in Table V, I reminds us that many exceptionally bright B stars may be short-period binaries, with lines so widened by rotation that their c-character, if any, is obscured. For this reason the number of highly luminous early B stars that would be inferred from spectra is probably too low. Therefore a spectroscopic survey of individual B stars, while it will select undoubted c-stars, is certainly incomplete, probably seriously so. General conclusions therefore are better based on the material[3] of Section 40.

38. The Bright-line B Stars.—The present section discusses the title of the B stars with bright lines to be included in the

[3] See p. 107.

high luminosity group. Their important spectroscopic characters are discussed in a later section.

The numbers of Be stars in the various classes are as follows:

Class	Number of Stars	Class	Number of Stars	Ratio
(O)	.	...	.. ..	(0.50:)
B0e	14	B0	376	0.037
B1e	15	B1	76	0 20
B2e	37	B2	369	0.10
B3e	49	B3	1,041	0 047
B5e	27	B5	1,348	0.020
B8e, B9e	29	B8, 9	4,500:	0 006
(A)	..	...	.	(0.00:)

Evidently there is a strong tendency for emission line B stars to occur in the spectral classes B1, B2, and B3. In extension of the table at both ends we may refer to the extremely frequent presence of emission for O stars, and its almost total absence from Class A.[4]

The tendency to occupy a narrow range of spectral class may represent a definite physical condition particularly favorable to the type of emission shown by bright-line B stars. Before committing ourselves to this tempting view, to which we shall return later, it is well to examine whether there is a tendency in classifying spectra to place an emission B star in Classes B1, B2, and B3. If there is such a tendency it may carry with it greater luminosity than normal for the class assigned, as was actually found for B3e stars by Gerasimovič.[5]

Class B3 represents effectually such B2 stars as are too hazy in spectrum to show certain specific lines distinctly; it shows spectroscopically just the effects that would be expected from motions and distribution (Class B2 is far more concentrated galactically than Class B3). Bright lines are not necessarily correlated with haziness; they occur in the spectra of c-stars (such as ζ Scorpii and α Cygni), and in fact are confined in Class A to such spectra, but they also occur in the hazy spectra of Alcyone (B5e) and γ Cassiopeiae (B0e). It therefore seems

[4] The few Ae stars are very abnormal objects such as α Cygni.

[5] H. B. 849, 1927.

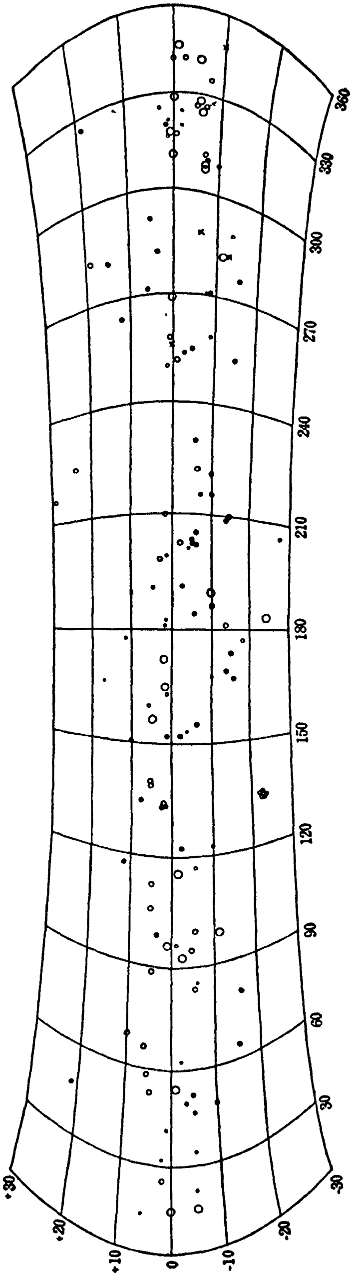

FIGURE VIII, 1.

Galactic distribution of bright-line B stars. Successively smaller circles and dots represent classes from B1e to B8e. P Cygni stars are shown by crosses. Five scattered stars of galactic latitude greater than +30°, and six in latitudes greater than −30°, are omitted.

improbable that they have in themselves a systematic effect on classification. This view is strengthened by the relatively large number of spectra classified B2e—larger in proportion to class B3e than the number of B2 stars relative to the rather large class B3. We may therefore conclude that the crowding of emission B stars into Classes B1, B2, and B3 is real and that the spectral classes of absorption and emission B stars are comparable in ionization, which is the criterion of spectral class used for the Draper classification (see Section 86) of B stars.

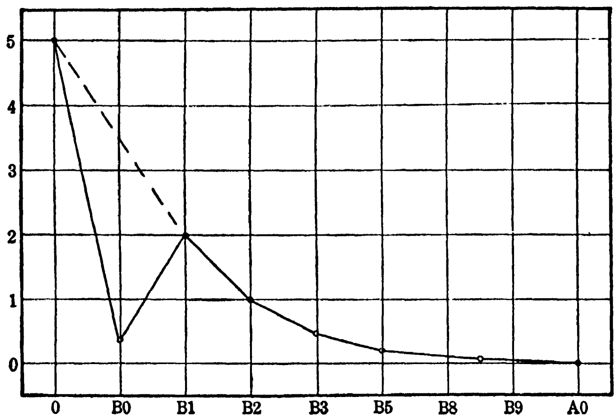

FIGURE VIII, 2.

Percentages of bright-line stars in various classes from O to A.

The classifications of Be stars being accepted as representative of their atmospheric condition, the conclusion of Gerasimovič that the stars of Class B3e are brighter by $2^m.4 \pm 1^m.1$ than absorption stars of Class B3 becomes of significance in connection with the general problem of the bright-line B stars. There is nothing in the distribution of the Be stars to contradict it—rather the reverse,[6] since they are probably not complete to so faint an apparent magnitude as the absorption B stars, hence

[6] An unpublished spectroscopic survey by Miss Hughes of southern Be stars shows that the bright lines tend to be associated with the spectroscopic features that distinguish the brightest B stars.

their rather small galactic concentration. There seems to be no reason to regard the classifications as systematically in error.

Gerasimovič mentions the most serious problem that must be faced—the fact that the emission B stars in certain clusters are of intermediate and not extreme brightness.[7] The cluster method is the essential method of attack for the whole problem of the relative luminosities, and the next section but one will be concerned with it.

39. Spectra and Temperatures of High Luminosity B Stars.—The spectroscopic qualities of the cB stars are definite and simple. The hydrogen and helium lines are sharper than in normal spectra, and their total absorption is smaller, though because of their definite "edges" they are very conspicuous. The K line is always very strong (in these spectra it is of course interstellar). The lines of $O+$, $Si++$, $N+$, and He are stronger than in normal stars of the same spectral class. In the c-stars of Class B5 (defined by the relative strength of hydrogen, helium, and silicon), the oxygen lines, absent even from Class B3, are prominent (as in η and o_2 Canis Majoris[8]).

The tabulation that follows contains all the available spectrophotometric measures of the temperature of stars of Class cB. The sources are Hertzsprung's compilation,[9] Gerasimovič's[10] measures, and the recent work of Greaves, Davidson, and Martin.[11] For completeness the cA stars also are included in this table, which is discussed in part in Section 52.

Two obvious tendencies are shown by the data in Table VIII, I: (1) the c-stars are abnormally cool; and (2) the temperatures tend to be correlated with apparent magnitude. These findings are of such significance that it is worth spending a little space on considering the matter from a rather different aspect.

[7] Trumpler, P. A. S. P., **38**, 352, 1926.

[8] Miss Cannon, H. A., **28**, 1897.

[9] Ann. Leiden Obs., **14**, 1922.

[10] H. C. 339, 1928.

[11] M. N. R. A. S., **90**, 104, 1929.

TABLE VIII, I.—TEMPERATURES OF cA AND cB STARS

Boss	Star	Temperature	Apparent Visual Magnitude
..	Mean B1	° 11000	....
5208	P Cyg (cB1)	5700	4 88
....	Mean B2	12000	....
8279	9 Cep (cB2)	{ 7100 / 6800	4.87
1507	χ_2 Ori (cB2)	6100	4.71
5361	55 Cyg (cB2)	5500	4.89
.	Mean B5	12300	....
4548	67 Oph (cB5)	9200	3.92
457	53 Cas (cB5)	6400	5 62
.	Mean B8	11000	....
1250	β Ori (cB8)	10000	0.34
5779	4 Lac (cB8)	8400	4.64
....	Mean B9	10500	....
781	H. R. 1035(cB9)	6600	4.42
1310	H. R. 1804(cB9)	6400	5.72
....	Mean A0	10000	....
2694	η Leo (cA0)	9800	3.58
1657	.... (cA0)	9000	4.50
5469	σ Cyg (cA0)	7700	4.28
786	H. R. 1040(cA0)	5900	4.76
724	H. R. 964 (cA0)	5100	5.92
....	Mean A2	8500	....
5320	α Cyg (cA2)	8450*	1.33
534	i Per (cA2)	6200	5.22
478	H. R. 618 (cA2)	6000	5.90
5608	ν Cep (cA2)	5700	4 46

* Mean from Hertzsprung (8300) and Greaves, Davidson, and Martin (8600°).

Hertzsprung[12] lists the stars that have color equivalents too large for their spectral class by at least $0^m.4$ (and that are therefore abnormally cool), and also the stars that have color equivalents smaller than 1.60 (thus being uncommonly hot); the color equivalent is defined as usual by c_2/T, where T is the temperature and $c_2 = 14{,}600$. The next tables contain these stars, their temperatures (derived from color equivalents), and remarks on their spectra, compiled from an examination

[12] B. A. N. 37, 1923.

of Harvard plates. Temperatures marked with an asterisk and a dagger respectively were determined by Gerasimovič and Miss Williams; the stars thus indicated are not in Hertzsprung's list.

TABLE VIII, II.—REMARKS ON THE SPECTRA OF ABNORMALLY COOL STARS OF GROUPS B AND A

Boss	Star	Temperature	Spectrum	Remarks
		°		
103	κ Cas	7400	B0	All lines somewhat weak, hazy. Perhaps a tendency to hydrogen emission. Maury division b
149	H. R. 189	6700	B3	Normal B3 spectrum
152	o Cas	8500*	B2	H and He lines extremely wide and shallow; no other lines; no emission. Maury division a, b
457	53 Cas	6400	cB5	Lines strong and sharp; Mg+, Si+, K, abnormally strong
534	i Per	6200	cA2	Hydrogen and K line show the c-character; Si+ strong
742	29 Per	7500	B3	Lines very wide and hazy; shallow; He strong; no Si+ Maury division a, b
744	31 Per	7400	B3	H and He strong, very wide, shallow. No Si+, C+, or N+. Maury division a, b
781	H. R. 1035	6600	cB9	Striking c-character. Williams line character‡ −1, M = −5;
786	H. R. 1040	5900	cA0	Spectrum strikingly c-character. Several metallic lines, He, Si+. Williams line character −1, M = −5;
844	o Per	6700 8500*	B1	Lines wide and hazy; shallow; perhaps verging on emission. Maury division a, b
865	Merope	9400*	B5e	Lines shallow and hazy; Balmer emission. Maury division a
894	ζ Per	7800*	B1	Lines rather wide and hazy. Maury division a
1195	11 Cam	7800	B3e	Strong bright H lines; He lines sharp absorption, but not c
1249	H. R. 1712	7100	B0	4686 and Pickering series stronger than in Class B. Maury division a-b
1333	χ Aur	6600	B1	Hydrogen weak and blurred; Hβ weak
1378	26 Aur	5500	A2	H lines very wide and hazy
1507	χ_2 Ori	6600	cB2	H and He with strong c-character
4548	67 Oph	7600* 9300†	cB5	Strong c-character. Williams line character‡ 0
....	H. R. 7081	7600	B3	H and He very wide and hazy
5004	ι Aql	8900*	B5	H strong, He weak, Mg+ just seen. Si+ strong, K weak. Not a c-star
5170	b_2 Cyg	8100*	B2e	Hβ bright on hazy dark line, other H hazy; He hazy but strong; K extremely shallow or absent
5208	P Cyg	6000	cB1	For remarks on this important spectrum see Section 75
5361	55 Cyg	5800 5500*	cB2	Mg+ strong; C+ conspicuous; Si+ just seen; H very sharp
5410	f_1 Cyg	7200	B0e	Bright lines
5471	υ Cyg	9000*	B3e	Lines extremely broad; Hβ bright; Maury division b, L
5512	69 Cyg	7400	B0	Closely resembles ε Orionis
5563	9 Cep	6700	cB2	He strong; Mg+, C+, N+, strong; K strong
5608	ν Cep	5500	cA2	Spectrum as α Cygni. Williams line character −1; M = −5:.
...	H. R. 9005	7400	B3	H and He wide and rather weak
5969	2 Cas	6400	A3	Lines wide and hazy; Maury line division a, b

‡ See H. C. 348, 1929. Line character is defined on p. 25 of that paper.

TABLE VIII, III.—REMARKS ON THE SPECTRA OF B AND A STARS WITH COLOR EQUIVALENTS GREATER THAN 1.6

Boss	Star	Temperature	Spectrum	Remarks
		°		
27	γ Peg	11900	B2	Normal; Maury line division a
122	..Cas	11000 13900*	B3	K shallow; H and He stronger than normal; Maury division a
1204	η Aur	10800	B3	Lines wide, hazy. Maury division ab
1303	γ Ori	12200	B2	Normal spectrum. Maury division a
1375	ζ Tau	11000	B3e	Bright lines
3281	κ Dra	11000 11600*	B5e	Bright lines. Maury division b, L
3566	H. R. 5191	10600 10200*	B3	H and He wide, hazy. Mg+ barely seen. Maury division b
4162	τ Her	11300 10300†	B5	Lines extremely wide. Maury division a. Williams line grade 3
4368	ζ Dra	10800 10200†	B5	Lines very wide and strong. Maury division a; Williams grade 2½
4479	ι Her	11200 12100*	B3	He, H wide and strong; Mg+ hardly seen; K line shallow
4590	102 Her	11400	B3	H and He strong and sharp, but not c. Maury division a
4897	..Lyr	12100 11200*	B3	H, He wide and strong; K line absent
5068	12 Vul	10600	B3	H, He wide, shallow; no others
5102	22 Cyg	10400	B3	Almost c. Mg+ imperceptible
5532	β Cep	11200	B1	Lines strong; medium width
5844	10 Lac	10600	Oe5	Lines weak, fine but not c
5944	α Peg	10800 10300†	A0	Lines very wide and hazy Hβ in emission

The uncommonly hot stars evidently tend to have strong, wide lines. For the cool stars the case is not so clear, but the majority are either c-stars or bright-line stars (real or potential). Moreover all the stars in Table VIII, II have lines which Miss Maury assigned to divisions a and b; in the group of cool stars fall all the c-stars, and also a number of stars which Miss Maury designated a, b. I suspect that an uncertainty as to line division arose from the unusual contour of these lines, which are on the verge of emission, and appear flattened. The reality of the division a, b is indicated not only by the line forms

in the stars assigned to the class but also by the next two groupings of the data. The first obtains the mean values of c_2/T and hence of temperature for the stars of Miss Maury's various divisions in the list used in Tables VIII, II and VIII, III:

Division	a	ab	b	c	a, b
c_2/T	1.28	1.35	1.41	2.41	2.11
Number of stars	7	1	7	7	6
Temperature	11400°	10800°	10400°	6100°	6900°

The next compares the same line qualities for different intervals in c_2/T, weights being assigned as follows: $c = 5$; $ac = 4$; $b = 3$; $ab = 2$; $a = 1$.

Interval in c_2/T	1.05 to 1.35	1.35 to 1.65	1.95 to 2.25	2.25 to 2.55	Above 2.55
Mean line quality	1.8	2.7	4.0	4 0	5.0

These two tabulations embrace only the data for the abnormally cool and hot stars. Using *all* the B stars in Hertzsprung's catalogue for which Miss Maury has determined the line character (not only those of extreme temperature) we obtain the following means:

Division	a	ab	b	c	a, b
Mean c_2/T	1.28	1 17	1.25	2.22	1 45
Number of stars	13	3	11	2	8
Temperature	11400°	12400°	10700°	6600°	10000°

We are now in a position to ask and answer several very important questions: (1) Are all cB stars of low temperature? (2) Have all low-temperature B stars the c-character? (3) Are the temperatures real in the sense that they are adequate to produce thermally the observed spectra? or (4) are agencies other than temperature and pressure required to produce the spectra observed?

(1) All the c-stars in group B (Bo to B5) whose temperatures have been measured are of low temperature; moreover not one entry in Table VIII, III (high temperatures) is a c-star.[13]

[13] For Class A, discussed in Chapter X, there seem to be both low and normal temperature stars with the c-character. The former are exemplified by Boss 781 and 786, the latter by α Cygni and β Orionis. Absence of such exceptions for group B is probably only a matter of observational selection, especially as the temperature of the c-stars of Table VIII, II seems to be correlated with apparent magnitude, and stars as bright as Rigel are uncommon.

(2) By no means all of the low-temperature B stars show the c-character, as is very evident from Table VIII, II. Nor is it possible to refer the hazy lined spectra in question to rotation, or the motion of a spectroscopic binary; the lines are not only too hazy but also intrinsically too wide for their depth; their quality suggests a state of "incipient emission." But the case of bright-line stars (real and potential) is rather different from that of the c-stars—we need only to recall that there are several bright-line stars in Table VIII, III (high temperature).

(3) It can be shown that the temperatures are not real in the sense of being adequate for the thermal production of the observed spectra; the matter is taken up in Section 42.

(4) Another agency may be invoked to produce either the high ionization or the low energy temperature: an exciting and ionizing effect at the surface of the star; or a reddening agent external to the star—perhaps, though not of necessity, near to it.

Before discussing the general conditions underlying the spectrum of the cB star, we shall summarize the data that can be obtained from the best coordinated source of astrophysical data—the star cluster. These important conclusions on luminosities and the source of bright lines are needed before the general discussion can be undertaken.

40. Spectra of High Luminosity B Stars in Galactic Clusters.—There are two bright galactic clusters that contain considerable numbers of bright-line B stars—the Pleiades and the double cluster in Perseus, and the latter contains in addition the most concentrated group of c-stars in the sky. The clusters have points of similarity in their spectrum-magnitude curves (both are classed Ib by Trumpler[14]) and in their structure, with the brighter early-type stars crowded toward the center. They present a contrast, in that the Pleiades are enmeshed in nebulosity, and the Perseus clusters free of it.

[14] P. A. S. P., **37**, 307, 1925.

The Perseus group seems to divide into a high luminosity section (stars brighter than the ninth magnitude), containing the c-stars; an emission line section (stars between 9.5 and 10.5, approximately); and an absorption line section of normal stars (fainter than 9.5). The Pleiades lack the c-star section; their emission line stars extend over the brightest magnitude and a half, and the remainder of the members have normal absorption spectra.

The adopted parallax of $0''.015$ for the Pleiades[15] allows us to conclude that the emission line section extends from absolute magnitude -2.4 to -3.8. Unfortunately the distance of the Perseus clusters is not known, but by superimposing their spectrum-luminosity curve on that for the Pleiades at Class A we obtain about the same range of absolute magnitude for the emission stars, and from about -4 to -7 for the c-stars.[16]

The range in mean absolute magnitude for B1 to B3 stars (Table VII, I) is from -0.97 to -2.29; converted into emission-star absolute magnitudes by adding Gerasimovič's $-2^m.4$ (derived, it is true, for only one of these spectral classes, but probably similar for all) these become -3.4 and -4.7. The range for the emission stars in the clusters, from -2.4 to -3.8, is near enough and similar enough to be suggestive. The uncertainty of Gerasimovič's value is large enough to permit them to coincide. We conclude that the occurrence of bright lines in B stars may be governed by luminosity.

The plot of the distribution of emission B stars over the sky shows that there are no other conspicuous clusterings, so that a further test of the luminosities cannot be made in this way. Therefore we should hesitate before drawing positive conclusions. It can be said, however, that there is no reason for doubting the high luminosity of emission B stars because in

[15] Trumpler, L. O. B. 420, 1930.

[16] This value is not absurdly high when we compare it with the brightness of Canopus, or the A and K stars in the Magellanic Cloud. See Section 17, where the highest accredited luminosities are discussed. Shapley's modulus (17.0) gives these values (H. Mon. No. 2, 1930); Trumpler's distance leads to a modulus of 15.6, making all the stars $1^m.4$ fainter (L. O. B. 420, 1930).

some galactic clusters there are still brighter stars without bright lines, since the latter are of enormous luminosity.

The temperatures of the Pleiades have been repeatedly measured[17]; apart from some minor but well-accredited differences they are not abnormally low—Merope is the coolest of the B5 spectra, and has in addition the strongest emission. The Perseus clusters need a serious spectrophotometric study; there are indications in the work of Balanowsky[18] that the c-stars (the brightest members of the clusters) are abnormally cool, but quantitative estimates of the temperature cannot be made from his discussion. This observation is in line with the data of Table VIII, I, and it encourages us to believe that the low temperature of the cB star, whatever its cause, is physically connected with the condition of the atmosphere or the surroundings, and is not an effect of space reddening or such process. If it were otherwise the fainter stars of the cluster would inevitably be abnormally red also—which is apparently not the case.

It is not easy to estimate the degree to which the two clusters on which our data are fullest are typical; they are members of a large group, about half the clusters with known spectra being of the Pleiades class, and about one fifth closely resembling the Pleiades in spectral makeup. We cannot assume that the spectrum-magnitude relation is quantitatively similar for all these clusters because the general shape of the relation is the same; but the two specimens at present examined are similar in this respect, (although they differ in the important matter of visible nebulosity), so that a general similarity seems not unlikely.

Two of the bright galactic clusters thus furnish, without need of generalization, a definite value for the absolute brightness of their emission B stars and cB stars—the only assumptions made have been the legitimacy of superimposing the common portions of their spectrum-magnitude curves, and the

[17] See, for instance, Hertzsprung, Mem. Dan. Acad., 4, 349, 1923.

[18] Pulk. Bul., 9, 277, 1924.

correctness of the adopted parallax. If we can go further and consider them other than abnormal, we can draw a parallel between the range in luminosity that produces emission lines and the range in spectral class in which emission lines most readily occur. This is a valuable step, for it enables us to compute the values of surface gravity most favorable to the maintenance of emission lines, on various assumptions as to temperature; for if emission lines occur preferentially in one spectral class (or at one luminosity, which comes to the same thing statistically in Class B), the surface conditions of the stars must be immediately responsible.

41. Spectrophotometric Data for Class B.—The physical picture of the B star contained in the previous sections must be supplemented by spectroscopic measures. The tabulations of the present section represent the few measures at present available for stars of group B. The procedure, and the general significance of the entries, are outlined in Chapter III. The mean percentage light losses are directly converted into logarithms of *NH* (Table VIII, V) by the method of Section 16.

TABLE VIII, IV.—MEAN PERCENTAGE LIGHT LOSSES FOR B STARS

Class	O (4)	B0 (1)	B3 (2)	B5 (6)	cB0 (2)	cB5 (7)
Hβ	18	30	36	41	32	28
4686	.	..	..	.	23	..
4481		..	..	..	4	..
4471	.	..	..	..	14	..
Hγ	29	44	46	49	33	37
Hδ	31	43	46	56	48	39
4026		26	..	17	16	18
Hε	31	37	50	57	58	44
K	(18)	.	(19)	..	(18)	(21)
Hζ		34	44	59	47	45

The entries for the K line are enclosed in parentheses because the ionized calcium is (at least preponderantly) stationary in stars of these classes.

TABLE VIII, V.—MEAN LOG *NH* FOR STARS OF GROUP B

Class	O	Bo	B3	B5	cBo	cB5
Hβ	17.24	17.60	17.78	17.94	17.66	17.54
4686	. .	.....	.....	.. .	17.38	.....
4481	.	.....	.....	..	16.9	.....
4471	.... .	.....	.. ..	.....	17.12	.....
Hγ	17.56	18.02	18.08	18.18	17.70	17.82
Hδ	17.63	18.00	18.08	18.39	18.16	17.88
4026	.....	17.48	.. ..	17.22	17.19	17.24
Hε	17.63	17.82	18.21	18.42	18.46	18.02
K	(17.24)	.....	(17.28)	.....	(17.24)	(17.34)
Hζ	.....	17.72	18.02	18.49	18.12	18.06

Contour data also are available for a number of stars of group B, as summarized in Table VIII, VI.

TABLE VIII, VI.—MEAN HALF WIDTHS OF LINES OF STARS OF GROUP B

Class	Hγ			Hδ			Hε		
	r = 4	17	31	4	17	31	4	17	31
O (4)	4.3	1 8	1.0	4.6	2.3	1.2	4.5	2.2	1.4
B3 (2)	11.9	6.6	4.4	11.2	5.2	2.3	13.2	6.0	3.3
B5 (5)	18.1	8.2	4.5	16.0	7.6	4.2	18.2	9.4	5.7
cB5 (6)	10.5	3.8	1.1	8.7	4.3	1.8	11.0	4.8	2.5

The numbers of atoms deduced from the measured contours are tabulated in Table VIII, VII. The obvious differences between the numbers of atoms tabulated in this table and Table VIII, V are caused in part by real changes in the contours between Classes F, G, and K (where line depth was calibrated) and Class B, and in part by the small resolving power used.

TABLE VIII, VII.—MEAN LOG *NH* FOR STARS OF GROUP B, FROM CONTOURS

Class	Hγ	Hδ	Hε
O	17.54	17.64	17.66
B5	18.82	18.74	18.87
cB5	18.03	18.16	18.28
η CMa	17.85	17.96	18.13

The matter is discussed in Chapter IX in the study of the Stark effect, and reference should also be made to Section 11 in Chapter III, which deals with instrumental influences on the depths and contours of lines.

42. Physical Condition of Supergiant B Stars.—The condition of a stellar atmosphere is governed by (1) the temperature; (2) the surface gravity (the pressure, both total and partial electron, is a function of the surface gravity); and (3) other possible but unknown effects. If we know the effective temperature and the absolute magnitude of a star, we can pass to its surface gravity by way of the relation

$$L = \pi acR^2T_e^4$$

where L is the luminosity, a is the Stefan constant, R the radius, and T_e the effective temperature. Table VIII, VIII shows the value of the surface gravity for a series of different masses and temperatures, mass and luminosity being connected empirically by the mass-luminosity law. For the present table the data for the mass-luminosity law were replotted, and the best curve drawn.

The surface gravities for the apparent upper and lower limits of the c-stars and the emission B stars in the Perseus clusters are given for several temperatures in the following tabulation. It is noteworthy that the range of luminosity of three magnitudes for the c-stars produces no greater range in surface gravity than the range of a magnitude and a half for the bright-line stars.

Temperature °	c-stars (−7 to −4)	Bright-line Stars (−4 to −2.5)
8000	2.5 to 2.7	2.7 to 2.9
10000	2.9 to 3.1	3.1 to 3.4
12000	3.1 to 3.3	3.3 to 3.6
15000	3.5 to 3.8	3.8 to 4.1
20000	4.0 to 4.3	4.3 to 4.6

A general evaluation of an upper limit to the temperatures of the c-stars may be made by considering the "Stark effect," discussed in Chapter IX[19]; it is shown there empirically that

[19] P. 132.

TABLE VIII, VIII.—SURFACE GRAVITIES FOR STARS OF VARIOUS MASSES AND EFFECTIVE TEMPERATURES

Mass	⊙×0.3	0.5	0.8	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	20.0
M (bol)	+9.2	+7.2	+5.3	+4.4	+1.6	+0.1	−0.7	−1.4	−1.9	−2.2	−2.5	−2.7	−2.9	−4.3
eff. T °														
3000	4.63	3.95	3.39	3.13	2.32	1.90	1.68	1.51	1.40	1.33	1.31	1.23	1.19	1.11
4000	5.13	4.45	3.89	3.63	2.82	2.40	2.18	2.01	1.90	1.83	1.81	1.73	1.69	1.45
5000	5.51	4.84	4.28	4.02	3.21	2.79	2.56	2.39	2.29	2.22	2.20	2.12	2.08	1.84
6000	5.83	5.15	4.60	4.34	3.52	3.10	2.88	2.71	2.60	2.54	2.52	2.44	2.40	2.15
7000	6.10	5.42	4.86	4.61	3.79	3.37	3.14	2.98	2.87	2.81	2.78	2.70	2.66	2.42
8000	6.33	5.65	5.09	4.84	4.02	3.60	3.38	3.21	3.10	3.04	3.02	2.94	2.90	2.65
9000	6.54	5.86	5.30	5.04	4.23	3.81	3.58	3.42	3.31	3.24	3.22	3.14	3.10	2.86
10000	6.72	6.04	5.48	5.22	4.41	3.99	3.77	3.60	3.49	3.43	3.40	3.32	3.28	3.04
13500	7.24	6.56	6.01	5.75	4.93	4.51	4.29	4.12	4.01	3.95	3.93	3.84	3.80	3.56
15500	7.42	6.74	6.19	5.93	5.11	4.69	4.47	4.30	4.20	4.13	4.11	4.03	3.99	3.75
17500	7.69	7.01	6.46	6.20	5.38	4.96	4.74	4.57	4.46	4.40	4.38	4.30	4.26	4.01
20000	7.92	7.24	6.69	6.43	5.61	5.19	4.97	4.80	4.70	4.63	4.61	4.53	4.49	4.25

the lines suffer a (Stark) widening when the surface gravity is greater than 10^4 centimeters per second per second. As the Stark effect is absent for the cB stars their temperatures are probably not above 15000°; 17000° is the formal limit, but the ionization of hydrogen, the preponderant element, will somewhat decrease it by contributing free electrons and increasing the partial pressure. In the same way a lower limit of 14500° can be set for the normal B star of absolute bolometric magnitude −2.5, as the Stark effect is present in its spectrum, and the lower limits for the means of the B classes are found on the same basis to be:

Class	B0	B1	B2	B3	B5
M	−2.65	−2.29	−1.94	−0.97	−0.66
M_{bol}[20]	−3.41	−2.93	−2.58	−1.61	−1.10
T_{ll}	15000°	13500°	13500°	10000°	10000°

[20] We need to know the temperature to correct from the absolute visual magnitude of Table VII, I to the absolute bolometric magnitude, and the result is thus approached by successive approximations. The corrections to absolute bolometric magnitude were obtained by interpolation from Eddington's table (The Internal Constitution of the Stars, 137, 1926), assuming 13200° for B0 and 12300° for the other four classes. A second approximation would raise the lower limiting temperatures a little, but the effect would be inappreciable to the order of accuracy to which the table is expressed.

If a star of Class B has a temperature lower than the limits just assigned for the corresponding class, it must on this basis be (*a*) a c-star or (*b*) of low luminosity for its spectrum. Seventeen of the thirty stars of abnormally low temperature in Table VIII, II have hazy lines, and therefore fall in the second category. On this criterion ζ Persei, if really of temperature 5800°, would have an absolute magnitude of about +1.6. But it seems fairly clear that the luminosities thus reached are not plausible, and a third alternative presents itself—one already commended by observations of other kinds—that the low temperatures are only apparent. Before discussing this hypothesis at length I shall indicate some other considerations that also point to it.

43. Ionization in the Atmospheres of Supergiant B Stars. Can the spectrum of a B3 star be produced under any combination of normal ionization circumstances at a temperature of 7000°? Using the ordinary formulae the required degree of ionization is reached at about 10^{-13} atmospheres,[21] a pressure that seems improbably low. The required conditions can furthermore be shown to be not only unlikely but also impossible by the consideration of the measured line intensities and the approximate values of log NH given in Table VIII, V and VIII, VII.

The spectral class B3 has been chosen for argument because the helium lines have their maximum near that class. The ionization curves for helium are reproduced in Figure VIII, 4; the corresponding curves for the lines of any substance (H, C+, O+) with an excitation potential comparable to the ionization potential are similar. At a temperature of 15000° the helium lines have their maximum at a pressure of $10^{-5.5}$, and the fractional concentration of helium giving the observed spectrum is about $10^{-8.4}$. At a temperature of 7000° the maxi-

[21] All the elements entering into the B3 spectrum are *below* their maximum at 7000°, so that Milne's suggestion of the absence of an absolute magnitude effect does not come into play. The discussion of the normal B spectrum should be compared and contrasted with the present one.

mum occurs at the low pressure mentioned, with a fractional concentration of about $10^{-14.5}$; the ratio in fractional concentration for stars at 15000° and 7000° is about 10,000,000. If (as we usually assume) the helium content of the atmospheres of stars is always the same, we should then expect the helium

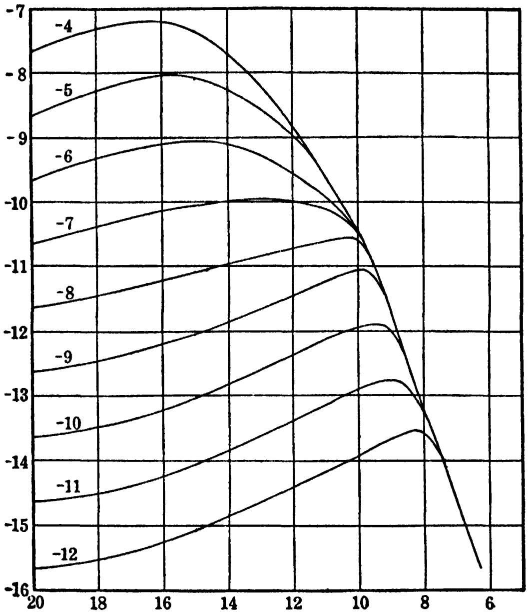

FIGURE VIII, 3.

Ionization of helium. Ordinates are values of log n_r, where n_r is the fractional concentration of neutral helium (4471); abscissae are temperatures. The various curves correspond to the attached values for log P_e, the partial electron pressure. The curves are computed on the basis of the old Fowler-Milne theory.

lines to be enormously more intense for the normal star than for the cool one—but any possible observed effect in this direction is not of the required order.

Besides this comparison the matter may receive a direct test. For the normal B star, the value of *NH* for hydrogen is of the order of 10^{19}, and therefore for the abnormally cool star

NH should be about 10^{12}. It has been shown in Section 16 that the lower limit for the detectability of a line on the one-prism Harvard plates is about 10^{16} atoms per square centimeter of surface, so that the abnormally cool spectrum should show no lines of hydrogen—or indeed any other substance. For η Canis Majoris, the cB5 star with the sharpest lines hitherto observed, *NH* is $10^{17.85}$ for H δ; this cool star with sharp lines has apparently a million times as many atoms of hydrogen as can be expected from the temperature and pressure.

It may be concluded with fair certainty that the observed low temperatures of cB stars are inadequate to produce the corresponding spectra by thermal ionization. The inference is (1) that the temperatures are real, but there is some additional exciting and ionizing cause; or (2) that thermal ionization only is operating and that the observed temperatures have suffered distortion. The next section discusses these alternatives.

44. Possible Interpretations of the Cool B Stars.—A problem so central in modern astrophysics cannot at present be conclusively discussed; crucial data are not available. I can only discuss the reality of the observed effect, and point out the bearings of my own data on the two main suggestions: (1) the measured temperatures are not real, and (2) there is some additional ionizing and exciting effect.

There can be no doubt that the measures quoted above of apparent temperature for the cool B stars have been accurately made. We may refer to the numerous determinations of the temperature of ζ Persei and P Cygni. The coolness of the stars appears both from measures of color equivalent and of spectral energy distribution. In both cases the temperatures are derived on a black-body assumption, and in neither case have measures been made outside the range from 3600A to 6400A. The most effective portion of the spectrum—the ultra-violet—is not measured, and thus the resulting information on the relative deficiency of this highly potent radiation is indirect.

There is, however, a source of information that is based on a spectral region much further to the violet—the color indices photoelectrically determined by Bottlinger.[22] Twenty-two stars are common to his list and that of Gerasimovič,[23] and the values of c_2/T (scale of Wilsing) for the former are compared in Figure VIII, 3 with the latter's gradients (based on Ao = 10,000°). The values are definitely correlated, but the very

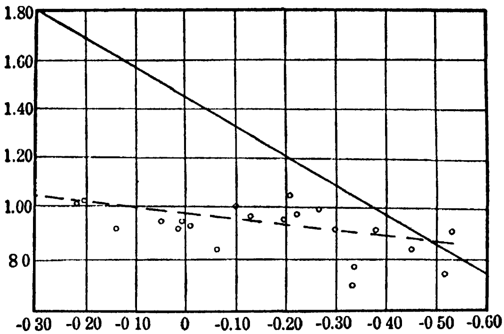

FIGURE VIII, 4.

Comparison of the temperatures derived by Gerasimovic and Bottlinger for abnormally cool B stars. Abscissae are Gerasimovič g gradients; ordinates are Bottlinger's values of c_2/T. The full line represents the expected relation if the temperatures are the same by the two methods; the broken line represents approximately the observed relation.

low temperatures inferred by Gerasimovič for the coolest of the B stars are represented in Bottlinger's list by values consistently greater than for Class Ao (putting the comparison in this way avoids the numerical uncertainty of Gerasimovič's zero point).

The interpretation of this diagram has already been pointed out by Miss Williams[24] for the A stars: it means that the blue portion of the "cool" spectra is depressed with respect not only to the redder portion of the spectrum but also to the violet

[22] Veröff. Berlin-Babelsberg, 3, No. 4, 1923.

[23] H. C. 339, 1929.

[24] H. C. 348, 1929.

regions.[25] The maximum dip in the energy curve (which must be regarded as established by this comparison of the colors of stars determined at Harvard and Babelsberg) does not extend to the violet beyond about 3600A (cf. diagram, given by Gerasimovič, H. C. 339, 1929), since the sensitive point of Bottlinger's cell is rather to the violet of this. The implied distortion of the spectrum should be detectable, therefore, for well-exposed spectrum for the violet, and it was actually found by Gerasimovič in the "ultra-violet appendage," having previously been noted in P Cygni and other bright-line B stars by C. S. Yü.[26]

On my view there is no *appendage* in the ultra-violet; the spectra suffer a "violet depression." When this depression is strong the apparent temperature (as derived from the comparison of the violet and the yellow) is lowered—an observed fact which Gerasimovič, with his differing view of the meaning of this phenomenon, urged against a connection with temperature. Evidently the maximum of the depression is to the red of Bottlinger's sensitivity maximum, but the depression is still in effect at that wave length, hence the slight correlation between his temperatures and those of Gerasimovič.

The correlation of the ultra-violet appendage with spectral class is pointed out by Gerasimovič as follows: "In the spectra of stars of Class B5 the appendage is not present; it is just visible in Class B3, and reaches a full development in Class B0 to B2 . . . the extension is very noticeable for O stars . . ."

Our view of the true spectrophotometric temperatures of the B stars is reached by successive approximations. We are now able to envisage the B classes increasing somewhat in temperature from B8 to B0, and displaying in their spectra an "ultra-violet depression" that sets in at B3, and is strongest

[25] A similar conclusion is reached by Öpik (Publ. de l'Obs. Astr. de l'Univ. de Tartu, **27**, No. 1, 1930): ". . . most complicated seems to be the relation between photoelectric and ordinary color index of early type stars. The complications are to be attributed probably to selective absorption in different spectral regions which make the intensity distribution in the spectrum deviate from a black-body distribution."

[26] P. A. S. P., **39**, 112, 1927.

at B0. This depression seemingly increases in strength with high luminosity (both along the sequence of B stars and also within each spectral class)[27]; it is probably present also in the spectra of some of the A stars with very low spectrophotometric temperatures. Unfortunately the true spectrophotometric temperatures are approached, so to speak, asymptotically with decreasing depression, and we are ignorant of the smallest amount of depression present in a stellar spectrum of Class B0. Therefore the highest temperatures recorded for such stars may be regarded as lower limits. For an estimate we compare the lower limit thus derived from the violet depression (14600°) and from the Stark effect (14500°); the agreement is accidental, but the order of the result is encouraging, and about 15000° offers a useful compromise for the temperatures of stars of Class B0. As was shown in the previous chapter, no greater temperatures are required to account for the observed spectra.

It seems best to leave the violet depression as an empirically established fact, and not to attempt to interpret it; the concomitant conditions have been shown to be high temperature and low surface gravity. Both Gerasimovič and Miss Williams have considered as an interpretation the hydrogen absorption beyond the Balmer limit, but its maximum at B0 is then perhaps surprising.

The abnormal reddening of stars of early type has also been discussed by several in its connection with the reddening of light in passing through interstellar space. The distribution of the reddened stars is not closely parallel with that of recognized obscuring nebulosity, though both are, of course, concentrated to the Milky Way. The suggestion has been made by Greaves, Davidson, and Martin[28] that the reddening might be caused by a nebula associated with the Eddington-Struve[29] calcium cloud. The stars are not, however, distributed in the

[27] Unpublished material at Harvard on the white dwarf Class A companion of o_2 Eridani shows an almost complete absence of the depression.

[28] M. N. R. A. S., **90**, 104, 1929.

[29] Gerasimovič and Struve, H. Repr. 56, 1929.

sky in the same way as Struve's stars with strong calcium; there is no concentration of them in his most densely populated spot, nor in Lacerta.[30] If the association were with anything we might expect it to be with the nebulosity that is suspected to lie behind the calcium cloud; we note, however, that there is as yet no observational evidence that stars seen *through* nebulosity are reddened[31] and that theoretically almost any form of spectral distortion can be induced by particles of appropriate size. But the thought most fatal to space reddening is that if the effect is so large for fourth- and fifth-magnitude stars, faint blue stars in the Milky Way would be unobserved; and this is contrary to the facts.[32] The decision seems to place the cause of the observed reddening at the surface of the star itself.

The change of view in regard to the reddening as a result of a violet depression has one important effect on theory—it does away with the need of the superexcitation phenomena suggested by Gerasimovič in explaining the observed spectra; but of course it does not demonstrate that such phenomena do not and cannot occur.

[30] Struve, Ap. J., **67**, 388, 1928.
[31] Russell, Dugan, and Stewart, **2**, 823, 1926.
[32] Shapley, Mt. W. Comm. 37, 1917.

CHAPTER IX

THE NORMAL A AND F STARS

STARS of group A (Classes B8 to A3) and group F (Classes A5 to F2) comprise between them more than one third of all stars brighter than apparent magnitude 9; there are about three times as many stars in group A as in group F brighter than this limit.[1]

45. Occurrence and Distribution.—Physical data describing the means of the classes in groups A and F are summarized in Table IX, I. The space number is the computed number[2] per 1,000,000 cubic parsecs.

TABLE IX, I.—STARS OF GROUPS A AND F

Class	Absolute Magnitude			c_2/T	Number of Stars		Space Number[7]
	Shapley and Cannon[3]	Luyten[4]	Pannekoek[5]	Hertzsprung[6]	in H. D.	[8.25	
B8	0.0	. . .	−0.2	1.44	. . . .	1,604	
B9		. . .	0.7	1.50	. . . .	2,752	
Ao	0.6	0.0	0.7	1.48	. . .	6,320	250
A2	1.2	0.8	1.5	1.59	. . . .	5,208	
A3		1.2	2.4	1.80	. . . . .		
A5	1.7	2 0	. . .	1.89	2,503	1,352	
Fo	2.4	. . .	. . .	2.20	5,377	3,208	680
F2	2.8	. .	. . .	2.47	3,576	1,976	

[1] Cf. Miss Cannon, H. B. 862, 1928.
[2] H. Repr. 6, 1924.
[3] *Ibid.*
[4] H. B. 797, 1922.
[5] Pub. Astr. Inst. Amsterdam, **2**, No. 2, 1929.
[6] Leiden Ann., **14**, 1922.
[7] Shapley and Cannon, H. Repr. 6, 1924.

The brighter A stars are not so conspicuously grouped as the brighter B stars,[8] and they are not irregularly distributed in galactic longitude, but they show distinctly the influence of the local cluster.[9] Fainter A stars (7.0 – 8.25) are more strongly concentrated to the Galaxy proper, though still fairly uniformly distributed in longitude, and they show regional vacancies. The galactic concentration of the A stars classified in the zone +50° to +60° (more than one third fainter than the ninth magnitude, and one half fainter than magnitude 8.7) is even more conspicuous, and is well illustrated by a diagram given by Miss Cannon.[10]

The tendency to clustering shown by the fainter A stars is associated with their numerous membership in galactic clusters, such as the Pleiades, N. G. C. 3532, and Messier 7. Pannekoek[11] calls attention to some less obvious associations of faint A stars, such as the one in Orion. All the details of the distribution of the A stars are the natural consequence of their moderate brightness and comparative commonness.

The stars in Group F,[12] though slightly concentrated in the Milky Way, display no regional vacancies, and their infrequent clustering is associated with very nearby galactic clusters like the Hyades. These features point to luminosities lower than for the A stars, as shown in Table IX, I, and therefore most of the classified F stars must belong to the main sequence.[13] From directly measured distances it appears furthermore that there are probably no "normal giant" F stars; the most luminous F stars are supergiants, conspicuous in the sky, and well represented in catalogues, but statistically negligible in space.[14]

The A stars are important for many reasons: their position in the spectral sequence is strategic, linking giant and dwarf,

[8] Shapley and Miss Cannon, H. C. 239, 1922.
[9] Shapley and Miss Cannon, H. C. 229, 1922.
[10] H. B. 862, 1928.
[11] Pub. Astr. Inst., Amsterdam, 2, No. 2, 1929.
[12] Shapley and Miss Howarth, H. C. 285, 1925.
[13] Shapley and Miss Cannon, H. Repr. 6, 1924.
[14] θ_2 Tauri, often quoted as a giant F star, is actually of Class A5.

and also forming a natural boundary between "hot" and "cool" stars. Their occurrence in galactic clusters[15] gives them a potential value in determining parallaxes, if their brightness can be established. The whole scale of stellar temperatures has moreover come to hinge on the A stars—for the ionization scale because the hydrogen maximum was first used to fix its zero,[16] and for the energy temperature scale because the A stars form the most convenient standards of magnitude (they are bright, common, and of smooth and unambiguous background in most of the photographed spectral region). They also fix the zero point of the system of color indices. The cluster type variable, fundamental in the measurement of cosmic dimensions, is an A star of peculiar properties and problems.

The important problems of Class A are thus the question of their luminosities, of their temperatures, and of the physical condition of their atmospheres, and each of these questions will now be briefly reviewed.

46. Luminosities of Class A Stars.—The mean absolute magnitudes of the A stars, given in Table IX, I, can be used in the estimation of spectral parallaxes only if the dispersion in M is small, or if a large number of A stars are present in a group, and the form of the luminosity curve is known for the group, so that a distance modulus can be derived.

It can be seen at once that the dispersion in M is not small. Table IX, II contains the relevant data for five bright galactic clusters; under each spectral class are tabulated the brightest and faintest magnitude found among the cluster stars with the corresponding spectrum, and the number of stars included. The data are taken from Raab's compilations,[17] which he attempted to free from non-cluster stars; the danger is rather that they are under-representative than that they include

[15] Shapley, H. Mon. No. 2, 1929.
[16] Fowler and Milne, M. N. R. A. S.,
[17] Lund Medd. Series 2, No. 28, 1922.

TABLE IX, II.—CLASS A STARS IN GALACTIC CLUSTERS

Cluster	B8		B9		A0		A2		A3		A5	
	No.	Limits	No.	Limits	No.	Limits	No.	Limits	No.	Limits	No.	Limits
		m *m*		*m* *m*		*m* *m*		*m* *m*		*m* *m*		*m* *m*
N. G. C. 3532	4	8.0– 8.9	12	8.2– 9.7	50	8.2–10 3	26	8.4–10.2	..	.. ..	..	.. ..
N. G. C. 2516	5	6.4– 8.4	4	7 0– 8.6	17	7.7– 9.4	..	.. .	..	..	..	. ..
Praesepe	0	.. ..	0	.. ..	9	6.5– 9.1	7	6.3– 9.5	7	6.2– 9.5	8	6.7– 9.5
N. G. C. 3114	..	. ..	14	8.3–10 1	13	8.3–10.0	..	.. ..	..	.	..	. ..
Messier 7	3	6.0–10.7	19	6.1–10.9	42	6.9– 8.9	8	8.7–10.7	5	8.7–10.0	..	.. ..

interloping stars. They suffice amply to prove the point that the dispersion in absolute magnitude within Classes B8 to A3 is at least two magnitudes. For several of the entries the maximum of the magnitude-frequency curves has evidently not been reached at the faintest available magnitude, but it seems as though the data are fairly complete for N. G. C. 3532 and Messier 7; at A0 these clusters are very populous and the large dispersion is in no doubt at all. Color-magnitude arrays for globular clusters show a comparable scattering and dispersion in the brightness of the A stars.

The apparent brightness of individual A stars will obviously not suffice to tell of their distances; we require either to know the frequency distribution on apparent magnitude for a group plausibly all at the same distance, and an assurance that our adopted mean absolute magnitude refers to its maximum (or is removed a definite distance from it),[18] or to have a more refined method of telling the brightness of an individual A star (such as the spectroscopic method of Miss Williams, referred to later).

The large dispersion in brightness of A stars is in sharp contrast to the undoubtedly small dispersion in the magnitudes of the cluster type variables—a matter with important practical consequences for measurement of distances, and of some theoretical significance in the matter of stellar variability.

47. The Temperatures of the A Stars.—Ionization theory has (somewhat uncertainly) assigned a temperature of 10000° to Class A0, and that temperature (sometimes coupled with an alternative 13000°, considered as giving a measure of uncertainty) fixes the zero point of our present color-index and energy-distribution temperatures. A critical investigation of the rectitude of this temperature is greatly to be desired; without it we can discuss only relative temperatures.

[18] Cf., for instance, the double maxima found for A stars by Malmquist (Lund Medd. Series 2, 46, 1927) and Wallenquist (Proc. Fourth Pac. Cong., Java, 1929; Pub. Bosscha Obs., 3, Part 2, 1929.)

The completest compilation of temperatures of A stars is that recently made by Miss Williams.[19] It is reproduced in part in Table IX, III, where successive columns give the class, number of stars, c_2/T on Hertzsprung's scale, and the corresponding temperature.

TABLE IX, III.—TEMPERATURES OF A STARS

Class	Number of Stars	c_2/T (Hertzsprung's Scale)	Temperature
			°
B5	8	1.47	9950
B8	8	1.49	9800
A0	33	1.56	9400
A2	17	1.73	8400
A5	7	1.89	7700
F0	10	2.10	6950

Miss Williams finds that the classes of A stars are very homogeneous in temperature—an effect strongly contrasted with the absolute magnitudes of the same stars. The boundaries of temperature (which are respected, almost without exception, by the non-peculiar A stars of her list) are summarized in the following tabulation: the first column names two spectral classes, the second, the value of c_2/T that forms the lower limit of the first, the upper limit of the second.

Limiting Classes	Boundary in c_2/T
F0, A5	2.00
A5, A3	1.84
A3, A2	1.81
A2, A0	1.64
(A0, B8)	(1.51)
(B8, B5)	(1.46)

The two boundaries in parentheses are less certain, and more stars fall outside them than for the first four entries of the table (perhaps intrinsically, perhaps from observational errors). At least within group A, the Henry Draper classification is a temperature classification. It is noteworthy that Bottlinger's

[19] H. C. 348, 1929.

color equivalents also have a very small dispersion within one spectral subclass.

48. The Spectra and Physical Condition of the A Stars.—The strong lines of hydrogen which characterize the class are wider and more intense than the absorption lines of any other substance—indicating, on the Unsöld scale,[20] about 10^{20} effective atoms per square centimeter of surface. Lines of such breadth are difficult to estimate accurately, and so it comes about that the spectral criterion in the A classes is chiefly the next most conspicuous line—the K line of ionized calcium.[21] In addition the A stars show faintly the stronger metallic lines that are present in the spectra of the F stars, helium lines are present in Classes B8 and B9, and the Mg+ line throughout the group. In certain exceptional stars the helium lines persist also in classes later than B9.

The most complete quantitative spectroscopic study of the A stars is one recently published by Miss Williams.[22] Its chief points of significance here are the temperature and pressure effects, discussed on the basis of measures of the contours and total absorptions of the hydrogen and calcium lines.

The K line of ionized calcium is found to be the determining criterion of spectral class. From comparison with measured energy temperatures the resulting spectral classes appear to have very small dispersions in temperature. Recalling the large dispersion in luminosity within one A class we see that the intensity of the K line must be independent of the luminosity (thus of surface gravity and pressure), and a function of the temperature only. We note that constant intensity of the K line does not necessarily mean constant ionization of calcium—in the present case the ionization is much higher for the brighter stars at any one temperature.

[20] See Chapter III.

[21] This restriction turns out to have been a happy accident, for classification by the strength of the hydrogen lines would not have been a temperature classification.

[22] H. C. 348, 1929.

The effects of differences in luminosity appear in the form and intensity of the lines of hydrogen. Miss Williams published detailed contours of the lines for a large number of stars, showing statistically that the brighter the star the sharper and narrower the lines of hydrogen and the less their total absorption. Attempts had already been made by Adams and Joy,[23] by Miss Fairfield,[24] and by Miss Douglas[25] to obtain spectroscopic criteria for absolute magnitudes of A stars, and Miss Williams' measures place such attempts on a definite quantitative basis. The relation between line contour and luminosity within a given spectral class must at present remain empirical, and cannot be considered finally established until it is demonstrated for individual stars (such as the members of a cluster) instead of statistically. But there can be little doubt of its actuality.[26]

Miss Williams has shown very satisfactorily that the A stars represent a series of spectral classes each with small range of temperature and considerable dispersion in brightness, size, and surface gravity. The insensitivity of the K line to the latter factor is interpreted by Milne's theoretical "null effect."[27] Even the mysterious strength of the helium lines for A stars, not only for c-stars like α Cygni but also for stars like α Andromedae that are peculiar in other ways, also finds an explanation in the greater thickness of the helium atmosphere for bright than for normal stars at temperatures low compared to the maximum temperature for the helium lines.

The physical conditions at the surfaces of the stars of the spectral classes comprising group A may be summarized as follows:

[23] P. N. A. S., **8**, 173, 1922.

[24] H. C. 264, 1924.

[25] J. R. A. S. Can., **20**, 265, 1926.

[26] In confirmation of this analysis of line contour in its relation to absolute magnitude we note Moore's description (P. A. S. P., **10**, 229, 1928) of the spectrum of Sirius B (absolute magnitude +11); it is of later type than Sirius (A3 or A4), *but the hydrogen lines are broader;* Fe+, Ti+, Sr+, and particularly Mg+ are weakened. Unpublished Harvard material shows a similar effect for the dwarf companion of o_2 Eridani.

[27] H. B. 870, 1929.

Class	Adopted Temperature °	Adopted Absolute Magnitude Upper	Lower	Logarithm of Surface Gravity Upper	Lower
B8–9	9800	−1.0	+1.0	4.14	3.61
A0	9400	−0.6	+1.6	4.29	3.71
A2–3	8200	+0.2	+2 2	4.23	3.66
A5	7700	+0.7	+2.7	4 56	3.98

It is interesting that the range in surface gravity within one spectral class is far larger than the difference between the surface gravities for the mean of Class B8–9 and of A5 stars. The large and striking absolute magnitude effects are of another order of magnitude from the differences (caused by temperature contrasts) in spectral class.

49. The Cluster Type Variable.—Considered as a variable star, the cluster-type variable is properly found in Chapter XIV. But as part of its significance is missed by not considering it as an A star I shall summarize here some points not explicitly connected with the variability.

a. The exceedingly small dispersion in absolute magnitude is an interesting contrast to the dispersion of the normal A stars. The following brief tabulation summarizes the frequency of apparent median magnitude (in tenth-magnitude intervals) for the variable stars given in ω Centauri by Bailey[28] with periods less than a day.

Magnitude Interval	Number of Variables	Magnitude Interval	Number of Variables
13.0–13.1	1	13 5–13.6	30
13.1–13.2	0	13.6–13.7	27
13.2–13.3	1	13 7–13.8	12
13 3–13.4	6	13.8–13.9	1
13.4–13.5	11	13 9–14.0	1

Effectively all the variables are within the interval 13.3–13.8, or half a magnitude. Similarly the apparent median magnitude for the cluster type variables in Messier 72[29] is 16.80 ± 0.05 (p.e.), and for Messier 68,[30] 15.90 ± 0.02 (p.e.). The

[28] H. A., **38**, 1902.

[29] Shapley, Mt. W. Contr. 195, 1920.

[30] Shapley, Mt. W. Contr. 175, 1919.

dispersion is negligible, which forms the basis for the use of cluster variables, even when few in number, for the measurement of stellar distances.

For the galactic cluster variables the same fixity of median magnitude must be assumed. The dispersion shown by those toward the galactic center may all be plausibly attributed to the radial depth of the clouds they occupy.[31]

b. While very restricted in luminosity, the individual cluster type variable has a considerable range in color.[32] There are no extensive data on the relation of color and spectral class for these stars, but evidently their spectra also have a wide dispersion (as must be expected if the dispersion in color in clusters is real, and extensible to the galactic variables, and if the spectra of the latter are classified according to the usual criteria).[33] The data on the spectra of cluster type variables are meager, and restricted by the faintness of the galactic specimens, which are of course the only members of the class with attainable spectra.[34] The following table contains details on the classified spectra of galactic cluster-type variables. Stars marked with an asterisk were classified at Mount Wilson[35] at an unspecified phase. The (remaining) Harvard values[36] refer to adopted median spectrum; the variations of the spectra of some of the brighter cluster type variables are similar to those of Cepheids of longer period. There is no obvious relation between spectrum and period; no relation of color to period has moreover been noted in the cluster data available.[37] It

[31] Shapley and Miss Swope, H. Repr. 52, 1928.

[32] Seares and Shapley, Mt. W. Contr. 159, 1918.

[33] It is quite possible that the strength of the hydrogen lines plays a disproportionately large part in the spectral classification of such faint stars, especially on Mount Wilson plates.

[34] Spectra of variables in Messier 13 have been given by Pease (Mt. W. Rep. **9**, 219, 1914; **10**, 268, 1915) and by Adams (P. A. S. P., **25**, 260, 1914); but these are not cluster type variables, but probably classical cepheids, which are not uncommon in clusters and are far brighter.

[35] Adams, Joy, and Sanford, P. A. S. P., **36**, 139, 1924.

[36] Shapley and Miss Walton, H. C. 313, 1927.

[37] Shapley, H. C. 315, 1927.

TABLE IX, IV.—SPECTRA OF CLUSTER TYPE VARIABLES

Star	Period	Spectrum	Star	Period	Spectrum
	d			*d*	
SU Dra	0.66	A2.5	SU Aur	0.47	F5*
X Ari	0.65	A2.0	RV UMa	0.47	F0*
RX Eri	0.59	F0*	UX Her	0.46	A3*
U Lep	0.58	A4*	SW Aqr	0.46	A3*
SW Dra	0.57	F4*	RV Cap	0.45	A8*
RR Lyr	0.57	A5.5	RR Leo	0.45	A6 (A0-F2)
UY Cyg	0.57	F0*	SW And	0.44	F1*
RR Cet	0.55	F0*	RW Dra	0.44	A5*
W CVn	0.55	F0* (A7-F5)	RR Gem	0.40	A8*
V LMi	0.54	A	RS Boo	0.38	A2.5 (A3-F4)
RZ Lyr	0.51	A2*	WY Tau	0.36	A
SW Boo	0.51	A	TV Boo	0.31	B9
XZ Cyg	0.47	A3	RZ Cep	0.31	A

appears that surface gravity must be much the same for all cluster type variables, for differences of luminosity within Class A have a great effect on this quantity—differences in temperature (color) comparatively little.

c. The spectra of the cluster type variables in general do not show the c-character.[38] The lines are however weaker than in the normal star—at least in the spectrum of RR Lyrae,[39] the only one on which extensive material is available, so the luminosity is not unusually low. This observation is in keeping with the moderate absolute magnitudes derived for the cluster type variables, in their occupancy of the lower portion of the period-luminosity curve.

About 300 galactic cluster type variables[40] are known; they thus form less than one per cent of all known stars of corresponding spectral class. But those bright enough to classify number only about one fifth of one per cent of the known stars

[38] Though SU Aurigae was assigned to the "pseudocepheids" with the spectrum cF5, uncommonly late for its period, by Adams, Joy, Stromberg, and Burwell (Mt. W. Contr. 199, 1921).

[39] Spectra by Hogg. Cf Section 69.

[40] This estimate includes about 200 unpublished Harvard discoveries.

of group A down to the same magnitude.[41] Whether this fewness corresponds to a range of absolute magnitudes where conditions are necessary and sufficient for the maintenance of cluster type variability will be considered in Chapter XIV.

50. Line Form in Class A.—In Chapter III the preliminary assumption made was that all the lines encountered were of sensibly similar contour. From the first, hydrogen was recognized to be an exception—its appearance in the Mount Wilson sunspot map is quite abnormal, and Unsöld's measures of solar hydrogen contours[42] showed large deviations from the relation to which other lines conformed. The abnormality of the contour and consequently of the behavior of the hydrogen lines in the spectral sequence were further brought out by the investigations of the writer and Miss Williams.[43] The abnormal shape of the hydrogen lines has been attributed by Struve,[44] and by Elvey,[45] to a "mol-electric" or Stark effect.

There is a simple empirical way of detecting the presence of a widening and shallowing effect (such as the Stark effect) on the basis of the observations of contour and line depth summarized in Chapter XV. It has been shown empirically in Chapter III that over the range of classes F0 to M5 the typical ultimate line, K, of ionized calcium, and its neutral line 4227, show a close correlation between measured line depth and NH derived from contours. The use of this correlation, on the assumption of similarity of contour, in deriving NH for lines of which the contours cannot be measured, is described in Section 15. As the method is empirical, the finite resolving power of the spectrograph is unimportant if all lines are similar and the same resolving power is used throughout.

On the other hand, if the line contours are not all similar, the relation of line depth to log NH will show deviations from

[41] The question of vestigial variation in Class A is noted in Chapter XIV, p. 247.

[42] Zs. f. Phys., **46**, 765, 1928.

[43] H. Repr. 55, 1929.

[44] Ap. J., **69**, 173, 1929.

[45] Ap. J., **69**, 237, 1929.

the relation found for calcium, and such deviations, if detected, indicate some effect that changes the contours of the lines concerned. It is easily shown that the hydrogen lines are not all of similar shape.

The following table contains a test of the suggestion; the first column gives spectral classes, the second the observed percentage light loss for $H\gamma$ with one objective prism on the 11-inch telescope. The next four columns represent the values of log NH for which the line would have the corresponding depth dl at the half breadth (in Angstroms) that heads the column—they represent several different resolving powers, the tabulated half breadth representing the half breadth of the "instrumental line." The next column contains the observed value of log NH, and the last, the difference between the fifth and seventh columns. The latter is a measure of the widening effect for hydrogen at the spectral class in question.

TABLE IX, V.—DETECTION OF LINE WIDENING IN A STARS

Class	Mean dl	Computed Half Breadth				Mean Log NH	Difference
		1A	2A	2.2A	5A		
O	31	17.4	18 0	18 3	18 8	17.6	−0.7
B2	36	17 5	18.1	18 4	18 9	. . .	. . . . .
B3	44	17 7	18.3	18 6	19 1	18.4	−0.2
B5	56	17 9	18.5	18.8	19 3	18.8	0
B9	62	18.0	18.6	18 9	19 4	19.0	+0 1
A0	67	18.1	18 7	19.0	19 5	19 2	+0.2
A2	60	17 9	18.5	18 8	19 3	19 2	+0.4
A5	62	18 0	18.6	18 9	19.4	19 4	+0.5
F0	57	17 9	18.5	18 8	19 3	18 8	0
F5	48	17.7	18 3	18 6	19.1	18 6	0
F8	45	17 7	18.3	18.6	19.1	18 5	−0.1
G5	32	17 4	18 0	18 3	18 8	17 7	−0.6
K0	32	17 4	18.0	18 3	18 8	17.7	−0 6
K5	31	17 4	18 0	18 3	18 8	17 9	−0.4

The weaker lines appear more strongly from the measure of line depth; the stronger, from the measure of contour. The resolving power now becomes of importance; for lines such

that not only the central portion but also the *whole contour* is instrumentally produced, any relation between depth and contour will be entirely obscured. The resolving power of the arrangement used is about 2,000, so that the width of the artificial line at $H\gamma$ is about 2.2A. Hence the column under 2.2A is the one to compare with the observed contours (as has been done). The agreement for the lines of Class B5 and F0 to F8 is good, and deviations that lead to a negative value in the final column are merely the effect of the finite resolving power distorting the whole line instead of simply flattening the central peak. The effect on the dl/NH relation depends entirely on the shape of the "instrumental contour."

There remain the outstanding positive entries in the last column, and they correspond to a place where the contours of the lines are abnormally broadened. The effect is evident for Classes B9, A0, A2, and A5. Presumably it would obtain for Class A3, and probably for Class B8, but measures are not available. These data are not competent to identify the widening as a Stark effect.

It appears that the widening sets in at some definite surface gravity. The next table contains the spectral class, the difference, from Table IX, V, for the classes unaffected by resolving

TABLE IX, VI.—STARK EFFECT AND SURFACE GRAVITY

Class	Difference (Table IX, V)	Logarithm of Surface Gravity	Assumed	
			M	T
				°
B3	−0.2	3.8++	−0.97	12300
B5	0	4.0++	−0.66	12300
B9	+0.1	4.0++	0.0	10500
A0	+0.2	4.0+	+0.6	10000
A2	+0.4	4.0+	+1.2	9000
A5	+0.5	4.1+	+1.7	8400
F0	0	3.9	+2.4	7000
F5	0	3.1	0.0	6500
G5	−0.1	2.7	0.0	5000

power, and the surface gravity for the corresponding stars. The last two columns give the data used to obtain the third. One + sign after log g indicates that the number should be increased by about 0.1; two, that the increase should be about 0.2, both on account of the progress of the ionization of hydrogen and the probable preponderance of that element in the atmospheres.

It seems that the effect comes into measurable operation for hydrogen at a surface gravity of about 4.1. We note that this number is very near the value for the surface of the sun, where the Balmer lines are conspicuously broadened. Results for other yellow and red dwarfs would be of great interest. This result is of a very general nature; use has been made of it in an earlier chapter in evaluating temperatures.[46]

[46] See p. 113.

CHAPTER X

THE HIGH LUMINOSITY STARS OF CLASS A

51. Numbers and Distribution.—One hundred and sixty-nine c-stars of group A (B8, B9, A0, A2, and A3) are in Appendix A, together with remarks on the individual spectra. The compilation is made from the Remarks to the Henry Draper Catalogue. Down to $8^m.25$ there are more c-stars within division A than in any other.

The grouping in apparent magnitude and spectral class is instructive.

Class	Magnitude]4	5	6	7	8	9	10	[11	Total
B8	1	4	4	5	8	8	0	1	31
B9	1	2	1	4	5	2	1	0	16
A0	3	2	5	8	10	10	1	0	40
A2	5	9	12	4	5	4	1	0	40
A3	2	2	1	1	2	1	4	0	13
B8, B9, A0	5	8	9	17	22	20	2	1	87
A2, A3	7	11	13	5	7	5	5	0	53
Total	12	19	22	22	29	25	7	1	140

The large number of stars in Class B8p and A2p is probably a classification effect, as mentioned in Chapter III. We note that two-thirds of the early cA stars are of magnitude eight and fainter, and the late cA stars are preponderantly bright, three-fifths being brighter than the seventh magnitude. This is undoubtedly caused by the necessity of basing estimates of line intensity on total absorption for small dispersion plates (see Chapter III for details); since the *total absorption* is about the same for late A stars and cA stars, few faint cA stars are detected. For the brighter stars the actual line quality can be examined and narrow-line spectra detected. The early cA

stars have much weaker spectral lines than normal stars of the same class, so that their spectra are evident even with small dispersion.

The supergiant A stars are of course closely confined to the galaxy but do not cluster very conspicuously like the O and B stars. Their galactic concentration makes them appear more grouped than normal A stars, and in some places (such as h and χ Persei),[1] they actually occur in condensed star groups. But individual cA stars do not appear to form associations. The prominent pair of c-stars ι_1 and ι_2 Scorpii, which suggest close association in space, are classed by Miss Cannon as cF5 and cA2; their apparent magnitudes are 3.14 and 4.88.

52. Temperatures of cA Stars.—The few cA stars for which color equivalents are determined are mentioned in Table VIII, I, in the section dealing with the cB stars.

The data lead us to the conclusion that faint cA stars tend to be yellow, but the two very bright cA stars (α Cygni and β Orionis) are both rather white, and the inference is that the phenomenon is statistically correlated with luminosity rather than physically connected with it. The same conclusion is reached by Miss Williams.[2] The results of previous chapters, discussing the similar phenomenon for the B and O stars, may be recalled.

In discussing the spectroscopic features of the cA stars I do not intend to consider the very low temperatures derived for some of the stars as real. It can be shown, exactly as for the cB stars in Chapter VIII, that if they are real, there must be physical conditions both unknown to us and working in what seems to be a very unlikely manner for some but not for all of the luminous stars in a given class, or the observed spectra could not be produced. Alternatively, the temperatures within one spectral class have but a small dispersion, and the limits within which they may differ and yet produce the

[1] See Chapter VIII, p. 107.

[2] H. C. 348, 1929.

observed spectra can be measured spectroscopically and found to lead to reasonable luminosities, on lines that are already fairly well established.

53. Spectral Characteristics.—The well-known spectra of Rigel and Deneb define and represent the cA stars very well. The special abnormalities shown by supergiants in each of the subdivisions contained within Class A are mentioned in the tabulation following:

Class	Remarks
B8	Lines narrow. K line too strong for class, sometimes as strong as in Class A2 (H. D. 6226). Si+ tends to be strong.
B9	Lines narrow. K line too strong for class, sometimes as strong as in Class A3 (H. D. 165246). Si+, Mg+ tend to be strong.
A0	Lines narrow. Helium sometimes seen.[3] K line sometimes strong for class. Si+, Mg+, Sr+ may be strong. Many fine metallic lines may be seen (this partly an effect of sharpness). Bright edges may appear to red of low-frequency lines.
A2	Lines narrow. Helium sometimes seen. Sr+, Si+, Mg+, Fe+ tend to be strong. Bright edges may appear to red (cf. α Cygni).
A3	Lines tend to be narrow (chiefly refers to hydrogen). Lines of Sr+ and Fe+ tend to be strong.

In summary, the K line tends to be abnormally strong, especially in the earlier classes; helium is often stronger than normal; ionized iron is conspicuous; and ionized silicon and strontium tend to be strong. Most striking of all, the lines of hydrogen are far weaker and narrower in supergiant A stars than in normal A stars—a very conspicuous and important phenomenon bearing on the constitution of the stellar atmosphere, and the relative conditions in giant and supergiant atmospheres, and crucial for the theory of the stellar absorption coefficient. These problems are mentioned in Chapter XV and XVI. Further, it has very important indirect results in having a systematic effect on the classification of short-dispersion spectra and on the interpretation of the color indices of faint stars of Classes A and F. In order to make clear the points

[3] 4026 and 4471 are commonly observed in cA stars; Campbell (L. O. B. 332, 1921) records the D_3 line in the spectrum of α Cygni. All three are of the $1^2P - n^2D$ series—the strongest stellar helium lines.

of contact between these matters and the strength of hydrogen in the spectra of A stars I shall first present the actual material in detail.

a. Line Intensities in A Stars; Results from Contour Measures.—The tables that follow contain the mean half breadth of the lines in the various spectral classes named, measured at four values of the percentage light loss; and the mean values of

TABLE X, I.—CONTOURS OF LINES IN A STARS

Class	Line	Percentage Light Loss				Log *NH*
		4	17	31	42	
B9	Hγ	21.0	12.0	7 9	6.0	19.24
	Hδ	21.0	12 2	8.0	6 2	19 21
	Hε	22.8	14.2	10 3	8 2	19 38
cB8	Hε	8 0	3 0	1 9	1 6	17.99
A0	Hγ	27.0	15 1	9 1	6 5	19 45
	Hδ	25 5	12.9	9.4	6.9	19.37
	Hε	22.4	13.1	9 4	7 0	19.32
cA0	Hγ	6 4	2 4	0 3	.	18.09
	Hδ	9.0	3 9	2 0	1.3	17.74
	Hε	4.8	3 4	2.4	1 8	18 35
A2	Hγ	28 2	14 4	9 2	6 2	19 54
	Hδ	26.8	13 6	9 2	5 8	19 42
	Hε	24.4	14 1	9 2	6 2	19 34
cA2	Hγ	6.0	2 2	0 2	...	17.80
	Hδ	8.2	3.4	1.0	...	18.13
	Hε	10 8	6 0	2 8	1.4	18 36

TABLE X, II.—PERCENTAGE LIGHT LOSSES, CLASS B8 AND B9

Line	Normal B8 (5 Stars)	cB8 (2 Stars)	Normal B9 (4 Stars)	cB9 (1 Star)
Hβ	63	28	60	23
Hγ	59	40	71	30
Hδ	60	45	74	39
4026	15	..	..	..
Hε	64	60	75	52
K	6	24	11	19
Hζ	53	45	67	45

TABLE X, III.—NUMBERS OF EFFECTIVE ATOMS, CLASSES B8 AND B9

Line	Normal B8	cB8	Normal B9	cB9
Hβ	18.62	17.54	18.52	17.40
Hγ	18.48	17.90	18.86	17.60
Hδ	18.52	18.06	18.94	17.96
4026	17.16	.....	.....	.....
Hε	18.64	18.52	18.98	18.24
K	16.88:	17.41	17.04	17.28
Hζ	18.30	18.06	18.74	18.06

TABLE X, IV.—PERCENTAGE LIGHT LOSSES, CLASS A0

Line	Mean A0 (9 Stars)	Mean A0, Dunham (11 Stars)	Mean cA0 (3 Stars)
Hβ	59	..	37
Hγ	66	65	46
Hδ	71	69	52
Hε	75	69	57
K	23	22	28
Hζ	72	..	54

TABLE X, V.—NUMBERS OF EFFECTIVE ATOMS, CLASS A0

Line	Mean A0	Mean cA0
Hβ	18.48	17.81
Hγ	18.70	18.08
Hδ	18.86	18.29
Hε	18.98	18.42
K	17.38	17.54
Hζ	18.88	18.33

log NH for each line discussed, based on the contour fit at the depth considered. The procedure of fit and measurement has been described in Section 16, Chapter III.

b. Line Intensities in A Stars; Results from Depth Measures.—The mean percentage light losses at the centers of lines on plates made with one objective prism are next tabulated. The very few two-prism measures included are corrected to

one-prism scale by the reduction curve given in Harvard Circular 301 for γ Cygni, another star with sharp lines comparable in quality to those found for supergiant A stars. The deduced numbers of effective atoms are derived from the percentage light losses by means of the correlation substantiated in Chapter III.

TABLE X, VI.—PERCENTAGE LIGHT LOSSES, CLASS A2

Line	Mean Normal A2 (4 Stars)	Mean Broad A2 (2 Stars)	Mean cA2 (5 Stars)	α Cyg	ν Cep	Mean A2	
						Dun	Wms
Hβ	55	48	36	38	33	..	.
Hγ	61	78	44	35	42	62	70
Hδ	64	84	48	51	58	66	73
Hε	75	86	57	61	50	71	..
K	50	28	36	36	38	33	51
Hζ	63	71	41	44	46	..	..

TABLE X, VII.—NUMBERS OF EFFECTIVE ATOMS IN CLASS A2

Line	Normal A2	Broad A2	cA2	α Cygni	ν Cephei
Hβ	18.36	18 16	17 80	17.84	17 68
Hγ	18.56	19 07	18.02	17.76	17.96
Hδ	18.64	19 24	18 16	18 24	18.45
Hε	18.98	19.31	18 42	18 55	18 21
K	18 55	17.54	17 79	17 79	17 84
Hζ	18.62	18 86	17 94	18 02	18 09

54. Empirical Effects on Spectral Class and Color Index of High Luminosity in A Stars.—Before attempting the analysis of the atmospheres of the stars observed I shall call attention to empirical effects of the observed line intensities.

The effect on classification is of importance. It may be seen from comparison of the data of this chapter and the one that follows that the hydrogen lines of supergiant A stars are comparable in intensity to those of normal F stars, and the hydrogen lines of supergiant F stars more nearly like those of

normal A stars. This point will be more clearly seen by consulting Figure XV, 2. If then the spectra are taken with very short dispersion so that line quality is obliterated, and only the total absorption of the lines appears, the supergiant A star would be placed in Class F on the basis of its hydrogen, and *vice versa.*[4] Spectra that show the K line would be saved from misclassification, but the tendency would in any case be in the direction stated, and for spectra that did not run beyond 4000 A it would probably be fatal to homogeneity of classification.

The effect of the strong lines of normal A stars on color index as determined from integrated color is the result of the distortion of the continuous background by the confluence of the Balmer lines, discussed a year ago by the writer. The amount of the effect has been roughly evaluated by Gerasimovič.[5] It is mentioned here with the reminder that colors are best determined *spectrophotometrically* rather than by integrated light; for the effect mentioned, though it may roughly be evaluated, has an obvious dependence on exposure time and may well be a serious source of error, especially for stars that are *not* very luminous.

55. Analysis of the Supergiant A Spectrum.—Analysis will be of three kinds: (*a*), general, on the basis of the appearance of the spectrum; (*b*), qualitative, on the behavior of the different kinds of lines and the general forms of the contours, and (*c*) quantitative, on the basis of the determined numbers of atoms.

a. General Aspects of the Spectra.—The spectroscopic peculiarities of the supergiant A have been summarized earlier in the chapter and mentioned in detail for the brighter stars; the behavior of atoms of different kinds relative to the same atoms in normal stars is now summarized, and the possible explanation of the differences noted.

[4] H. Repr. 48, 1928.
[5] H. C. 339, 1929.

Atom	Behavior	Possible Interpretations
H	Weakened and narrowed	Lowered pressure and consequent decrease of fractional concentration.[6] Decreased Stark effect from lowered pressure (probably a small factor).
He	B8, weakened? Ao on, strengthened	
Ca+	B8, 9 strengthened Ao on, weakened or unaltered	Possibly interstellar line superposed. Lower pressure decreases fractional concentration?[6] Not weakened in same ratio as the hydrogen, see below, so the interpretation must be different in part.
Fe+ Ti+ Cr+ Ni+ Sc+	B8, just seen; Ao on, greatly strengthened	Partly due to sharpness and definiteness of lines, but this is not a very large factor; see below. Partly due to lowered pressure and consequent increased ionization.

Some of the features receive their explanation in terms of lowered pressure—dependent on lowered surface gravity; any effects of lowered pressure will also enhance the effects of radiation pressure, in proportion to the tendency to support by radiation pressure of the atoms concerned. If this explanation is to be adduced to account for the relative enhancement of different lines, we should have to consider that radiation pressure is most effective in supporting helium, that ionized metallic lines come next, and that calcium and hydrogen are least supported—not entirely what would be expected from the solar chromosphere.

The great intensity of helium in the spectra of some A stars (it occurs in η Leonis, in α Cygni, and very strongly in the anomalous spectrum of υ Sagittarii, where it undoubtedly arises from the same body as the metallic lines[7]) has been explained by Miss Williams,[8] who points out that depletion of helium by low ionization is outbalanced here by atmospheric extent.

The lines of ionized metallic atoms, at their strongest at or near Class F5, appear in Classes cA2 and cAo, and also more faintly

[6] See Section 80: the theoretical treatment of the effect depends on the adopted relation of the general absorption coefficient to temperature and pressure.

[7] J. S. Plaskett, M. N. R. A. S., 87, 31, 1926.

[8] H. C. 348, 1929.

in Classes A2 and A0; even in the spectrum of β Orionis a number of metallic lines of ionized atoms are found.[9] A remark may here be made about the ionized iron lines which appear bright in the spectra of some B stars,[10] at ionization temperatures far above those at which they are last seen in the spectral sequence. It is well known that bright lines tend to be more conspicuous than absorption lines due to the same substance—witness the enormous strength of the carbon and helium bands in the O stars,[11] of the 4686 line even in the absorption O stars, and its survival into Class B unaccompanied by other hallmarks of Class O; the great intensity of the bright hydrogen lines in the spectra of the long period variables; the enormous intensity of the bright bands in novae and even the reversal of the dark hydrogen lines in B stars, where the bright line is obliged to overpower an absorption line, which appears as dark wings and often as a dark center. It is not difficult to believe that the number of atoms required to produce a certain total absorption (relative to continuous background) is far larger than the number required to produce, under the appropriate conditions, a total emission of the same strength, measured in percentage of the corresponding continuous background. This would save us from part of the difficulty of reconciling the spectra of Wolf-Rayet stars with their observed temperatures, and from having to assume ridiculously large quantities of helium and oxygen in their atmospheres, and of hydrogen in the atmospheres of the long period variables. We then see no reason to think that there is more hydrogen in the atmospheres of these latter stars than in those of Betelgeuse and Antares—a large enough quantity to cause theory considerable discomfort.[12] A quantitative check on the total emission in the centers of the solar H and K lines, and a comparison with the floccular area repre-

[9] Lockyer, Pub. Sol. Phys. Com.; "Catalogue of 470 of the Brighter Stars."
[10] Merrill, Mt. W. Contr. 334, 1927; Gerasimovič, H. B. 851, 1927.
[11] See Figure VI, 2.
[12] See Section 81.

sented, would be instructive in this regard; but I have no facilities for making them.

b. Qualitative Aspects of Line Intensity and Shape.—Reference to the collected data of Chapter XVI shows that within Class A the hydrogen lines have their maximum; but from Chapter XVI it appears that for supergiant stars the maximum is at Class F5. The bearing of this observation on the classification and color indices of faint stars has been mentioned above.

It will be recalled that the hydrogen maximum at A0 formed the zero point of the earlier Fowler-Milne system and that if the class has the temperature of 10000°, a partial electron pressure of about 10^{-4} atmospheres results. If the c-stars be treated as another series with a constant partial electron pressure (not necessarily at all a justifiable assumption), a pressure of about 10^{-8} atmospheres accords with a maximum in the neighborhood of 7000°. The logarithm of the ratio in surface gravity for stars of absolute magnitudes 0 and -3 at these two temperatures is about 1.25; this should be compared with the ratio in P_e. For the predicted relation between surface gravity and pressure preliminary theory[13] derived strict proportionality; more refined theory[14] deduced $P \propto \sqrt{g}$; observation[15] gave roughly $P \propto g^2$; but from other considerations Struve[16] finds that $P \propto \sqrt{g}$ is the most satisfactory relation. The surface gravities for normal and supergiant A stars suggest $P \propto g^2$.

The generalization of the Saha theory presented by Milne[17] refined the determination of maximum by referring it to a certain value of *dl*. The examination, by the writer and Miss Williams, of the available data on the contours of the hydrogen lines near the maximum, confirmed the suspicion that the maximum would differ at different depths for the normal stars, but found it in the sense opposite to what was predicted.[18]

[13] Pannekoek, B. A. N. 19, 1922.

[14] Milne, M. N. R. A. S., **89**, 3, 17, 1928.

[15] Payne and Hogg, H. C. 334, 1927.

[16] Ap. J., **69**, 187, 1929.

[17] M. N. R. A. S., **89**, 3, 17, 1928.

[18] Payne and Williams, H. Repr. 55, 1929.

Miss Williams[19] has concluded that this difference is referable almost entirely to a general distortion of the lines within Class A, so that as a test of the generalized Saha theory the data are irrelevant. Incidentally the differences between different authors[20] as to the precise class at which the hydrogen lines are at maximum receive a justification in the fact that different measures actually lead to different maxima.

That a similar distorting phenomenon does not occur for the c-stars is very probable, though it is unfortunately impossible to determine this certainly with the dispersion now used. The point is nevertheless very important and should be examined without delay. There is a further possibility that the hydrogen maximum at Class A0 is not a pure Saha phenomenon, the true maximum being at F5, and the maximum at A0 being entirely an effect of the line widening just mentioned. This does not seem very likely, because the *depth* of the hydrogen lines in the normal stellar sequence runs through a maximum at about A0, and a widening effect such as the Stark effect might cause greater broadening there, but would probably not affect the depth.

c. Quantitative Comparison of Three cA Stars.—Spectra of the three bright stars, ι_2 Scorpii, α Cygni, and υ Sagittarii, all approximately of Class A2, were available with sufficient dispersion for a more detailed study. All are obviously very bright; we consider that ι_2 Scorpii is the faintest, α Cygni of intermediate brightness (perhaps of absolute magnitude -5), and υ Sagittarii the brightest. Table X, VIII summarizes the mean measured percentage light losses for the lines mentioned at the heads of the columns: 4 of hydrogen, 6 of helium, 2 each of ionized calcium, ionized strontium, ionized silicon, and ionized nickel, 20 of ionized iron, 5 of ionized titanium, 4 of ionized chromium, and three of ionized magnesium.

[19] H. C. 348, 1929.

[20] Miss Cannon, Preface to the Henry Draper Catalogue; Menzel, H. C. 258, 1924; H. Mon. No. 1, 56, 1925: "The writer is inclined to believe that no significant maximum can in fact be derived for the Balmer lines."

TABLE X, VIII.—MEAN PERCENTAGE LIGHT LOSS FOR SUPERGIANTS, CLASS A2

Atom	ι_2 Scorpii	α Cygni	υ Sagittarii
H	40	68	24
He	..	8:	20
Ca	5	..	..
Ca+	40	69	32
Sr+	15	21	12
Si+	16	26	28
Fe	9	11:	..
Fe+	14	24	17
Ti+	11	12	14
Ni+	5	10	18
Cr+	8	12	14
Mg+	17	22	23

The data tabulated in X, VIII were derived from two-prism spectra, and reductions based on investigation of the same spectrum with different dispersions were applied to bring them to the one-prism scale. The resulting values of *dl* and the corresponding values of log *NH* derived from them by the method of Section 16 are given in Table X, IX.

The types of line behavior may be summarized in five classes:

Line Type	ι_2 Scorpii to α Cygni	α Cygni to υ Sagittarii
Neutral ultimate (Ca, Fe)	Weakened	Weakened
Hydrogen	Strengthened	Weakened
Ionized ultimate (Ca+, Sr+)	Strengthened	Weakened
Helium	Strengthened	Strengthened
Ionized Subordinate (Si+, Fe+, Ti+, Ni+, Cr+, Mg+)	Strengthened	Strengthened

There are five classes of line, and three classes of behavior: that of the neutral ultimate lines; the ionized ultimate lines and hydrogen; and the ionized subordinate lines and helium. The reason for these relations may lie in the higher mean ionization potential of the ionized subordinate than the ionized ultimate lines.

The most striking feature of the spectra is the occurrence of helium, which by itself points to great luminosity for α

Cygni, and enormous brightness for υ Sagittarii. The behavior of the helium lines, the ionized subordinate lines, and the neutral ultimate lines are all consistent with each other and with the idea that the stars are arranged above in order of increasing brightness. The behavior of hydrogen and ionized ultimate lines is less easily interpreted. The former is well

TABLE X, IX.—REDUCED PERCENTAGE LIGHT LOSSES FOR SUPERGIANTS AND RELATIVE LOGARITHMS OF *NH*

Atom	ι_2 Scorpii		α Cygni		υ Sagittarii	
	dl	Log *NH*	*dl*	Log *NH*	*dl*	Log *NH*
H	34	17 72	57	18.42	20	17.30
He	..	<16.8	7:	16.92	17	17.22
Ca	4	16.80:	..	<16.8	..	<16.8
Ca+	34	17.72	58	18.45	27	17.51
Sr+	13	17 10	18	17.24	10	17.00
Si+	13	17.10	22	17.35	24	17.41
Fe	8	16.94	9:	16.97:	.	<16.8
Fe+	12	17.07	20	17.30	14	17.12
Ti+	9	16.97	10	17.00	12	17.08
Ni+	4	16.80	9	16.97	15	17.15
Cr+	7	16.92	10	17.00	12	17 08
Mg+	14	17.12	27	17 50	28	17.54

below its maximum, and the latter well above theirs. Perhaps it is best to leave this observation an empirical one; it is evidently connected with the problem of the absorption coefficient discussed in Chapter XV. We note that hydrogen at this particular spectral class is very insensitive to absolute magnitude effects and also that the intensity of calcium is changing very rapidly between A2 and F0 as a function of class. As the spectra are all three somewhat abnormal, the exact placing in a spectral class is not easy, and perhaps the irregular behavior of the ionized calcium is a result of the slightly inaccurate assumption that the spectral classes of the three stars are the same.

The increasing luminosity from ι_2 Scorpii to υ Sagittarii may be expressed in another way. Both calcium and iron

occur in measurable quantities in both ionized and neutral states, and by comparing the relative numbers of neutral and ionized atoms we may express the ionization in the atmosphere. The logarithms of the ionization ratios thus derived are as below:

Ratio	ι_2 Scorpii	α Cygni	υ Sagittarii
Ca+/Ca	0.9	$\gtrless$ 1.1	>0.4
Fe+/Fe	0.1	0.3	>0.3

The spectrum of υ Sagittarii shows no neutral lines, so that for it we can derive only an upper limit. It is clear that the three stars have been arranged in order of increasing ionization, and hence probably of increasing luminosity.

CHAPTER XI

THE NORMAL STARS FROM F TO K

THE second-type stars in whose spectra the metallic lines are the chief feature (Classes F, G, and K) present a large number of spectroscopic problems. By far the greater number of these stars are dwarfs; giants are uncommon, and supergiants exceptional.[1] The spectral classification of second-type stars is examined in the present chapter as a basis for the study of the corresponding supergiants.

Physical conditions at the surface of the average second-type giant and dwarf are summarized below; the temperatures have been compiled from the measures of Seares,[2] Hertzsprung,[3] Russell, Dugan, and Stewart,[4] and Hufnagel;[5] the luminosities used in computing the surface gravities are those given by Russell, Dugan, and Stewart.[6]

TABLE XI, I.—SURFACE CONDITIONS OF SECOND-TYPE STARS

Class	Effective Temperature		Logarithm of Surface Gravity	
	Dwarf	Giant	Dwarf	Giant
	°	°	cm./sec.^{2}	cm./sec.^{2}
F5	7200	6800	4.5	3.8
G0	5900	5500	4.4	2.9
G5	5500	4700	4.3	2.2
K0	5000	4100	4.1	1.9
K5	4100	3400	4.1	1.5
M0	3420	3200	4.6	1.4

[1] Shapley and Miss Cannon, H. Repr. 6, 1924.
[2] Ap. J., **55**, 198, 1922.
[3] Ann. Leiden Obs., **14**, 1, 1922.
[4] Astronomy, **2**, 753, 1927.
[5] H. C. 343, 1929.
[6] Astronomy, **2**, 740, 1926.

From Class Go onward the giant is cooler than the dwarf by an amount corresponding to half a spectral class; from F5 to Mo the ratio in surface gravity between giant and dwarf increases from less than 10 to over 1,000.

A given spectral class implies a certain degree of average ionization. Each line of Table XI, I therefore summarizes the physical condition of two stellar atmospheres that are on the average equally ionized, although at very different temperature and pressure. The implications of this view are the subject of Chapter XV.

56. Classification of Second-type Stars.—The spectroscopic features by which the classes may be recognized are given in Table XI, II, which is condensed from the preface to the Henry Draper Catalogue.

TABLE XI, II.—CLASSIFICATION OF SECOND-TYPE STARS

Class	Criteria
Fo	K = H + Hε; G band weak; H = 0.5 × Sirius
F2	Same, but G band stronger
F5	H = 0.2 × sun; 4326 = 0.1 × Hγ; 4308 stronger than 4315
F8	Like solar spectrum,
Go	Hγ = 1.5 × 4326; 4077 = 4026; G band continuous
G5	Hγ + 4337 = 4326; cooler than Go
Ko	Hγ = 0.5 × 4326; G band stronger than 4227
K2	4227 stronger; cooler than Class Ko
K5	H, K, and 4227 the strongest lines; absorption bands appear

The composition of the spectral classes appears to vary considerably; all are preponderantly composed of dwarfs, but the relative proportion of giants and supergiants differs much from class to class in the catalogues, and presumably in space. Classes Fo, F2, F5, and F8 appear to contain only dwarfs and supergiants. Capella is one of the very few gGo stars; "normal" stars appear in any numbers only at classes Ko, K2, K5, and M.

It may be regarded as established within the spectral sequence that

a. The mass luminosity law holds, at least to a first approximation.

b. There is a correlation between mass, temperature, and surface gravity, expressed by

$L = acR^2T_e^4$, where $a = 7.64 \times 10^{-15}$, c is the velocity of light, R the radius of the star, T_e the effective temperature, and L the luminosity.

c. For stars of *any one temperature* there is a tendency to a clustering of the masses around two values (giant and dwarf); we note, however, that these do not correspond to the same spectral class; the giants are "earlier."

For a given effective temperature it is the surface gravity that governs the partial pressure, and therefore the degree of ionization and the spectral class. In order to produce a certain degree of ionization at a given temperature a certain partial pressure must occur; at any one temperature the surface gravities cluster around two values (giant and dwarf), and thus for certain surface gravities, certain degrees of ionization appear uncommonly if at all. If there are gaps in luminosity there must also be gaps in spectral class. The rarity of giant stars with temperature 8000° to 6000° leads, as a natural consequence, to gaps in the spectral sequence for giant stars. As our accurate knowledge of surface gravities is limited to a few binary stars, which are difficult to observe spectroscopically and almost inaccessible for temperature measurements, the suggestion just put forward cannot as yet be tested in detail.

57. Spectrophotometric Analysis of the Classes.—The spectra of 141 stars brighter than the sixth magnitude have been analyzed with the microphotometer in making the present survey of normal second-type stars; the best one-prism spectrograms in the Harvard collection were chosen, no attempt being made to represent completely any one spectral class or region of the sky. About half the plates were standardized by means of the apparent magnitudes of the stars on the plates (method 4, Table II, I), and the whole series were also analyzed by the

mean reduction curve of Hogg (method 12, Table II, I). The agreement of the results from the two methods was very close.

From the comparatively simple spectra of the hot stars the transition to rich metallic spectra occurs at about Class F2; for cooler stars the metallic spectra become increasingly conspicuous and, with the confluence of wings, and the incidence of band spectra, detract from the significance of measured intensities. If spectrophotometric intensities are to be measured for the cooler stars, we must be assured that the "tangent continuous background" is not grossly in error, that the lines we measure are reasonably free from blending, and that heavy bands are avoided. The larger the dispersion the easier it is to pick unblended lines; but at the same time the tracing of the continuous background becomes more difficult. Or, again, we must confine our work to lines so strong that they are unaffected by small resolving power (especially for measures of contour), and for these lines the wings tend to be so wide and strong that the background is almost impossible to trace. It seems as if accurate spectrophotometry were observationally indeterminate, unless some independent method is devised for putting in the continuous background—perhaps one based on temperature, relying on a zero point of the continuous background as free as possible from disturbing lines.[7]

There are many examples of disturbing effects of blending and background; for instance the apparently abnormal strength of $H\beta$ in some K classes, owing to the incidence of a band of titanium oxide; the strengthening of the line at 4215 by the neighboring cyanogen band; and the great apparent intensity conferred on the ultimate lines of aluminum by their proximity to the H and K lines.

The choice of lines to be measured in the G and K stars is limited in an obvious way by these considerations. The following lines were finally selected:

[7] Cf. Miss Williams, Ap. J., *in press*.

TABLE XI, III.—LINES SELECTED FOR MEASUREMENT

Wave Length	Atom	Wave Length	Atom
4861	Hβ	4101	Hδ
4340	Hγ	4077	Sr+
4326	Fe (Sc)	4046	Fe
4300	CH	3970	Ca+
4227	Ca	3933	Ca+
4215	Sr+		

The lines, measured in terms of percentage light losses in the way outlined in Chapter II, were used to derive the following ratios:

Designation	Ratio
a	4340/4326
b	4300/4326
c	4300/4340
d	3970/4227

The data on which the discussion is based are not tabulated but are presented in diagrammatic form, and certain mean results are given in full.

More than half of all spectral classes are between 7000° and 3000° in surface temperature. The lower the temperature the more rapidly do the spectra change with temperature; between Fo and Go the interval is 2000°, between Go and Ko, about 1000°, and between Ko and Mo, about 600°. But even though the cooler classes are increasingly crowded, on a temperature scale, toward Mo, the existing classes (Go, G5, Ko, K2, K5) have been found to admit of further subdivision for a number of bright stars; they have been placed in decimal subclasses by Adams, Joy, Strömberg, and Burwell[8] and by Shapley and Mrs. Shapley.[9] The Mount Wilson reclassifications were made on the basis of estimated line ratios. In the following paragraphs is described an attempt to assemble the observed G and K stars in significant groups on the basis of measured line ratios.

[8] Mt. W. Contr. 199, 1921.

[9] H. C. 232, 1922.

The ratios *a*, *b*, *c*, and *d* were plotted against Henry Draper class, as shown in Figure XI, 1. The quantity *a* is the most closely correlated with spectral class down to and including Class Ko, beyond which it cannot be used; it was adopted as a first criterion of class. Its mean values, for every tenth of a spectral class, were used to define the revised classes, as shown

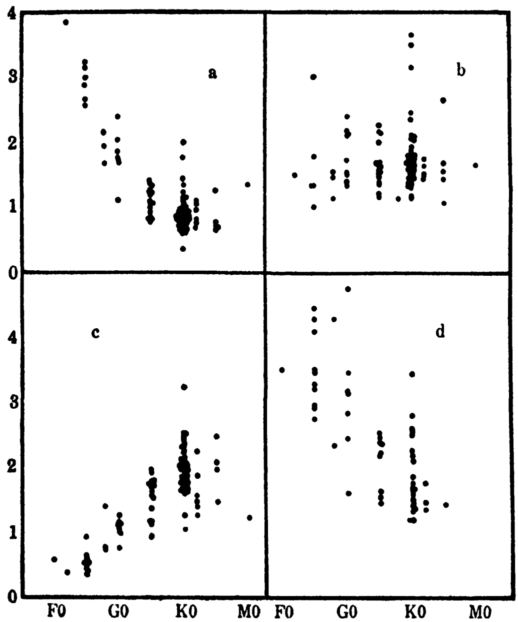

FIGURE XI, 1.

Correlation of Henry Draper class (abscissa) with the four ratios (*a*) 4340/4326, (*b*) 4300/4326, (*c*) 4300/4340, (*d*) 3970/4227.

in Table XI, IV. The ratio 4340/4326 is used by Miss Cannon in defining the G and K classes (cf. Table XI, I), so the closer correlation of H. D. class with this quantity when measured is not unexpected; our ratio puts her estimates on a quantitative basis.

As a test of the homogeneity of the new classes, the correlation with four leading features of the spectrum was examined, and Figure XI, 2 compares the correlation of both Henry Draper

TABLE XI, IV.—THE RATIO 4340/4326 FOR CLASSES F TO K

Interval of Ratios of *dl*	Class	Interval of Ratios of *dl*	Class
2.96–2.80	F5	1.12–1.03	G6
2 80–2.61	F6	1.02–0.95	G7
2.60–2.45	F7	0.94–0.89	G8
2.44–2.29	F8	0.88–0.83	G9
2.28–2.16	F9	0.82–0.79	K0
2.15–1.97	G0	0.78–0.75	K1
1.96–1.77	G1	0.74–0.72	K2
1.76–1.57	G2	0.71–0.69	K3
1.56–1.39	G3	0.68–0.67	K4
1.38–1.25	G4	(0<.67)	K5
1.24–1.13	G5	0.83	K5

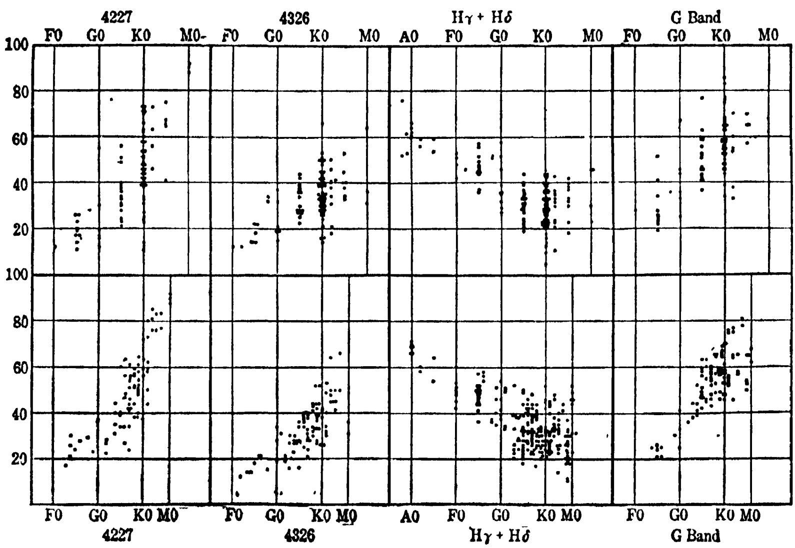

FIGURE XI, 2.

Comparison of Henry Draper and revised spectral classes; the former are above, the latter below. Each plot shows the relation of the depth of individual lines (indicated above the diagram) to spectral class. Line depths are expressed in percentage light loss at the center; with the dispersion used this quantity is a direct function of total absorption. For the revised classes, Mo follows directly upon K5, as it should do physically. It is evident that the scatter is smaller for the revised classes; see Table XI, V.

and revised classes for the lines 4227 (Ca), 4326 (Fe), the G band, and a mean of $H\gamma$ and $H\delta$. There can be no doubt that the correlation is improved by the reclassification in all four cases. The most striking improvement is for the line at 4227, which has a conspicuous dispersion in the Henry Draper classes; these measured classifications therefore represent improvements in homogeneity, though the results are not perfect; but it is not profitable to attempt to make a one-parameter classification in greater detail.

The qualitative results shown in the diagram are tested numerically in Table XI, V which gives, in successive columns, the spectral classes, the extreme percentage light losses, the corresponding extreme values of log NH, the difference between the extremes of NH; a second set of columns contains the same data for equivalent intervals in revised classes. Data for three lines are tabulated, and at the foot of each "difference" column a mean is taken for the line concerned. It appears that the dispersion is in every case greater for the H. D. than for the revised class. The dispersion in revised class is least for 4326, almost the same for 4227, and greatest for hydrogen; the improvement in dispersion for hydrogen is very small. The inferences are that (1) the measured classes are more homogeneous than the H. D. classes as far as the three features examined are concerned, (2) the hydrogen lines are more sensitive, and their changes with conditions are larger, than for the other lines—as has long been known in other connections (for instance, the red supergiants have stronger hydrogen than the sun).

It is noticeable in Table XI, IV that the ratio 4340/4326 changes very slowly for classes later than K0, and even appears to turn up again at K5; the iron line no longer strengthens with advancing class, although it does not noticeably weaken. The changes in the sensitive hydrogen line govern the ratio from this point on, and hence the stars with strong hydrogen tend to have early "measured" classes. An example of this tendency is found in Mount Wilson Contribution 199, where

TABLE XI, V.—COMPARISON OF REVISED CLASSES WITH HENRY DRAPER CLASSES

H. D.	*dl*, 4227		*NH*, 4227		Diff.	Revised Class	*dl*, 4227		*NH*, 4227		Diff.
	High	Low	High	Low			High	Low	High	Low	
Fo	12	12	17.06	17 06	0.00	Fo –F2.5	.	..	.. .	....	...
F5	26	11	17 47	17.04	0 43	F2 5–F7.5	20	07	17.30	16.90	0 40
Go	30	12	17.60	17.06	0.54	F7.5–G2.5	27	12	17.51	17.06	0 45
G5	56	21	18.39	17.33	1.06	G2.5–G7.5	43	14	18.00	17.12	0.88
Ko	73	11	18.92	17.04	1.88	G7.5–Ko.5	62	27	18.59	17.51	1.08
K2	73	46	18.92	18.09	0.83	Ko.5–K3.5	75	34	18.97	17.72	1 25
K5	75	41	18.97	17.73	1.24	K3.5–K7.5	73	66	18.92	18.71	0.21
Mo	92	88	19.49	19.37	0.12	M	92	88	19.49	19.37	0.12
					0.76						0.55
	dl, 4326		*NH*, 4326				*dl*, 4326		*NH*, 4326		
	High	Low	High	Low			High	Low	High	Low	
Fo	12	12	17.06	17 06	0.00	Fo –F2.5	12	12	17.06	17.06	0 00
F5	22	14	17.35	17.12	0 23	F2.5–F7.5	21	14	17.33	17.12	0.11
Go	37	15	17.81	17 15	0 66	F7 5–G2 5	32	15	17.66	17.15	0.51
G5	44	23	18.03	17.38	0 65	G2.5–G7 5	40	16	17.91	17.18	0.73
Ko	66	14	18.71	17.12	0.59	G7.5–Ko 5	50	26	18.21	17 28	0 93
K2	50	18	18.21	17 24	0 97	Ko 5–K3 5	53	30	18.30	17 60	0.70
K5	53	33	18 30	17 69	0 61	K3.5–K7.5	64	41	18.64	17 93	0.71
M	64	31	18.64	17.64	1 00	M	36	31	17.78	17.64	0.14
					0 71						0.48
	dl, Hydrogen		*NH*, Hydrogen				*dl*, Hydrogen		*NH*, Hydrogen		
	High	Low	High	Low			High	Low	High	Low	
Fo	53	42	18.30	17.97	0.33	Fo –F2.5	60	42	18 52	17.97	0.55
F5	57	36	18.42	17.78	0.64	F2.5–F7.5	57	36	18.42	17.78	0.64
Go	58	27	18.46	17 51	0.95	F7.5–G2.5	58	33	18.46	17.69	0.77
G5	44	19	18.03	17.27	0.76	G2.5–G7.5	50	18	18 21	17.24	0.87
Ko	46	05	18.08	16.84	1.24	G7.5–Ko.5	52	17	18 28	17 21	1.07
K2	43	11	18.00	17.04	0.96	Ko.5–K3.5	44	14	18.03	17.12	0.91
K5	42	18	17.97	17.24	0.73	K3.5–K7.5	44	10	18.03	17.00	1.03
M	46	23	18.08	17.38	0 70	M	52	23	18.28	17.38	0 90
					0.79						0.72

all M stars with supergiant tendencies are classed under "measured" spectrum as G—the strong hydrogen is responsible for the anomaly. Many of the K5 stars considered here are in the same situation, and the rise in ratio *a* at K5 is due to the fortuitous inclusion among the very few K5 stars investigated of some intrinsically bright ones. There do not seem to be any K5 stars of intermediate absolute magnitude, and therefore the mean absolute magnitude of a group of K5 stars (which, if they are bright, will not include dwarfs) will be higher than for a similarly chosen group of Ko stars.

TABLE XI, VI.—INTENSITY OF 4227 AND SURFACE GRAVITY WITHIN CLASS G9

Absolute Magnitude	Surface Gravity	*dl*	log *NH*
−1.1	1.90	61	18.56
−0.4	2.10	59	18 49
−0.1	2.20	48	18.12
+0.5	2.50	51	18.25
+0.7	2.52	51	18 25
+1.0	2.80	51	18 25
+1 6	2 86	52	18 28
+5.6	4.41	36	17 78
+6.1	4.60	21	17 33

The ratio 4340/4326 cannot therefore be used for determining spectral class later than Ko, and another must be sought. In order to determine a satisfactory one, the effects of luminosity will first be described. The ratio 3970/4227, the fourth plotted in Figure XI, 1, is the most obvious one to relate to absolute magnitude, as it must give the degree of ionization of calcium in the atmosphere. The intensity curves for the lines of ionized calcium, however, pass a maximum within Class K, and their ratio to that of another line will therefore be both insensitive and ambiguous. In addition to the effect of ionization, however, absolute magnitude is related to a factor that may be called "amount of atmosphere" (see Chapter XVI), which may militate against and even reverse a weakening by ionization. The matter is at present therefore

best considered empirically. The line 4227 has been found in practice to be related, within any one measured class, to the brightness of the star. In most of the classes the data are somewhat incomplete, owing to a scarcity of dwarfs, but for Class G9, as tabulated in Table XI, VI, they appear fairly definite. The neutral calcium line is far stronger in the bright stars than in the faint ones. The surface gravities given in this table are based on several assumptions; the absolute magnitudes, from the Yale catalogue, are converted into masses by the mass-luminosity curve, and the surface gravities are then computed from the temperatures by means of the usual formulae.[10] The temperatures are based upon the data of Seares[11] for giants and dwarfs, the change being assumed to be linear with brightness.

Within Class G9 the line 4227 increases in strength with increasing brightness, but its behavior changes rapidly in later classes. It grows strong with lower temperature (see Table XI, VI, and Chapter XV, Table VI) and is perhaps the best single criterion of *class* for giant stars of Classes K2, K5, and M, apart from the titanium oxide bands. That is, if a giant star is known to be of Class K2 or later, it can be placed by means of the calcium line. The absence of intermediate stars for these spectral classes (and the apparent faintness and rarity of dwarfs) makes the criterion a practical one for the brighter stars. The change of 4227 with absolute magnitude *for giant stars* is probably smaller than its change from one class to the next.

The behavior of the line 4227 in dwarf stars is very striking; in 61 Cygni, class dK7, it is relatively as strong as H and K,[12] as might be expected from the small degree of ionization there; though absolutely it is doubtless weaker than in some bright stars. In the spectrum of the companion to Castor it extends one tenth of the distance from $H\gamma$ to $H\delta$, or is about 25 Ang-

[10] Eddington, The Internal Constitution of the Stars, 136, 1926.

[11] Ap. J., **55**, 198, 1922.

[12] Russell, Dugan, and Stewart, **2**, 873, 1926.

stroms wide[13]—it is rather weaker than H and K in gK5 stars.[14] Data on the matter are somewhat outside my province, but I note the great strength of the 4227 line in the very bright star Mira and that in the spectrum of R Doradus, also undoubtedly very luminous, it is about as strong as H and K. Among the dwarfs, as among the giants, the line would probably be a good criterion of class; but the absolute strength would differ in the two cases; the possibility of using it for determining class depends here on the absence of intermediate magnitudes, and the possibility of using it for absolute magnitude in early K classes, justified empirically, depends on the presence of intermediate magnitudes.

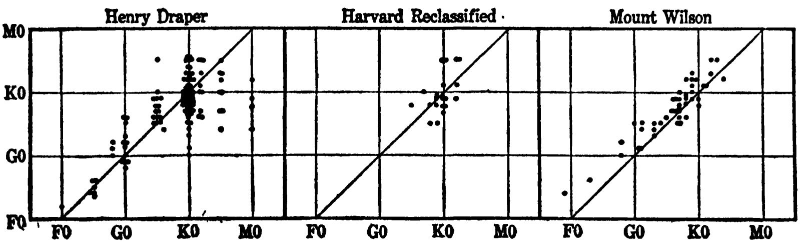

FIGURE XI, 3.

Comparison of revised classification (ordinate) with (1) Henry Draper classes; (2) reclassifications by Shapley (H. C. 232, 1922); (3) Mount Wilson classes (Mt. W. Contr. 199, 1921).

As an independent test of the reclassifications made by the measured values of *dl*, they are compared in Figure XI, 3 with reclassifications made at Harvard[15] and at Mount Wilson[16] for the same stars. The Harvard and Mount Wilson reclassifications are closely duplicated; as the other reclassifications were largely based on the ratio 4340/4326 this shows that ratios derived from percentage light losses compare in detail with careful eye estimates of relative intensity, over which, however, they probably have no great advantage for lines of

[13] Joy and Sanford, Mt. W. Contr. 320, 1926.

[14] I assume the width to be measured near the extreme wings; if it is measured for a percentage light loss greater than 13 this statement is no longer true.

[15] Shapley and Mrs. Shapley, H. C. 232, 1922.

[16] Adams, Joy, Strömberg, and Burwell, Mt. W. Contr. 199, 1921.

similar strength. The superiority of microphotometry appears in the measurement of the *shapes* of lines, and in the comparison of lines of very different intensities.

In summary, the present section contains a very general account of the spectroscopic qualities of the F, G, and K stars, and an examination of the methods of classifying them. The temperatures, on which there is at present little direct material, will undoubtedly be very important in classification in the future, at least as far as Class K2; from Table XI, VI we infer that a direct measure of energy temperature might well replace the ratio 4340/4326; and for a parameter expressing the surface gravity the line 4227 might be found practical. Before any such recommendation were put in force, however, it would be necessary to determine whether in Class K there are abnormally low (or high) temperatures, such as have been found in other classes; and this entails a laborious observational investigation.

Class Mo follows closely upon Class K5[17] which already shows the beginnings of the bands of titanium oxide.[18] The decimal classes into which M has more lately been divided place spectra with weak TiO at Class Mo,[19] which has the effect of transferring most of Class K5 and part of Class K2 into Class M.[20] The bands of titanium oxide are evidently more sensitive than any other feature of the spectrum to changing conditions—as is shown from the facts (1) that within one spectral class they rise from mere traces to being the strongest feature in any stellar spectrum of the normal sequence; (2) that they change with far greater range than any other absorption lines in the spectra of long-period variables, and (3) that they do not appear at all in the spectra of other stars of almost similar temperature. For this reason titanium oxide bands are rather difficult to use as a basis of classification in conjunc-

[17] Hogg, Pop. Astr., **36**, 236, 1928.
[18] Preface to the Henry Draper Catalogue, H. A., **91**, 1919.
[19] Trans. I. A. U., 1925; Merrill, Pop. Astr., **37**, 444, 1929.
[20] Cf. Adams, Joy, and Humason, Mt. W. Contr. 319, 1926.

tion with less sensitive criteria—the matter is even worse than it would be if the M stars were to be classified by the strength of the hydrogen lines. Hydrogen indeed forms a somewhat parallel case, though the conditions to which it is sensitive are not similar to those affecting titanium oxide. Anomalies that appear at the junction between Classes K and M are undoubtedly related to the change from a less to a more sensitive criterion (i.e., from lines to bands). The safest method of classing an M star would seem to be by estimating the strength of 4227, and the uncertainty introduced by luminosity is no greater than the uncertainty in resulting spectral class.[21] The use of 4227 in classifying M stars is suggested (since the present chapter was written) by Merrill.[22] The spectra of M stars, variable and constant, are still a fruitful field for analysis, and attention is drawn to the suggestions made in Section 66.

[21] Cf. Strömberg, Mt. W. Contr. 327, 1927; Luyten, H. Repr. 49, 1928.
[22] Pop. Astr., **37**, 452, 1929.

CHAPTER XII

THE HIGH LUMINOSITY STARS FROM F TO M

58. The Pseudocepheid.—The supergiant with metallic lines has received more study than the other subdivisions of c-stars. The division embraces all the Cepheid variables, and many other intrinsic variable stars, in addition to the typical "pseudocepheids" that are generally taken to define it. This chapter deals primarily with the invariable second-type supergiant, the variable being discussed in Chapter XIV.

a. Numbers and Distribution.—The recorded supergiants are distributed among the spectral classes as follows:

Class	Number of Stars (H. D.)	Number of Supergiants (H. D.)	Individual H-L Stars
A5	1,352	7	1
F0	3,208	23	..
F2	1,976	11	..
F5	3,504	21	..
F8	2,488	19	5
G0	2,784	28	5
G5	5,248	8	1
K0, K5	18,344	4	18

The H. D. numbers and spectral characteristics of 151 second-type supergiants are summarized in the Appendix.

b. Temperatures of Second-type Supergiants.—The few individual measures of the temperatures of supergiant stars show that they are cooler than normal (dwarf) stars of similar spectral class. The following temperatures have been compiled from the measures and discussions by Wilsing,[1] Hertzsprung,[2] Pettit and Nicholson,[3] Fessenkoff,[4] and Gerasimovič and the writer.[5]

[1] Pots. Publ. No. 72,

[2] Ann. Leiden Obs., **14**, 1922.

[3] Mt. W. Contr. 369, 1928.

[4] Rus. Astr. Journ., **4**, 169, 1927.

[5] H. B. 866, 1929.

TABLE XII, I.—TEMPERATURES OF SUPERGIANT F STARS

cFo		cF5		cF8		cGo	
	°		°		°		°
22 And	5890	α Per	5520	γ Cyg	5500	α Aqr	4460
14 Ori	7600	ϵ Aur	5300	ρ Cas	3850		
31 Cep	6560	φ Cas	5070	45 Dra	4260		
β CrB	7600	35 Cyg	5280				
		41 Cyg	6000				
Mean	6900	....	5400	....	4500	....	4450
Mean (normal stars)	6600	....	6100	....	5450	....	5100

The supergiant is about 700° cooler than the dwarf in these spectral classes, except at Class Fo, where the spectroscopic differences between normal and supergiant are at their smallest for the most conspicuous lines (those of hydrogen), and so no systematic difference of classification with temperature would be expected.

This difference of temperature should be compared with the mean difference of 300° found between the Cepheid variable and the stars of corresponding spectral class.[6] The suggestion is that the Cepheid is more luminous than the average, but not so luminous as the c-star of Classes F and G. This is in harmony with their spectral characteristics, which display a marked but not extreme c-character.

c. Variability of Radial Velocity.—A few Class F supergiants have been found to display variations of radial velocity that seem to be intrinsic, possibly indicating a vestigial pulsation that recalls the vestigial variations mentioned in a later chapter.[7] The following stars of Classes F, G, and K with these peculiarities are enumerated by Beer[8]: α Persei, ϵ Aurigae, ψ Aurigae, δ Canis Majoris, ρ Puppis, ξ Puppis, β Coronae Borealis, 105 Herculis, Boss 4817, 22 Vulpeculae, 35 Cygni, ξ Cygni, ζ Capricorni, γ Capricorni, ζ Cephei, Boss 5931, and

[6] See Chapter XIV, p. 209.
[7] See Chapter XIV, p. 247.
[8] Veröff. d. Universitäts-Sternw., Berlin-Babelsberg, 5, No. 6, 37, 1927.

56 Pegasi.[9] He points out that all are c-stars with the characteristic spectrum and a very large galactic concentration. ϵ Aurigae seems to have a slight variation in brightness[10] with a rather irregular period similar to that of the variations of radial velocity.

59. Spectroscopic Analysis of the Classes.—The general qualities of the c-stars of second type are summarized below. The arrangement is like that of Table XI, I, at the beginning of the previous chapter.

Class	Description	Interpretation
cA5	Hydrogen lines not particularly narrow, but total absorption low.[11] Metallic lines sharp	Halfway between maxima from normal stars (A) and for supergiants (F5)[12] In line with Class A2
cF0	Lines narrow. Ionized lines of metals relatively strong. Hydrogen rather strong	Low pressure
cF5 cF8 cG0 cK0	Lines strong. Ionized lines very strong; neutral weaker. Hydrogen strong. G band weak	Low pressure; extensive atmosphere
cK	Occasional bright lines	
cM	Hydrogen very strong; long-period variables, bright lines, especially strong for hydrogen	Low pressure

Measures of line intensity, and the consequent values of log NH, are contained in Tables XII, II to XII, IX, which are self-explanatory in form. Table XII, XII contains detailed data for four stars of Class F5, ranging from a bright supergiant to the dwarf star Procyon.

[9] β Doradus, included in the list, has been shown to be a genuine Cepheid variable (Shapley and Walton, H. C. 316, 1927), and the same is not out of the question for one or two others.

[10] Frl. Güssow, A. N., **232**, 207, 1928; Shapley, H. B. 858, 1928; Stebbins and Huffer, Pop. Astr., **36**, 306, 1928.

[11] The two stars for which the lines are stated as narrow are fainter than the eighth magnitude, and it is total absorption that is actually described. See Chapter III.

[12] See p. 269.

TABLE XII, II.—MEAN PERCENTAGE LIGHT LOSSES, CLASSES A5 TO F2

Line	A5 (7 Stars)	F0 (10 Stars)	cF0 (2 Stars)	c F2 (2 Stars)
Hβ	54	46	44	41
Hγ	55	49	48	58
4227	..	..	22	17
4215	..	12	..	..
Hδ	62	57	59	52
4077	..	16	..	..
4046	..	13	.	..
Hε	75	74	78	78
K	51	76	76	78
Hζ	71	..	..	57

TABLE XII, III.—MEAN LOG *NH*, CLASSES A5 TO F2

Line	A5	F0	cF0	cF2
Hβ	18.33	18.08	18.02	17.94
Hγ	18.36	18.18	18.16	18.45
4227	..	.	17.36	17.22
4215	..	17.07		..
Hδ	18.59	18.42	18.48	18.27
4077		17.19	.	..
4046		17.10		.
Hε	18.98	18.94	19.07	19.07
K	18.24	19.00	19.00	19.07
Hζ	18.86			18.42

TABLE XII, IV.—MEAN PERCENTAGE LIGHT LOSS, CLASSES F5 TO F8

Line	F5 (15 Stars)	cF5 (7 Stars)	F8 (4 Stars)	cF8 (6 Stars)
Hβ	41	44	36	38
Hγ	45	52	42	56
4227	24	33	34	38
Hδ	48	54	45	51
Hε	81	84	79	90
K	78	83	76	89
Hζ	..	56	..	..

TABLE XII, V.—MEAN LOG *NH*, CLASSES F5 TO F8

Line	F5	cF5	F8	cF8
Hβ	17 94	18 03	17.78	17 84
Hγ	18.06	18.29	17.96	18 39
4227	17 41	. .	18 06	18 24
Hδ	18 16	18 27	17 72	17 84
Hε	19 16	19 25	19 10	19 43
K	19 10	19 21	19 00	19 40
Hζ	. . . . .	18.39	. . . .	. . . . .

TABLE XII, VI.—MEAN PERCENTAGE LIGHT LOSS FOR CLASSES G0 AND G5

Line	Mean *dl*, G0 (10 Stars)	Mean *dl*, cG0 (4 Stars)	Mean *dl*, G5 (27 Stars)
Hβ	31	46	24
Hγ	33	52	33
4326	21	. .	31
G	34	. .	50
4227	27	55	39
4215	13	. .	29
Hδ	34	51	32
4077	22	. .	31
4046	23	. .	31
Hε	74	78	78
K	75	76	73

TABLE XII, VII.—MEAN LOG *NH* FOR CLASSES G0 AND G5

Line	Mean G0	Mean cG0	Mean G5
Hβ	17 63	18.08	17.42
Hγ	17.69	18 27	17.69
4326	17.33	. . . . .	17.63
G	(17.72)	. . . . .	(18.21)
4227	17.51	18.36	17.87
4215	17.10	. . . . .	17.57
Hδ	17.72	18.24	17.66
4077	17.36	. .	17.63
4046	17.39	. . . . .	17.63
Hε	18.94	19.06	19.06
K	18.97	19.00	18.91

TABLE XII, VIII.—MEAN PERCENTAGE LIGHT LOSS FOR CLASSES Ko, K2, AND K5

Line	Mean *dl*, Ko (64 Stars)	Mean *dl*, cKo (3 Stars)	Mean *dl*, K2 (10 Stars)	Mean *dl*, cK2 (1 Star)	Mean *dl*, K5 (11 Stars)	Mean *dl*, cK5 (2 Stars)
Hβ	23	26	16	34	19	30
Hγ	29	36	28	43	31	42
4326	36	..	35	..	41	40
G	57	.	50	..	58	60
4227	51	57	60	56	70	52
4215	37	..	38	..	37	..
Hδ	32	43	34	..	31	45
4077	34	..	36	..	37	..
4046	39	..	42	.	48	..
Hε	81	88	87	..	79	76
K	78	86	88	..	73	..

TABLE XII, IX.—MEAN LOG *NH* FOR CLASSES Ko, K2, AND K5

Line	Mean Ko	Mean cKo	Mean K2	Mean cK2	Mean K5	Mean cK5
Hβ	17.39	17.48	17.18	17.72	17.27	17.60
Hγ	17.57	17.72	17.54	17.99	17.63	17.96
4326	17.78	.. ..	17.75	. ...	17.93	17.90
G	(18.42)	....	(18.21)	. .	(18.42)	(18.52)
4227	18.24	18.42	18.52	18.39	18.82	18.27
4215	17.81	.. ..	17.83	. ..	17.81	. .....
Hδ	17.66	17.99	17.72	.. ..	17.63	18.05
4077	17.72	. ...	17.78	. ..	17.81	....
4046	17.87	...	17 96	. ...	18.12	....
Hε	19.15	19.36	19.33	.. ..	19.09	19.00
K	19.06	19.30	19.36	. ...	18.91	....

In Table XII, XIII are summarized the basic data for the physical comparison of the normal second-type star with the supergiant—data regrettably meager, but quantitative, though of only slight precision. It appears that the hydrogen lines are on the average about twice as strong in supergiants as in normal stars from Go to K5, not strengthening noticeably with advancing class, although in Class M the supergiant has about a hundred times more hydrogen. From the data on hydrogen

TABLE XII, X.—MEAN PERCENTAGE LIGHT LOSS IN CLASS M

Line	Mean *dl* (5 Stars)	Log *NH*
Hβ	18	17.24
Hγ	25	17.45
4326	44	18.02
G	57	(18.42)
4227	68	18.76
4215	40	17.90
Hδ	40	17.90
4077	37	17.81
4046	47	18.12
K	80	19.12

TABLE XII, XI.—MEAN FOR CLASS M FROM HOGG'S DATA, K LINE

	K2	K5	Mo	M1	M2	M3	M5
Mean *dl*	82	74	80	77	89	78	77
Log *NH*	19.18	18.94	19.12	19.03	19.39	19.06	19.03

TABLE XII, XII.—ANALYSIS OF STARS OF CLASS F5

Atom	ι2 Scorpii		b Velorum		ε Aurigae		α Canis Minoris	
	dl	Log *NH*	*dl*	Log *NH*	*dl*	Log *NH*	*dl*	Log *NH*
Fe	18	17.3655	18	17.3655	18	17.3655	16	17.3075
Fe^+	19	17.3979	17	17.2695	18	17.3655	14	17.0043
Ca	21	17.4771	18	17.3655	18	17.3655	17	17.2695
Ca^+	75	18.9777	75	18.9777	75	18.9777	75	18.9777
Sr^+	24	17.6021	22	17.5105	14	17.0043	12	16.90:
Log (Fe) + (Fe^+)	17.68		17.59		17.64		17.48	
Log Fe^+/Fe	0 03		−0.04		−0.04		−0.30	
Log Fe/Fe^+	−0.03		0.04		0 04		0.30	
Log (Ca) + (Ca^+)	19.00		18.99		18.99		18.95	
Log Ca^+/Ca	1 50		1.61		1.61		1.71	
Percentage ionization of calcium	96		97		97		98	
Percentage ionization of iron	52		48		48		33	

tabulated by Adams and Russell[13] we infer that there is *less* hydrogen in the atmosphere of α Persei (cF5) than in that of Procyon (dF5); and probably rather less in γ Cygni (cF8) than in the sun (dGo).[14]

The H and K lines differ but little, being stronger in the supergiants of Classes Go and Ko, slightly weaker in K5 (on the other side of the maximum). Neutral calcium is far the stronger in Class cGo, rather stronger in cKo, rather weaker in cK2, and much weaker in cK5.

The data of Adams and Russell permit the comparison of the cF5 and dF5 stars, and the cF8 and cGo stars, α Persei and Procyon, γ Cygni and the sun. The results, which are very tentative, are summarized below:

TABLE XII, XIII.—COMPARISON OF GIANT AND SUPERGIANT

Element	dF5 to cF5	dGo to cF8
Iron	Slightly weaker	Stronger
Titanium	Weaker	Slightly weaker
Calcium	Weaker	Slightly stronger
Magnesium	Slightly weaker	Slightly stronger
Manganese	About equal	Slightly stronger?
Chromium	Stronger?	Weaker?
Vanadium	Weaker	Weaker
Sodium	Weaker	Stronger
Iron+	Much stronger	Much stronger
Titanium+	Much stronger	Much stronger
Barium+	Stronger	Much stronger
Scandium+	Much stronger	Much stronger
Yttrium+	Much stronger	Much stronger

On the average we may take "slightly" to cover everything from twice to three times as strong; "stronger" or "weaker" to imply four to nine times as strong; and "much" to imply

[13] Mt. W. Contr. 359, 1928.

[14] The behavior of iron in the supergiant F5 spectrum should be compared (Chapter XVI, p. 270), and also that of the hydrogen lines in the spectra of Cepheid variables (Chapter XIV, p. 212); the maximum for hydrogen in normal stars is at a higher temperature, and for supergiants at a rather lower one.

ten times, or more. Summarizing the data, we find that the supergiant F5 star has neutral lines rather weaker than those of the corresponding dwarf; and ionized lines considerably stronger. The supergiant F8 star (notwithstanding its slightly earlier spectral class) has all lines stronger than in the sun—neutral lines slightly and ionized lines considerably strengthened. We note that α Persei (-1.8) is probably much less luminous

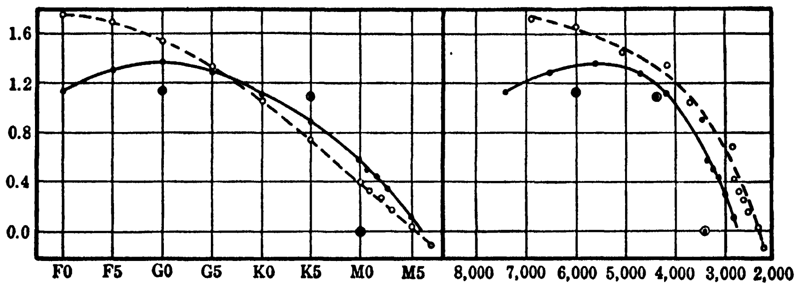

FIGURE XII, 1.

Ionization of calcium along the spectral sequence. Left side: ordinates and abscissae are log (ionized calcium/neutral calcium) and spectral classes. Circles, dots, and circled dots represent supergiants, giants, and dwarfs. Right side: ordinates and abscissae are log (ionized calcium/neutral calcium) and temperature. Note that at any one temperature, supergiants show the greatest ionization of calcium, dwarfs the least. The reason that the ionization of giants has a maximum at about 5400°, and that for supergiants shows no maximum, is not obvious; if the fall in the ratio with rising temperature were the result of the progress of second ionization, the supergiant should show a maximum at lower temperatures than the giant.

than γ Cygni for which an absolute magnitude of -3.5 may be assumed.

These quantitative data will at some time permit a quantitative test; but we must confine ourselves here to qualitative comments. The stronger hydrogen in the brighter stars is in good accordance with Milne's prediction[15] on the basis of an absorption coefficient varying with pressure. The behavior of the calcium lines is the product of the interplay of atmospheric extent and ionization—the former preponderating at higher temperature, the latter in Classes K2 and K5. We recall the abandonment of the 4227 line in Class K5 as an index of absolute

[15] M. N. R. A. S., 89, 3, 17, 1929.

magnitude for normal second-type stars, due to the subordination of pressure to temperature effects.

The second half of Table XII, XII analyzes the spectra of four F5 stars arranged in order of decreasing brightness. The two last lines give the percentage ionizations of calcium and iron, respectively—the former decreasing with luminosity, the latter increasing. Here, again, there are two effects playing each other out; temperature would seem to preponderate for calcium, pressure for iron. In further illustration of the ionization of calcium, the percentage ionization throughout the spectral classes for normal stars is shown in Figure XII, 1. Supergiant stars of any one spectral class should fall below the curve at low temperatures, above it at high ones.

For α Persei, another cF5 star, Adams and Russell[16] deduce about six times as much atmosphere of metallic atoms as the sun has; comparing Procyon with the supergiant F stars of similar class I find between twice and thrice as much iron atmosphere. The results by both methods are very rough at present, and the lack of precise agreement need not disquiet us.

60. Detailed Analysis of the Supergiant Spectrum.— The ultimate purposes of spectrum analysis can be served only by detailed and quantitative work. Though stellar spectroscopy is still in its pioneer days, we look forward to directness of purpose and freedom from problems of methodology. In clearing the ground for detailed study much laborious work in identification and measuring is essential. The first really detailed study of any star other than the sun has recently been published by Dunham[17] for α Persei.

The study consists of accurate wave lengths and arbitrary intensities of 1,300 stellar lines, and identification of the majority with laboratory lines. For the most part it is basic to further researches in furnishing accurate wave lengths of unblended or little blended lines, and a quantitative study of the star itself

[16] *Loc. cit.*

[17] Princeton Contr. No. 9, 1929.

must obviously await accurate intensities.[18] But some features of the discussion have a direct bearing on the subject in hand.

The lines, as a whole, are both *wider* and *deeper* than those of the solar spectrum. This, almost the only published observation of its kind, is of immense importance. It shows the inadequacy of the existing contour formulae; the measured residual intensity is less likely here than anywhere to be instrumental—least of all for the sun. Incidentally the lines of an ac star are conclusively shown to be wider than those of a dwarf of similar temperature.

Dunham's detailed remarks on the lines of the 26 elements definitely identified are of great interest. The lines of neutral carbon (excitation potential 7.45), nearer their maximum in α Persei than in γ Cygni, will ultimately provide another test of the laws of ionization. Neutral iron dominates the spectrum; ionized iron is almost as strong. Reference should be made to Table XII, XII, where a similar result is indicated. It is noteworthy that the zinc lines at 5894, 6214 are absent from the spectrum.[19]

A comment on the degree of ionization in the spectrum is worth quoting: "In the visible region neutral and enhanced lines are almost equally conspicuous. The limited band of spectrum which we can observe happens, however, to be so placed as to favor the neutral lines to a marked degree . . . Thus a casual estimate of the degree of ionization in a stellar atmosphere based on the relative intensities of enhanced and neutral lines in the available region will usually be much too low."

[18] The author provides a method of expressing intensities approximately in terms of the Rowland scale, which has been independently calibrated by Russell, Adams, and Miss Moore (Mt. W. Contr. 358, 1928).

[19] Zinc (ionization potential 9.35 volts) was observed by Adams and Joy (P. A. S. P., **34**, 177, 1922) in the Class A spectrum, presumably at or near its maximum. Menzel's doubts on this identification seem somewhat unfounded; with small dispersion the observations are difficult (H. C. 258, 1924).

CHAPTER XIII

THE RED SUPERGIANT

THE split in the spectral sequence[1] is represented almost entirely by supergiant stars; no evidence of dwarfs of Classes N and S has been found, and the N stars at least are among the most luminous known. Stars of Class R are somewhat less bright. Class S is too poorly populated for a definite estimate of absolute brightness to be possible, but the long mean period and the intimate spectroscopic affiliation with Class M both point to considerable luminosity.

61. The Supergiants of Class M.— In Class M, the accepted criteria for supergiants are still spectroscopic, as in earlier classes, but these criteria no longer involve the c-character. We are led statistically to associate certain abnormal strengths and weaknesses of line with very high luminosity in Class M. Excluding the long period variables (which are considered in Chapter XIV) we can summarize the outstanding spectral peculiarities very briefly.

Spectral Peculiarities of Supergiants in Class M.—The spectra are exceedingly difficult to discuss in detail, for they are riddled with lines, and so much blended that satisfactory separation is quite impossible except for very high dispersion; in addition they are cut up by several sets of absorption bands. Hydrogen is abnormally strong for them all; neutral calcium (4227) unusually weak—the early Harvard work[2] considered that both elements behaved as they do in the spectra of stars of

[1] The parallel course of the K-M, K-S, and K-R-N branches of the spectral sequence was first explicitly regarded as a split in the sequence by Rufus (Publ. Obs. Mich., **3**, 258, 1923).

[2] H. A., **28**, 1897.

the "previous group" (K5). H and K are not dissimilar in strength to the same lines in normal M stars;[3] iron (4046) and ionized strontium (4215) lines are somewhat stronger in the spectrum of the supergiant. The $a^5D - a^7F$ lines of neutral iron were noted by Baxandall and Stratton[4] as very strong in the spectrum of Betelgeuse, less so in those of Mira and R Doradus; the $a^5D - a^7P$ lines are also strong, though less conspicuously so.

The most detailed spectroscopic study of the M supergiants available at present is contained in the very condensed quantitative analysis made by Adams and Russell[5] on the basis of their calibration[6] of the Rowland scale. The calibration, which is of a semiempirical nature, was designed for converting Rowland intensities directly into relative numbers of atoms per square centimeter column (equivalent to the NH of the Unsöld determination), and hence also for converting into numbers of atoms estimates of line strength for stars other than the sun, when such estimates are made by direct comparison with a solar spectrum of the same dispersion. The method is one of great power, especially for the fainter lines that are irresolvable and instrumentally blurred by small instruments, and there is no doubt that the spectral analysis of the future will employ it. It is probable that the success of the method will depend largely on the slight exposure of the long-dispersion spectrograms used and the consequent dominance of line width over other factors affecting intensity estimates; and for the same reason the method cannot be applied with lesser dispersion.

The results of Adams and Russell are so important that I reproduce in the next table their data for M supergiants (mean of Betelgeuse and Antares; only for iron do the data for these two stars differ sensibly).

[3] From visual estimates; no great difference in line contour was found by Hogg (H. B. 859, 1928) for α Orionis and other M stars.

[4] Baxandall and Stratton, Obs., **52**, 148, 1929.

[5] Mt. W. Contr. 359, 1928.

[6] Mt. W. Contr. 358, 1928.

TABLE XIII, I.—DATA FOR SUPERGIANTS OF CLASS M, FROM ADAMS AND RUSSELL

Atom	E. P.	Log N_1/N_2	Atom	E. P.	Log N_1/N_2	Atom	E. P.	Log N_1/N_2
Fe	0.06	2.18	Ca	0.00	2.02	V	0.03	2.66
	0.94	1.26		1.88	0.26		0.28	2.31
	1.54	0.76		2.51	0.16		1.08	2.24
	2.17	0.38		2.81	−0.26		2.07	2.18
	2.56	−0.06						
	2.89	−0.11	Mg	0.00	1.15	Fe+	2.83	−0.67
	3.24	−0.22		2.70	0.05		3.56	−0.96
	3.47	−0.40		4.33	−0.80			
	4.03	−0.58				Ti+	1.16	0.16
	4.50	−0.82	Mn	0.00	2.65		1.57	0.38
				2.20	1.29		2.00	−0.07
Ti	0.03	3.10		2.94	0.02		2.93	−0.32
	0.83	1.34						
	1.02	2.00	Cr	0.99	1.38	Ba+	0.00	1.35
	1.44	1.86		2.89	0.27		0.65	0.90
	1.82	1.38					2.50	0.55
	2.09	1.28	Sc	0.00	3.26			
	2.26	1.10		1.44	2.05	Sc+	0.60	0.75
	2.50	0.88		1.93	1.65		1.50	0.16

The most important question that arises in applying these data to the analysis of the atmospheres is the uniformity of the calibration. The method used by Russell, Adams, and Miss Moore[7] depends for its scale on formulae for multiplet intensities which are only approximately checked in laboratory investigations, and whose extension to stellar absorption lines requires careful analysis before discussing the results derived from the process. The authors themselves have pointed out that the formulae are approximate but probably give "correct average results." When this is borne in mind the data given in the paper have considerable application.

The entries in Table XII, XIII present the logarithms of the number of atoms per square centimeter column in terms of the number in the solar atmosphere. As discussed in the orig-

[7] *Ibid.*

inal paper, the relative number appears to be a function of excitation potential, greater for small excitation potentials. If the data of Table XII are plotted, we can deduce graphically the effective excitation potential at which the number of atoms would be the same at the surface of the sun and the M supergiant; in many cases this excitation potential is within the observed range:

Atom	Effective Excitation Potential	Difference, I. P.-E. P.
Fe	2.68	5.63
Ti	0 60	5.90
Ca	0 58	5.51
Mg	2.85	4.76
Mn	3.00*	4.41*
Cr	3.13*	3 59*
Ba+	4.00*	5 96*
Ti+	1 92	11.68
Fe+	1 0:*	11.0:*
Sc+	2 0:*	11 5:*

* Extrapolated.

We note that assuming Milne's generalized Saha equations for the same element, at the same excitation potential, for two stars for which the pressures, temperatures, and surface gravities are designated by P', P'', T', T'', g', and g'', these quantities are related by the expression

$$\frac{g''P'^2}{g'P''^2} = e^{(I - E)/k(T' - T'')}\left(\frac{T''}{T'}\right)^{5/2}$$

Carrying through the computations, and assuming T' (sun) $= 5600°$, T'' (Betelgeuse) $= 2640°$, and using the excitation potentials just tabulated, we obtain

$$\frac{g'}{g''} = \frac{1.202P''^2}{P'^2}$$

for the neutral atoms, and

$$\frac{g''}{g'} = \frac{2.54P''^2}{P'^2}$$

for the ionized atoms (excluding Ba+, which rests entirely on extrapolation and is very uncertain).

Adams and Russell give the values of surface gravity (in terms of the sun) as 0.0005 for Betelgeuse and 0.00025 for Antares. Adopting 0.0004 for the M supergiant we derive a pressure of 1.66×10^{-2} in terms of the solar atmospheric pressure. The pressure derived for the ionized lines, as a result of their great strength, is about ten times as much. Adams and Russell commented on the great intensity of the ionized lines, which led in the hands of these authors to very low pressures. The pressures here derived are more nearly what would be expected on general grounds.

Incidentally it may be noted that the numerical results just obtained give pressures very nearly but not quite proportional to the square root of the surface gravity.

Adams and Russell consider that their measures afford evidence of very extensive chromospheres for the supergiants of Class M—a hundred times as many metallic atoms per unit area as for the sun. The behavior of hydrogen is even more striking; the lines of this element are abnormally strong in the M supergiants—as strong as in Procyon (dF5), according to Adams and Russell. They regard this as evidence of an enormous abundance of hydrogen in the atmospheres of the supergiants: "with the smallest plausible allowance for the effect of the ionization potential . . . the abundance of normal hydrogen must be thousands of times greater in Antares or Betelgeuse than in the sun." As about 96 per cent of the atoms in the solar atmosphere are found by Russell to be hydrogen,[8] this leads to the conclusion that all but 10^{-5} of the atmospheres of supergiants in Class M consists of hydrogen. This abundance is "almost incredibly great." Adams and Russell point out that two other unexplained observations would be explained if the abundance were real—the discrepancy between electron pressures derived from ionization and from numbers of atoms; and certain observations connected with the flash spectrum. The consequences of high abundance of hydrogen in the sun should of course be reproduced in greater intensity

[8] Ap. J., **70**, 11, 1929.

for M supergiants if the abundance of hydrogen is as great for them as estimated above.

From ionization, Adams and Russell derived pressures of about 10^{-8} dynes per square centimeter for iron, 10^{-7} dynes per square centimeter for titanium, and 10^{-6} dynes per square centimeter for scandium; the results from intensities of lines, derived as for the sun by Russell, give about 100,000 dynes per square centimeter—a value ridiculous in itself and also when compared with the value obtained from ionization, which is about 10^{-13} of it. The introduction of a factor of 10^5, introduced in the same way as the abundance of hydrogen was introduced for the sun by Russell, alleviates the discordance but does not remove it. Apparently the enormous abundance of hydrogen in the atmospheres of red giant stars is real. But it must be remembered that the occurrence of the hydrogen lines in any star (giant and dwarf) of so low a temperature is very puzzling—with any plausible value for the Boltzmann factor there would be too few atoms to show, if the number involved at Ao is about 10^{20}.

62. Number of M Supergiants.—The normal M supergiant (without emission lines) is apparently rather frequent. Adams, Joy and Humason,[9] in their list of 410 M stars for which spectroscopic parallaxes have been obtained, give 28 stars, which they consider supergiants, of absolute visual magnitude brighter than -1.0. Their list is reproduced in Table XIII, II.

It is of interest that two stars in the list have composite spectra; Antares and δ Sagittae present an interesting study in the violet, where the hydrogen lines—evidently of an A star—become prominent, almost washing out the K line completely.[10] As Shajn has apparently shown quite convincingly[11] the composite spectra emanate from pairs of stars, not one star under unusual conditions. It is conceivable that the strength

[9] Mt. W. Contr. 319, 1926.
[10] H. A., **28**, 99, 100, 1897.
[11] A. N., **228**, 337, 1926.

TABLE XIII, II.—SUPERGIANTS OF CLASS M (MOUNT WILSON)

Boss Number	Star	Spectrum	Absolute Magnitude	Boss Number	Star	Spectrum	Absolute Magnitude
. .	+61°8	M2ep	−1.3	1549	6 Gem	M2	−1.4
. .	+56°547	M3	−1 4	1606	ψ^1 Aur	Mo	−2.7
. .	+56°551	Mo	−1.4	1743	....	Mo	−2 1
.	+57°550	M2	−1.4	1810	σ CMa	Mo	−2 1
..	+55°597	M4	−1.3	1887	....	Mo	−2 2
.	+56°595	M1	−1.8	1985	... . .	M3ep	−1.5
. ..	+56°597	Mo	−1.7	4193	α Sco	M1	−3.5
....	+56°609	M1	−1.1	4373	α Her	M5	−1.6
765	Pi 27	Mo	−1.8	....	−15°4502	M1	−1.1
864	Pi 121	M1	−1 1	4800	δ^2 Lyr	M4	−1.4
....	+29°897	M1	−2.4	5052	δ Sge	M2	−1.0
1335	119 Tau	M2	−3.0	5458	Pi 61	M2	−1.6
1468	α Ori	M2	−4.3	5593	ω Cep	M2	−3.0
1479	π Aur	M3	−1.0	5650	Pi 360	M2e	−2.0
....	+23°1243	M3ep	−1.3	5804	5 Lac	Mo	−2.1

of the hydrogen lines is often to be referred to a companion[12] (the strength of the K line should reveal the fact if that were the case). The companion of Mira and the spectroscopic orbits of Antares and Betelgeuse[13] have suggested companions for three bright M stars, and possibly duplicity (which Shajn[14] shows to be associated with great brightness) assists in imposing a special mark on the spectra of many M stars—supergiants would not necessarily have companions, but those that had companions would be spectroscopically detected.

63. Variability of Supergiant M Stars.—All the observed variations of M stars are probably intrinsic; they need only be summarized here very briefly.

a. Variations of Brightness.—The long-period variables[15] have been shown to be supergiant stars, chiefly of Class M; their range in visual magnitude is in general large[16]—usually

[12] Cf. Miss Cannon's comment on the spectrum of Betelgeuse.

[13] Spencer Jones, M. N. R. A. S., **88**, 675, 1928.

[14] *Loc. cit.*

[15] Shapley, H. Repr. 53, 1928.

[16] Pettit and Nicholson, Mt. W. Contr. 369, 1928.

about five magnitudes—as against a bolometric range of about a magnitude. Their magnitudes and temperatures are thus summarized by Pettit and Nicholson:

Phase	Mean Radiometric Magnitude	Heat Index	Water-cell Absorption	Temperature from Heat Index °	Temperature from Water Cell °
Maximum	1.3	4.4	1.50	1990	2350
Median	1.8	6.6	1.84	1550	2060
Minimum	2.2	8.9	2.17	1350	1830

The irregular variables of Class M have also been shown[17] to be supergiants in luminosity; their spectra are discussed under "Variable Stars."

The recognized M supergiants raise the question whether they are intrinsically more variable in light than less luminous giant M stars which have been shown by Stebbins and Huffer[18] to be without exception variables. Of the stars listed as supergiant M stars in Table XIII, II (Boss 1335, 1468, 1616, 4800, 5593, and 8383) 6 were found by Stebbins and Huffer to have ranges greater than a quarter of a magnitude; out of the 30 stars enumerated there they must have examined 18—their limits were $-10°$ south, and the sixth visual magnitude—so about one third of the supergiants are definitely variable. Furthermore, of the objects on their program, which covered 164 stars, 24 proved to have ranges as large as a quarter of a magnitude—about seven per cent, a much smaller fraction. In other words the supergiants appear to be definitely more variable. Stebbins and Huffer state: "In Classes M2 and M3 the variables clearly average brighter than the constant stars. In all the classes, stars brighter than the absolute magnitude -1.0 are variables except the three stars of spectra M1 and M2, of which there are only a few measures. Either extreme luminosity or extreme redness seems to go with variability."

Aside from Stebbins and Huffer's measures, it is known that some of the red stars in the Perseus clusters are variable,[19]

[17] See Chapter XIV, p. 240.

[18] P. N. A. S., 14, 491, 1928.

[19] Waterfield, unpublished.

so that the percentage of variable supergiants is perhaps even greater than just suggested. But on the other hand, Antares, one of the more conspicuous red supergiants, has never been reported as variable; it has not been examined with precision, and might possibly be found to vary, although Spencer Jones notes,[20] in connection with its changes of radial velocity, that changes in its diameter have not been recorded either. There is a suggestion that the irregular variability of the M supergiant is, like that of other M stars, occasioned by changes of surface temperature which operate by strengthening or weakening the band absorption in the spectra.[21]

b. Variations of Diameter.—The interferometer has provided data on variations of diameter for only one M supergiant—Betelgeuse. The angular diameters and deduced radii (in terms of the sun's radius) that have been measured for red stars are summarized in the next table.[22]

Star	Angular Diameter	Radius	Date
Arcturus	0.020	27	
Aldebaran	0.020	38	
Betelgeuse[23]	0.047	300	1921
	0.055	350	1922
	0.034	210	1923
	0.041	260	1924
	0.044	280	1925
	0.034	210	1926
	0.041	260	1927
	0.037	230	1928[24]
Antares	0.040	450	
β Pegasi	0.021	40	
α Herculis	0.030	400	
Mira Ceti (maximum)	0.056	300	

The measured changes in the radius of Betelgeuse amount to 50 per cent, which seems enormous in comparison with the

[20] M. N. R. A. S., **88,** 676, 1928.

[21] Payne and ten Bruggencate, H. B. 876, 1930.

[22] Russell, Dugan, and Stewart, **2,** 749, 1927.

[23] Spencer Jones, M. N. R. A. S., **88,** 675, 1928.

[24] Pease, Mt. W. Rep., 1928.

radial changes of 10 per cent inferred for Cepheids.[25] The error of observation is probably considerable.

c. Variations of Radial Velocity.—Seven of the stars in Table XIII, II appear with variable radial velocities in the Third Catalogue of Spectroscopic Binaries:

Star	Range, km./sec.	Reference
Boss 864	16	Merrill, L. O. B., **6**, 143, 1911
Boss 1606 (ψ Aurigae)	6	Olivier, L. O. B., **6**, 145, 1911
Boss 5052 (δ Sagittae)	14	Campbell, L. O. B., **4**, 96, 1906
Boss 5593 (μ Cephei)	13	Lick, L. O. B., **7**, 102, 1912
Boss 5650	17	Lick, P. A. S. P., **34**, 169, 1922
Betelgeuse	5	Spencer Jones, M. N. R. A. S., **88**, 660, 1928
Antares	5	Spencer Jones, M. N. R. A. S., **88**, 660, 1928

Nine other stars of Class M occur in that catalogue: Boss 826, η Geminorum, p^2 Leonis, ϵ Muscae, F Centauri, 4 Draconis, R Lyrae, λ Ursae Minoris, and o Pavonis, none of which has the spectral peculiarities associated with M supergiants. We suspect that R Lyrae, at least, is of comparable brightness;[26] and the similarity of its range in radial velocity (11 kilometers) with those tabulated is also suggestive that the stars are similar. The ranges of the other stars mentioned are from 7 to 16 kilometers.

Here again it appears that M supergiants are more prone to variability of radial velocity than less luminous M stars; one third of the M supergiants listed are of variable velocity, and only one tenth of those that are not supergiants—out of all proportion to their relative numbers.

The most significant analyses of the radial velocities are those made by Spencer Jones for Betelgeuse and Antares.[27] He found periodic variations, with superimposed irregular variations of short period, and deduced orbital elements for both stars. The elements previously derived elsewhere had been shown to be unsatisfactory. He showed that the changes

[25] Chapter XIV, p. 201.
[26] See Chapter XIV, p. 241.
[27] M. N. R. A. S., **88**, 660, 1928.

in brightness could be definitely correlated with changes in radial velocity and that the changes in diameter of Betelgeuse were possibly related to the velocity changes. The resulting amplitude of radial variation is enormous, as the periods are five and seven years, respectively; Spencer Jones discussed these with reference to the period-density relation required by a pulsation hypothesis, which he regarded with favor.

The analogy with long period variables and Cepheids is suggestive, especially as Betelgeuse is variable. But it is surprising that these two stars do not vary more in brightness than is observed, when they are compared with long period variable stars, which undergo comparable radial changes; and that their spectra do not show the conspicuous emission lines of the latter. The spectral and radial velocity variations of the small-range M variables[28] are closely related.

It is likely that the radial velocities of other stars enumerated in Table XIII, II would yield results similar to those given by Betelgeuse and Antares, and a discussion of the problems raised would be more relevant if we knew how far these two bright supergiants are typical.

64. Stars of Class N.—The catalogues of N stars differ somewhat in content; Espin[29] enumerated a considerable number of N stars on colorimetric grounds, but many of the stars in his lists have been shown by Miss Cannon to be of Class K or R. There are 146 stars of Class N in the Henry Draper Catalogue, and the following paragraphs refer to them only, although probably some additional true N stars are contained in Espin's lists.

The N stars have a strong galactic concentration; 82 per cent of them are within 20° of the galactic plane. Of the related class of R stars but 50 per cent are within the same limits. Espin derived a very similar galactic concentration for the N stars in his lists.[30]

[28] See Chapter XIV, p. 240.
[29] M. N. R. A. S., **58**, 443, 1898.
[30] Ap. J., **10**, 169, 1889 (224 stars).

The distribution in galactic longitude is very uniform; for the four intervals 0–90, 90–180, 180–270, and 270–360, the numbers are 38, 31, 36, and 40, respectively. There is no great increase in numbers toward the galactic center—rather there is a slight deficiency between 90° and 130°. Here again there is a marked contrast with Class R, which shows concentration to the galactic center.

The luminosities of the N stars are undoubtedly very high, and no dwarf N stars are known. Eighty-four Class M dwarfs

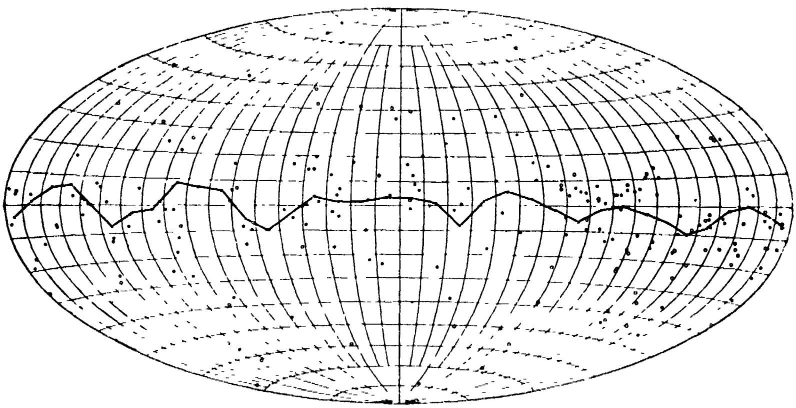

FIGURE XIII, 1.
Galactic distribution of N stars (dots) and stars of Class R (circles).

are known,[31] as against about sixteen hundred presumable giants, about five per cent of the whole; a similar proportion among the N stars would require seven N dwarfs. Thus the N dwarfs are either far less numerous in proportion (perhaps absent altogether), or absolutely much fainter than dwarf M stars.

The existing determinations of the absolute magnitudes of the N stars are summarized in Table XIII, III. The tabulated absolute magnitudes are visual.

Evidently −2.5 is about the right value for the absolute visual magnitudes; bolometrically the stars must be far brighter, as their temperatures are about 2600°.

[31] Luyten, M. N. R. A. S., **86**, 48, 1925.

TABLE XIII, III.—ABSOLUTE MAGNITUDES OF STARS OF CLASS N

Authority	Reference	Absolute Visual Magnitude	Remarks	No. of Stars
Luplau-Janssen and Haarh	A. N., **214**, 383, 1921	−2 6	Parallactic motion	
Kapteyn	Ap. J., **32**, 91, 1910	−2.6	Parallactic motion	120
Moore	L. O. B., **10**, 160, 1922	−1 5		
Moore	P. A. S. P., **35**, 124, 1923	−1 4	Class Na	20
		−2 4	Class Nb	62
		−4.2	Class Np	9
Wilson	A. J., **35**, 125, 1923	−1 1	Class Na	
		−2 4	Class Nb	
Galactic dip, D* = 33 pc	. . . . . .	−2 5	. . . . .	146
Galactic dip, D = 50 pc	. . . . . .	−3 5	. . . .	146
Variables, D = 33 pc	. . . .	−3 9	. . . .	119
Variables, D = 50 pc	. . . . . .	−4 9	. . . . .	119

* D = distance from the galactic plane.

The variability of the N stars is one of their most obviously characteristic features; probably all N stars vary to some extent, the ranges being from several magnitudes to a few tenths. Several classes of variables are represented by N stars, but most of the N stars are probably irregular variables of small range.

Variable Class	Number of Stars Class N	Class R
Long period	21	3
R Coronae	. .	2
Novae	1	.
Cepheids	. .	4 (probably spurious)
Irregular	36	3
Unknown type	61	7

The difference between M and N stars is apparently not fundamental, as the same types of variation occur for both. The N stars of long period class are more than usual subject to irregularity, and their periods on the average are far longer than for Me stars—this probably is a correlate of the low mean density that is suggested by their high luminosity and great redness.

The N stars are connected to the main series of classes by way of the R stars, and so a comparison of the two classes is appropriate:

	Class N	Class R
Numbers (H. D. C.)	146	66
Galactic concentration	82	50
Angular dip	0.89	10.3
Absolute visual magnitude	− 2.5	+ 1.7
	− 3.5	+ 0.7
Effective temperature[32]	2000° to 2300°	4000[33]
Mass (× sun)	20	2

65. Spectroscopic Analyses of N Stars.—Extensive studies of the N stars were begun by Hale, Ellerman, and Parkhurst[34] and have been pursued chiefly at the Lick Observatory by Moore[35] and by Shane.[36,37] The conspicuous feature of the N spectrum is the bands of "carbon," but the metallic spectra that occur are of more immediate value in examining the physical conditions. From the description and plates given by Shane[36] we select the following points as of importance:

Absorption lines of hydrogen are absent or very faint.

Emission lines of hydrogen, variable in intensity, occur for some (variable) N stars.

Neutral iron lines, especially low-temperature lines, are very prominent.

The D lines of sodium are stronger than in any other class (three or four Angstroms wide in 152 Schjellerup and 19 Piscium).

In the variable N stars (such as U Cygni) the D lines may be from 50 to 100 Angstroms in width.

The 4227 line of calcium is very strong.

The neutral lines of titanium, vanadium, scandium, and yttrium are prominent.

[32] Pettit and Nicholson, Mt. W. Contr. 369, 1928.

[33] The rough estimate of effective temperature is made on the basis of Merrill's estimates of relative intensity of the red and blue portions of the spectra of B. D. −24° 12084 and B. D. +42°2811 in Scientific Paper of the Bureau of Standards No. 318, 1918. "These members," he says, "show in a definite way the wide divergence from Class N and the similarity to Class K . . . The correspondence of these two stars is apparently to an early rather than a later subdivision of Class K."

[34] Publ. Yerkes Obs., 2, 253, 1904.

[35] L. O. B. 342, 1922.

[36] L. O. B. 329, 1920.

[37] L. O. B. 396, 1928.

The two most definite conclusions to be drawn are from the hydrogen and the sodium lines. Both facts point to temperatures lower than for normal M stars—a conclusion borne out in general by the color indices and radiometric temperatures, if we exclude the Me stars from consideration. If the D lines are 100 Angstroms wide (we assume that this width is measured for at least 4 per cent light loss, though a greater contrast factor is more likely) they are by far the strongest lines recorded anywhere in the stellar sequence—about twice as wide as hydrogen when most powerful. The corresponding number of sodium atoms per square centimeter surface is about 10^{21}, and even supposing sodium to be a common substance, as we know it is, this argues for a very low temperature— of the order of 2000°, in agreement with radiometric measures.[38] The contrast in width of the D lines for the small-range or constant stars 19 Piscium and 152 Schjellerup and the variables such as U Cygni, RS Cygni, and R Leporis suggests that the latter are at considerably lower temperatures, as are the Me stars when compared with absorption M stars. In other words, observation indicates that in the atmosphere of a red variable star, ionization and excitation are proportionately stronger than for an invariable or small-range star of similar temperature.

The great strength of the lines of titanium, vanadium, scandium, and yttrium is noteworthy but unelucidated.

66. Interrelationships between the Red Types.— In comparison of the spectra of N and R stars Rufus[39] gives the following arbitrary estimates for lines of neutral calcium:

Line	Excitation Potential	Class N (3 Stars)	Class R (10 Stars)
4227	0.00	...	7 7
4425	1 88	4 3	3 6
4435	1.88	8 3	6 3
4455	1.88	4.7	3.3

[38] Temperatures are given as follows by Pettit and Nicholson (Mt. W. Contr. 369, 1928); 19 Piscium (N0), 2360; U Hydrae (N2), 2360; X Cancri (N3), 2260; VX Andromedae (N7), 2010.

[39] Publ. Obs. Mich., 2, 103, 1916; Shane, L. O. B. 396, 1928.

The line 4227 is the ultimate neutral line, the other three are strong penultimate lines. All should strengthen with low temperature. Their relative intensities in Class R are as would be expected. The relative intensities of the penultimate multiplet in the N and R classes point distinctly to higher temperature for the latter. There is no doubt that 4227 should be exceedingly strong in Class N, but attempts to photograph it have apparently failed.

Visual comparison with Class K shows that 4227 is about as strong in Class R as in the K giants; the R stars also have very strong H and K lines which associate them with Class K2, where the ionized calcium maximum occurs. Although distinguished by their band spectra, Classes R and K are very similar as regards their metallic absorption spectra.

The band spectra in Classes R and N have been completely summarized by Shane on the basis of an analysis of the spectra with very low dispersion.[40] The salient points in the investigation are:

The cyanogen bands (4216, 3883, 3590) increase in strength from Ro on, have a maximum at R5, and disappear at about N4. (We recall the parallel changes in Classes K and M with a maximum at about K2.)

The 4606 cyanogen band has a different behavior—increasing from Ro to R8 and remaining constant thereafter.

The Swan band at 4737 increases in strength from Ro to R5, decreases from R5 to R8, and then increases again throughout the N classes.

The energy distribution in the Ro spectrum is not greatly different from Class Go; the violet grows progressively fainter so that in Y Canum Venaticorum, Class N, the relative intensities at 4800A and 3900A bespeak a temperature of only 950°! This temperature is of course spurious.[41]

The G band increases from Ro to a maximum of unusual strength in R3, and then fades almost to nothing at about

[40] L. O. B. 396, 1928.

[41] See p. 235.

N3; there are some irregularities in its intensity. (We recall the changing intensity along the normal sequence with a maximum near G5.)

The behavior of the band spectra in N stars is a fruitful field for theoretical work; obviously the changes in the Swan band point to something quite different from anything encountered for spectral lines. No possible combination of ionization and excitation could produce a double maximum, but the additional possibility of molecular dissociation at a temperature comparable to that required for ionization provides disposable conditions for tackling the problem.

Both the cyanogen and the G bands relate the R stars definitely to the K stars in surface conditions. This point, the similarity of the metallic spectra, and the erratic differences of the band spectra from star to star[42] suggest that in examining the red stars physically it may be worth while to consider the bands abnormal and superimposed on the physically significant metallic spectrum. There is a suggestion that the substance giving the bands really is superimposed in some cases.

I again enumerate the common features of Classes N and M:

a. Temperatures; and the difference of temperature between large-range variables and virtually constant stars.

b. Luminosities, masses, and dimensions.

c. Metallic spectra.

d. Type of variability, at least for variables of large range.

Their chief points of difference are:

a. Galactic concentration (greater for N stars).

b. Distribution in galactic latitude (the M stars are commoner toward the center, the N stars uniformly placed).

c. Numbers.

d. Details of variability, the N stars tending to have more superimposed irregularities.

And, of course,

e. Band spectra.

[42] Pettit and Nicholson, Mt. W. Contr. 369, 1928.

On the whole the two classes of stars are similar in the more fundamental respects, dissimilar in superficial matters. A study of their relationships must keep the distinction between the two types of phenomenon clearly in view, recollecting especially that the most significant feature of a star is not necessarily the most obvious.

CHAPTER XIV

THE VARIABLE STAR

WITH few exceptions the stars that are intrinsically variable are bright; indeed we may almost say that even the apparent exceptions are not genuine. For instance there seems to be little doubt that the variables in the Orion nebula are dwarf stars;[1] but their situation and the manner in which they vary[2] suggest that after all they are not intrinsically variable—they are caused to fluctuate by the enmeshing nebulosity. Though the surface variability of the sun may be typical of some other dwarf variables, it probably differs greatly from the deep-seated changes that characterize more luminous variable stars. Variability of the type now to be described is undoubtedly confined to the brighter stars. Almost all variable stars probably satisfy our adopted criterion of high luminosity. The cluster type stars are a little fainter.

67. Types of Variable Stars.—The types of variable stars that can be conveniently described together are summarized in the following list. References are to sections of the present chapter:

1. The principal sequence.
   a. The Cepheid variable (69).
   b. The long period variable (70).
   c. The semiregular variable (71).
2. The intermediate group (72).
3. The irregular variable (73).
4. The SS Cygni type (74).
5. The R Coronae Borealis star (75).

[1] Shapley, H. C. 254, 1924.
[2] H. B. 868, 1929.

6. The Nova (76).
7. Special cases (e.g., RT Serpentis) (77).

68. Statistical Spectroscopic Data.—Although each class of variable has its own spectroscopic tendencies, the relations are by no means unique. The classes of variable and types of spectrum are summarized in Table XIV, I, which presents a representative, though probably somewhat incomplete, body of data chiefly compiled from Prager's catalogue. Only actual spectra, not color classes or other measures of temperature, are represented.

TABLE XIV, I.—SPECTRAL CLASS AND TYPE OF VARIABLE STAR

	O	B	A	F	G	K	M	N	R	S	Pec.	Total
Long period	..	.	..	.	6	4	472	32	4	26	1	545
Cepheid and semi-regular	.	..	22	34	84	11	11	.	1			163
R Coronae type	..	2	1	.	3	2	1		2		.	11
Irregular	..	3	6	2	3	26	43	32	3	..	1	119
Novae	2	.	.	.			1	.	2	.	9	14
Peculiar	2	..	.		.	.			1	.	.	3
Totals	4	5	29	36	96	43	528	64	13	26	11	855

No one type of variability seems to be uniquely associated with any particular class of spectrum; R Coronae and irregular variables seem to be especially random in their distribution. There is a *tendency* for the long period variables to be late in spectral class, and for the Cepheid and semiregular variables to be early. Likewise any given spectral class may contain one of several types of variable: Class R (uncommon as it is) is of interest in this regard.[3]

69. The Cepheid Variable.—The celebrated Cepheid variable has received more universal attention than most of the matters discussed in this monograph; and yet it probably

[3] Cf. Section 75.

presents more unsolved problems than any of them. The extreme difficulty and complexity of its problems discourages any pretensions to finality or even completeness.

As a preliminary I shall attempt to summarize the observed facts respecting the Cepheid variable proper (including, on special mention, the cluster type variable), and to indicate the bearing of a few of them on suggested interpretations of the variation, or on the criticisms of the explanations. For convenience I tabulate the data under the following subheads:

a. Stellar data.
b. Data connected with variability in light.
c. Variations of velocity.
d. Spectroscopic data.
e. Temperature changes.
f. Physical conditions.
g. Affiliations.
h. Theoretical considerations.
i. General discussion.

Many of the subsections overlap, but these categories include most types of observation bearing on the Cepheid.

a. Stellar Data (1) Parallaxes.—The available data on directly measured parallaxes are summarized by Shapley[4] in his discussion of the period-luminosity curve, and the material need not be repeated. It points to high luminosity, which, however, it is inadequate to evaluate.

Spectroscopic parallaxes recently derived at Mount Wilson[5] by extrapolating the reduction curves for normal giant stars are in close agreement with trigonometric parallaxes; the absolute magnitudes differ systematically by $+1^m.0$ from the values given by the period-luminosity curve. The discrepancy is no larger than might have been expected.[6]

(2) Proper Motions.—The proper motions of 84 stars, regarded as Cepheids, were tabulated and discussed by R. E

[4] H. Mon. No. 2, 1930.
[5] Mt. W. Rep., p. 112, 1929.
[6] See Section 69*d*.

Wilson[7]; it was found that the 19 with periods less than a day included all those of proper motion greater than 10″ per century. "It appears" he wrote "that we have to deal with two dissimilar groups of stars, one scattered more or less at random over the sky, and having a wide range both in magnitude and peculiar motion, the other lying close to the Galaxy with a more moderate range in magnitude, and exceptionally small peculiar motion." Proper motions of eight more cluster type variables have since been measured by Luyten.[8]

(3) Radial Velocities.—The Cepheid variable has on the average a small mean radial velocity, the cluster type variable a large one; representative values are 12 kilometers per second and 100 kilometers per second. Data on mean and individual radial velocities are tabulated by Ludendorff.[9] Reference should be made to the use of these data in examining luminosities, velocity distribution, and galactic structure.[10] It must be recalled that the space velocities of cluster type variables are not distributed at random, as these stars share the motion of the high velocity stars. Therefore velocities cannot be used to obtain a mean parallax of the cluster type variables.

(4) Space Velocities.—The peculiar velocities of the cluster type variables are high, averaging about 70 kilometers per second, and those of the Cepheids of longer period are about 12 kilometers per second.[11]

(5) Galactic Concentration.—The classical Cepheid (of period greater than a day, and generally less than 50 days) is strongly concentrated to the Milky Way. The cluster type variable, on the other hand, is but very slightly concentrated.

[7] R. E. Wilson, A. J., **35**, 35, 1923. Wilson's list contained, however, SS Geminorum, U Monocerotis, R Sagittae, V Vulpeculae, and some others, now known not to be regular Cepheids. Their inclusion did not affect his results appreciably.

[8] Luyten, H. B. 847, 1927; See also Mrs. Bok (Miss Fairfield), (in preparation).

[9] Handbuch der Astrophysik, **6**, 214, 1928.

[10] R. E. Wilson, A. J., **35**, 35, 1923; Strömberg, Mt. W. Contr. 293, 1925; Oort, B. A. N. 120, 132, and 159.

[11] Russell, Dugan, and Stewart, Astronomy, **2**, 762, 1926.

The contrast is of course connected with the differences in velocity and brightness between the two classes of stars.

(6) Galactic Dip.—If the angular dip of the system of Cepheid variables is determined with respect to the Milky Way, for a small enough range in period and apparent magnitude, it is possible to deduce the corresponding brightness (cf. the treatment of the N stars in Chapter XIII). Historically the process has been reversed, and the Cepheid variables used for determining the distance of the sun from the galactic plane. On this basis the following determinations of E (the distance, in parsecs, of the sun above the plane) have been made:

Authority	Reference	Number of Stars	E
Hertzsprung	A. N., **196**, 207, 1913	68	+37
Shapley	Mt. W. Contr. 157, 1918	87	+60:
Gerasimovič and Luyten	H. Repr. 37, 1928	100	+34

The wide differences between these results have led the writer to reexamine the data, which were brought up to date by the inclusion of all the available material. The following results were obtained:

Material	Number of Stars	E
All available.	240	+25
Stars within 5,000 parsecs and ±20°.	116	+28
Classical Cepheids only.	144	+74
All stars within 5,000 parsecs.	211	−16

Evidently the method is not a reliable one for the evaluation of E. It merely emphasizes the high luminosity of the classical Cepheids and the lack of concentration of the cluster type stars to the galactic plane.

(7) Number of Cepheid Variables.—Of the 240 known Cepheid variables in the galactic system more than half are classical Cepheids; the others are cluster type variables. As the book goes to press a number of additional cluster type variables are announced by Miss Hoffleit.[12] They raise the proportion of cluster type variables, but as the population of variable stars differs in different regions, the actual numbers are

[12] H. B. 874, 1930.

probably not important. In the direction of the galactic center there are considerable numbers of the latter[13] and relatively few classical Cepheids. The two classes of variables are evidently not uniformly represented in all districts.

The distribution of Cepheids and cluster type variables in clusters and external systems is also of great interest. The majority of variable stars in the globular clusters are of short period, and classical Cepheids are exceedingly uncommon there. In the Magellanic Clouds the whole range of periods is represented—the short periods as yet very sparingly. In the more distant systems only classical Cepheids have been detected, but the short period Cepheids would be too faint to show. It seems clear that the relative numbers of short period and classical Cepheids differ in different districts; in the globular clusters the former preponderate enormously and in the galactic system they are appreciably more numerous (allowance being made for selection resulting from different luminosities); we cannot as yet pronounce on the different balance in different regions of the galactic system, nor on the proportion of long to short periods in external galaxies.

The stellar data on the Cepheid variable assure us of its high luminosity, and consequently of its large mass. They suggest that the Cepheid variable is uncommon in space. We recall the small dispersion in absolute brightness of the cluster type variable,[14] and the large dispersion for other stars of similar spectrum, and infer that cluster type variability is peculiar to, though not necessarily typical of, a particular luminosity (see Subsection 69*e*). The corresponding data for the classical Cepheid are not so explicit, because the luminosities are not all similar.

b. Variability in Light.—(1) The Period-luminosity Curve.[15] The relation between the period of variation and the brightness of the star has been empirically determined with great

[13] Shapley and Miss Swope, H. Repr. 52, 1928.

[14] See Table IV, IV.

[15] Miss Leavitt, H. C. 173, 1912; H. A., **60**, No. 4, 1908.

certainty. Theoretical problems arising from this relation are discussed in Subsection *g*. The basic material for the relation is presented by Shapley in his discussion of the distances of clusters.[16] The deviations of individual stars from the period luminosity curve are probably small.

(2) Form of the Light Curve.—The fall in luminosity is never steeper than the rise, and the light curves of most Cepheids are asymmetrical, though a few, such as those of ζ Geminorum and Polaris, are inappreciably so. Characteristic sinuosities are a feature of the visual and photographic light curves, especially for stars of certain periods. There are no data on the sinuosity of radiometric light curves. There is a general tendency of the light curves of stars of similar period to be alike,[17] and there is a progression in form of light curve with period. Finally, the light curves are subject to variability, especially those of some cluster type variables (e.g., RR Lyrae, which shows, at least sometimes, a 40-day period variation in the time of rise[18]), but also those of some classical Cepheids (e.g., TU Cassiopeiae,[19] Y Sagittarii,[20] and BF Ophiuchi[21]).

(3) The dispersion in range is considerable. Confining ourselves to bright classical Cepheids, we find ranges from $0^m.08$ (Polaris) to $2^m.5$ (X Puppis).[22] But though the dispersion is large, by far the greater number of ranges are about $0^m.7$. Among the cluster type variables we have β Cephei ($0^m.05$) and RR Lyrae ($0^m.85$); or examining Bailey's variables[23] in ω Centauri, we find ranges from $0^m.43$ to $1^m.45$. Seemingly large luminous ranges are accompanied to some extent by large spectral ranges—the spectrum of Polaris, for instance, varies only inappreciably, and X Lacertae, with small range, has small

[16] Shapley, H. Mon. No. 2, 1930.
[17] Hertzsprung, B. A. N., **3**, 115, 204, 1926.
[18] Shapley, Mt. W. Contr. 112, 1916.
[19] Robinson, H. B. 866, 1929.
[20] ten Bruggencate, Ann. Bosscha Obs., **2**, C3, 1928.
[21] Shapley, H. B. 875, 1930.
[22] Robinson, H. B. 872, 1930.
[23] H. A., **38**, 133, 1902.

spectral variation.[24] But among the stars that vary more conspicuously, range in light and spectrum are but little correlated as far as the available data go. The point requires more investigation.

(4) The variability of light is accompanied by a variation of color; the star is bluest at maximum, reddest at minimum. There are apparently no explicit data on the relation of range to temperature change, but we may expect the two to be correlated.

(5) The radiometric range appears to be of the same order as the visual range but rather smaller; the available data[25] refer only to δ Cephei and η Aquilae.[26]

(6) On the theoretically important variations of period for normal Cepheids few data are available. The variations of many of the stars have been better represented by sine and power terms than by linear formulae, as may be seen from the footnotes given by Prager to his list of variables with periods less than 100 days. Apparent variations of period may (perhaps commonly) be results of changes in the shape of the light curve rather than of a periodicity inherent in the stars; this has been shown to be so for BF Ophiuchi.[27] But other stars, for example κ Pavonis,[28] are apparently subject to real irregularities of period.

The two stars for which changes of period have been best determined are δ Cephei[29] and ζ Geminorum.[30] Both are decreasing in period—the former by 0.079 ± 0.0083 second a

[24] Shapley and Miss Walton, H. C. 313, 1927.

[25] Pettit and Nicholson, Mt. W. Contr. 369, 1928.

[26] Hopmann (A. N., **222**, 233, 1924) has stated that the bolometric ranges are small and the total radiation of Cepheids constant; but this result is exceedingly sensitive to temperature errors (especially at low temperatures, where the correction to bolometric magnitude is large), and the radiometric measures are more reliable.

[27] Shapley, H. B. 875, 1930.

[28] Jacobsen, L. O. B. 412, 1929.

[29] Hertzsprung, A. N., **210**, 23, 1919; Ludendorff, A. N., **212**, 185, 1920.

[30] Guthnick, B. Z. No. 13, 1920; Becker, Dissertation, 1924; Jacobsen, L. O. B. 379, 1926; Nielsen, Medd. fra Ole Rømer Obs. 5, 1930.

year, the latter by 3.6 seconds a year. (Hertzsprung points out in deriving the former value that the reality of the decrease stands or falls by the reality of the weight ascribed to the earliest isolated observations.)

It is interesting that both these changes are decreases. But the result is of little general significance, because it applies only to two stars, because the order of the decrease is very different for the two, and because less regular changes of period have undoubtedly been recorded for somewhat similar stars.[31]

Some existing theories of Cepheid variability take account of (1) and of (4), but most of the other observations mentioned in this subsection have not yet received an obvious interpretation.[32] The sinuosities of the light curve are especially interesting.

c. Variations of Velocity. (1) Correspondence with the Light Curve.—The radial velocity varies with the same period as the light and the curve displays similar asymmetry. Moreover, ten Bruggencate[33] has made the important observation that changes in the light curve of Y Sagittarii are accompanied by changes in the radial velocity curve.

(2) Maximum brightness almost coincides with maximum velocity of approach; minimum, with maximum velocity of recession. (If the radial motions are interpreted in terms of pulsation, maximum light is marked by high surface temperature, apparently due to compression; but actually it occurs much later than the time of maximum compression—a difficult theoretical point.)

(3) If the observed radial motions are interpreted in terms of a spectroscopic binary, the resulting orbit has always certain peculiarities; for instance, the mass function is small, the longi-

[31] For a summary of data see Ludendorff, Handbuch der Astrophysik, **6**, 198, 1928.

[32] A new attempt to interpret the coincidence of maximum luminosity with maximum velocity of expansion is given in a recent paper by Rosseland (Trans. Norwegian Acad., Oslo, Math.-Naturw. Kl. No. 6, 1929), where also the previously untouched succession of the forms of Cepheid light curves with advancing period is commented upon (see subsection 69*b* (2)).

[33] H. C. 351, 1930.

tude of periastron about equal to 90°,[34] and the value of *a* sin *i* small—the systems indicated are hard to visualize; seemingly they are physically impossible.

The observed features of the velocity variation show definitely that an orbital interpretation is inadmissible and that a pulsational variation is not of unreasonable magnitude. The observation described in Subsection 69*c* (2) is an important and difficult feature of the problem, and many attempts have been made to explain it, with only partial success.

d. Spectroscopic Data. (1) Spectral Character of the Cepheid.—The classical Cepheid has a spectrum that shows the c-character, though not to a very marked extent. Data on the spectroscopic absolute magnitudes of Cepheid variables have recently been discussed elsewhere.[35] Obviously the classical Cepheid can compete in brightness with the corresponding supergiant.

The cluster type variable is not a c-star; it does, however, seem to have rather narrower hydrogen lines than ordinary, and is therefore brighter than the average late A star of similar spectral class. This is well shown in the color-magnitude diagram of Messier 3, for instance. Actual evaluation of the brightness of a few typical Cepheids is made in a later section.

(2) The Period-spectrum Relation.—This important statistical relation, which had long been evident in general, was formulated on the basis of extensive data by Shapley and Miss Walton[36] (cf. also the compilation of data by Adams and Joy[37]). Applied at first only to classical and cluster Cepheids, it was later shown to extend unbroken to the semiregular and long period variables.[38] The period-spectrum relation is certainly less exact than the period-luminosity curve, but it represents an extremely important statistical tendency. The inexactitude of the relation is discussed in Section 69*e*.

[34] Another way of describing the characteristic asymmetry.

[35] Shapley and Payne, H. B. 872, 1930.

[36] Shapley and Miss Walton, H. C. 313, 1927.

[37] P. N. A. S., 13, 391, 1927.

[38] Shapley, H. B. 861, 1928.

(3) Bright Lines in the Spectra of Cepheid Variables.—The bright lines that have been observed in the spectra of Cepheids[39] occur for stars that are of comparatively long period (mean $P = 18$ days) and presumably very luminous. The observations are enumerated in Table XIV, II.

TABLE XIV, II.—CEPHEID VARIABLES WITH BRIGHT LINES IN THEIR SPECTRA

Star	Period	Note
	d	
W Serpentis	14	Adams and Joy, P. A. S. P., **30**, 306, 1918
W Virginis	17	Joy, P. A. S. P., **37**, 156, 1925
SV Monocerotis	15	Shapley and Walton, H. C. 313, 1927
V Velorum	4	
l Carinae	36	
VY Carinae	19	
X Crucis	6	
S Crucis	5	
U Sagittarii	7	
U Monocerotis	46	
RU Scuti	20	
YZ Sagittarii	10	
TT Aquilae	14	
X Cygni	16	
U Carinae	39	

(4) Spectroscopic Affiliations.—The apparently related stars on both sides of the classical Cepheids merge into them in spectrum as well as in period. The cluster variable SU Aurigae is given a "pseudocepheid" spectrum at Mount Wilson,[40] and the bright line spectrum of U Carinae (period 39 days) is very similar to that of the short period Mira variable SX Herculis, (period 102 days) with its K spectrum and bright hydrogen lines. The transition to the spectrum of the related long period variable is continuous. Affiliation of Cepheid spectra with the spectra of semiregular stars such as SS Geminorum and V Vulpeculae is also evident in the c-character, the spectral changes, and the variations of radial velocity.

[39] Shapley and Miss Walton, H. C. 313, 1927.

[40] Adams, Joy, Strömberg, and Burwell, Mt. W. Contr. 199, 1921.

(5) The Spectral Variations.—The characteristic spectral variation of the Cepheid variable is *across* the period-spectrum curve, not along it. In other words, while in the period-spectrum relation the redder stars are on the whole more luminous (inferred jointly from the period-luminosity curve and the period-spectrum relation), the individual variable is bluest, and earliest in spectrum, at maximum light.

Data on the variations of the spectra of three bright northern Cepheids are given in the accompanying diagrams; their quantitative aspect is still in its crudest stage. We may summarize them as follows:

The hydrogen lines are at their sharpest and deepest at maximum light.

The H and K lines are at their sharpest and deepest a quarter period after maximum light; their variation in contour is large.

The temperature is greatest at maximum, least at minimum light.

These observations, crude as they are, give some valuable information. They apply to the surface of one star (of unchanging mass) that undergoes known changes of brightness and temperature, so that the changes in surface gravity are determinate. With rising temperature and falling gravtiy, the hydrogen lines strengthen and sharpen—a typical absolute-magnitude effect for a star of spectral class later than A5, the apparent change in spectral class going with the temperature. But under the same conditions the ionized calcium lines, which also strengthen and sharpen, proceed in the opposite direction to their spectral class effect for the temperature change involved. For them the change in surface gravity is the more important, and for the hydrogen lines the change in temperature.

The spectral changes of other lines are various, complicated, and dissimilar for different stars (see Table XIV, III and XIV, IV). Reference should be made to the studies of bright Cepheids by Henroteau and Miss Douglas[41] and by Sanford.[42] Evi-

[41] Pub. Dom. Ap. Obs., 9, 163, 1929.

[42] Mt. W. Contr. 340, 1927; 352, 1928.

dently no complete explanation of the changes of intensity of different lines can be framed as yet, especially when we bear in mind the probable variety of radial motions for the lines of different kinds.[43]

TABLE XIV, III.—MEAN PERCENTAGE LIGHT LOSSES FOR η AQUILAE

Mean Phase*	Hβ	Hγ	Hδ	Hε	K	4227	4215	4077	4046	4326	G
0.64	38	51	53	90	92	30	14	37	28	20	25
1.22	37	54	46	94	94	28	20	28	26	12	42
3.65	36	37	30	79	72	31	28	31	20	21	30
5.0	37	39	46	72	68	41	26	49	46	20	46
6.35	42	47	50	87	82	37	22	43	29	19	31

* Reckoned from maximum.

TABLE XIV, IV.—MEAN PERCENTAGE LIGHT LOSSES FOR ζ GEMINORUM

Mean	Hβ	Hγ	Hδ	Hε	K	4227	4215	4077	4046	4326	G
1.05	38	51	42	94	96	36:	23	40	15	23	36
3.29	20	25	27	75	74	..	..	..	..	14	32
4.7	25	25	25	69	65	31	21	37	30	16	33
9.0	29	34	34	81	80	22	12	32	29	13	30

TABLE XIV, V.—PERCENTAGE LIGHT LOSS FOR THREE CEPHEID VARIABLES

Star	Line	Value of *dl* at			Value of *dl* for Invariable at Median Spectrum	
		Max.	Med.	Min.	Giant	Supergiant
δ Cep	4227	32	24	18	24	24
η Aql	4227	41	30	28	28	30
ζ Gem	4227	36	36:	22	31	34
δ Cep	K	82	81	54:	78	81
η Aql	K	94	89	68	75	89
ζ Gem	K	96	85	65	74	89
δ Cep	4340	56	..	..	46	54
η Aql	4340	54	..	..	33	58
ζ Gem	4340	51	..	..	32	52

[43] Rufus, P. N. A. S., 10, 264, 1924.

One quantitative attempt is of possible significance—the empirical evaluation of the brightness of the Cepheids from their measured line intensities at different points in their spectral variation. I utilize the measures of line depth for the three stars ζ Geminorum, η Aquilae, and δ Cephei; first singly, then in combination.

The following rough empirical conclusions can be drawn from this comparison at median magnitude:

Star	From 4227	From K	From Hγ
δ Cep	Equal to supergiant	Equal to supergiant	Equal to or brighter than supergiant
η Aql	Equal to supergiant	Brighter than supergiant	Fainter than supergiant
ζ Gem	Brighter than supergiant	Brighter than supergiant	Fainter than supergiant

At median magnitude, then, all these three Cepheid variables are appreciably brighter than the normal giant star, and comparable with the supergiant. ζ Geminorum, with the largest period of the three, seems to be slightly the brightest.

A further test combines the data for the three variables. The spectra were individually classified by Miss Cannon. The intensities of the lines in question, for all three stars, were then plotted against spectral class, and smooth curves drawn through the plots. (It will be noticed that this procedure neglects the possible, and indeed established, differences of detail between the spectra on the up and down grade.[44]) These intensities, and the smoothed intensities at the same spectral classes for giant and supergiant stars, are plotted in Figure XIV, 1 and tabulated in Table XIV, VI for the mean of Hγ and Hδ, and in Table XIV, VII for the mean of H and K.

From these data we find empirically that the Cepheid variable is considerably brighter at median than the normal giant, and

[44] In his interesting analysis of the spectral changes of δ Cephei, Reesinck (B. A. N., 4, 42, 1927) states, for instance, that "with decreasing temperature the ionisation is stronger than with equal but increasing temperature." Cf. footnote, p. 211.

slightly brighter at maximum than the corresponding supergiant star. The change in the lines at minimum is a shallowing, but not a very great weakening. The general impression of great luminosity that is obtained from the look of the spectrum

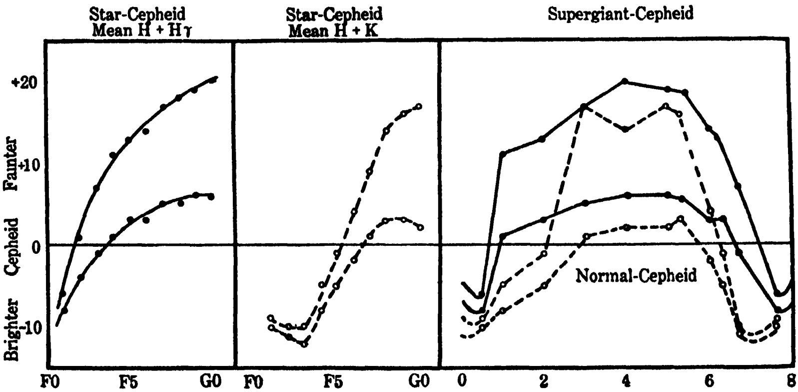

FIGURE XIV, 1.

Empirical comparison of Cepheids with giant (below) and supergiant stars of the same spectral class. On the left, the difference in percentage light loss for various spectral classes is plotted for hydrogen, and in the middle, for ionized calcium. On the right, the difference between η Aquilae and supergiant (above) and normal stars (below) is plotted against phase. Broken lines denote ionized calcium, full lines, hydrogen. We note that from the Ca+ lines, the Cepheid is brighter than the supergiant for 2.8 days of its 7-day period and brighter than the normal star for 4.2 days. From the hydrogen lines, the corresponding figures are 5.7 and 6.5 days.

TABLE XIV, VI.—MEAN INTENSITIES OF Hγ AND Hδ FOR CEPHEIDS, GIANTS, AND SUPERGIANTS

Class	Normal	Supergiant	Cepheid	N − C	S − C
F0	54	52	..	.	..
F1	52	54	60	−8	−6
F2	50	55	54	−4	+1
F3	48	56	49	−1	+7
F4	46	56	45	+1	+11
F5	45	55	42	+3	+13
F6	43	54	37	+5	+14
F7	42	54	37	+5	+17
F8	40	53	35	+5	+18
F9	39	52	33	+6	+19
G0	37	51	31	+6	+20

TABLE XIV, VII.—MEAN INTENSITIES OF H AND K FOR CEPHEIDS, GIANTS, AND SUPERGIANTS

Class	Normal	Supergiant	Cepheid	N − C	S − C
F0	75	77	..	..	..
F1	76	77	86	−10	− 9
F2	77	78	88	−11	−10
F3	78	80	90	−12	−10
F4	79	82	87	− 8	− 5
F5	80	84	85	− 5	− 1
F6	79	85	81	− 2	+ 4
F7	79	87	78	+ 1	+ 9
F8	78	89	75	+ 3	+14
F9	76	(89)	73	+ 3	(+16)
G0	74	(89)	72	+ 2	(+17)

at maximum is borne out by these empirical comparisons. The significance of the differences is more apparent when the physical conditions at the surface have been examined.

e. Temperature Changes of Cepheid Variables.—The temperatures and temperature changes of Cepheid variables will merely be summarized in the present section; the methods used by investigators quoted must be sought in their papers.

The simultaneous variation of color and spectrum is shown in Table XIV, VIII which summarizes the early compilation

TABLE XIV, VIII.—RANGES IN COLOR INDEX AND SPECTRUM FOR CEPHEID VARIABLES

Star	Range in Color Index	Limiting Spectra	Spectral Range
η Aquilae	0 42	F2-G9	1.7
δ Cephei	0 49	F4-G6	1.2
RT Aurigae	0.35	F1-G5	1 4
SU Cassiopaeiae	0.14	F2-F9	0 7
SU Cygni	0.35	F0-G1	1 1
T Vulpeculae	0.40	F5-G1	0.6
S Sagittae	0.53	F8-G7	0.9
U Vulpeculae	0.4	F8-K0	1.2
XZ Cygni	0.58	A0-A8	0.8
RS Bootis	0.6	B8-F0	1.2

of[45] ranges in color indices, and the more recent data[46] on the spectral ranges of the same stars.

The next table summarizes some actual determinations of color, and names the authors responsible for them. Hopmann's temperatures are determined colorimetrically; those of Pettit and Nicholson are radiometric; those of Gyllenberg are spectrophotometric and entitled to a greater weight than estimates based on color indices.

TABLE XIV, IX.—ESTIMATED TEMPERATURES OF CEPHEID VARIABLES

Star	Maximum	Minimum	Reference	Spectrum
	°	°		
δ Cephei	6700	4780	1	F4-G6
δ Cephei	6200	4580	2	F4-G6
T Vulpeculae	4670	3290	3	F5-G1
η Aquilae	5240	3960	4	F2-G9
η Aquilae	4950	3900	5	F2-G9
S Sagittae	5700	4300	6	F8-G7
SU Cassiopeiae	6400	5110	7	F2-F9

REFERENCES

1 Hopmann, A. N., **226**, 1, 1925.
2 Pettit and Nicholson, Mt. W. Contr. 369, 1928.
3 Hopmann, A. N., **221**, 337, 1923.
4 Hopmann, A. N., **222**, 1, 1924.
5 Pettit and Nicholson, Mt. W. Contr. 369, 1928.
6 Gyllenberg, Lund Medd., Series 2, 24, 1920.
7 Hopmann, A. N., **226**, 1, 1925.

We also recall the negligible color excess of RS Bootis,[47] which places the temperature rather closely, on the basis of spectral type.

The ranges in color are closely comparable to the ranges in spectrum, as Seares and Shapley pointed out long ago. In the mean the Cepheids seem to be about 500° cooler than giants of similar spectral class. The comparison is rough, and not numerically significant, as the redness undoubtedly differs for

45 Seares and Shapley, Mt. W. Contr. 159, 1918.
46 Shapley and Miss Walton, H. C. 313, 1927.
47 Seares and Shapley, Mt. W. Contr. 159, 1918.

stars of differing period and brightness; there is also a possibility that the color of a Cepheid variable does not correspond to its effective temperature.[48]

Okunev[49] has recently discussed color changes for Cepheid variables on the basis of comparisons of visual and photographic light curves and finds that the color curves show sinuosities similar to those shown by the light curves. It is possible that such fluctuations of color do occur within a period, and some secondary maxima of temperature are well known;[50] but the same effect would of course be produced by comparing light curves of dissimilar makeup—one smoothed of sinuosities, the other retaining them. The changes of color of Cepheids should be studied directly, not by comparing light curves, and until much material on secondary temperature fluctuations is available, Okunev's criticisms of the pulsation hypothesis remain of insufficient foundation.

f. Physical Conditions. (1) Changes of Pressure, Temperature, and Surface Gravity.—Adopting the pulsation theory to interpret the chief qualities of the Cepheid variable, we can draw the following rough and tentative picture of the physical conditions at different points in the variations:

Phase	Dimensions	Surface Gravity	Temperature (Observed)
Maximum	Intermediate	Intermediate	Maximum
Median decreasing	Maximum	Minimum	Intermediate
Minimum	Intermediate	Intermediate	Minimum
Median increasing	Minimum	Maximum	Intermediate

The interplay of conditions may produce practically anything in the way of spectral effects, depending on whether the atom

[48] This has been suggested to me by ten Bruggencate. Hopmann (A. N., **222**, 233, 1924) somewhat overemphasizes the coolness of the Cepheid variables because the spectral classes that he quotes are considerably earlier than those now adopted for the stars concerned, as shown in Appendix B.

[49] Okunev, A. N., **236**, 312, 1929.

[50] Hopmann, A. N., **222**, 1, 1924; ten Bruggencate, Ann. Bosscha Obs., **2**, C3, 1928.

under consideration is in a condition to respond more readily to changes of temperature or of pressure. In comparing maximum and minimum we are confronted with effects of temperature only, and in comparing the two median magnitudes we have effects related only to differences in surface gravity.[51]

The data for δ Cephei and η Aquilae may be analyzed qualitatively from these two standpoints. Tables XIV, X

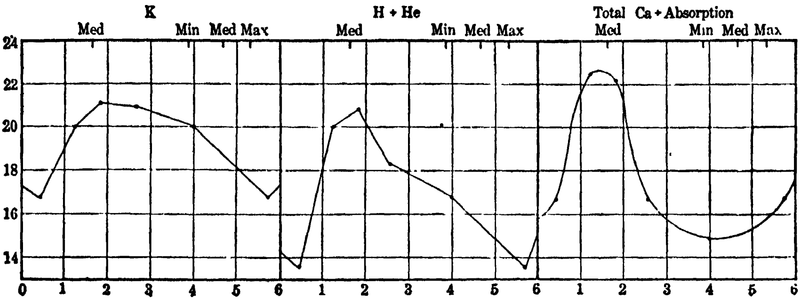

FIGURE XIV, 2.

Line breadth and phase for δ Cephei. For the first two sections ordinate and abscissa are width at a depth of 4 per cent, and phase, for the K line (on the left), and H + Hε. The K line is less than the expected factor 1.41 times wider than the H line, perhaps because of the effect of the hydrogen line on H. The third section shows the sum of the total absorptions of H and K.

and XIV, XI contain the differences, for the lines enumerated in the left-hand column, with different surface gravity, and with different temperature. The data are taken from Figure XIV, 2, XIV, 3, and XIV, 4. Spectral variations have been described for other stars, notably T Monocerotis[52] and U Vulpeculae,[53] but they are expressed in terms of ratios (as are most of the curves given by Henroteau and Miss Douglas), and ratios, though

[51] Both these conclusions are subject of course to second order corrections. For instance it is found that the temperatures are "higher at every point of the ascending branch of the photographic light curve than at points of corresponding brightness on the descending branch," for Y Sagittarii (ten Bruggencate, Ann. Bosscha Obs., 2, C23, 1928), δ Cephei (Reesinck, Dissertation, Amsterdam, 1926), and Y Ophiuchi (ten Bruggencate, Ann. Bosscha Obs., 2, B58, 1927). See footnote, p. 206.

[52] Sanford, Mt. W. Contr. 340, 1927.

[53] Sanford, Mt. W. Contr. 352, 1928.

TABLE XIV, X.—ANALYSIS OF THE SPECTRAL VARIATIONS OF δ CEPHEI

Line	Maximum	Minimum	Difference	Median Decreasing	Median Increasing	Difference
H + Hϵ (half breadth)	13.6A	16.4A	−2.8A	21.6A	15.4A	+6 2A
K (half breadth)	16.2A	20.2A	−4.0A	20.8A	19.0A	+1.8A
Sr + (per cent loss)	62	62	0	51	62:	−11:
G band (per cent loss)	40	48	−8	45	43	+2
Ca + /Ca (ratio in *dl*)	2 7	2.9	−0.2	3.2	2.8:	+0.4
Fe + /Fe(ratio in *dl*)	1.2	1.1	+0.1	0.9	1.1	−0.2

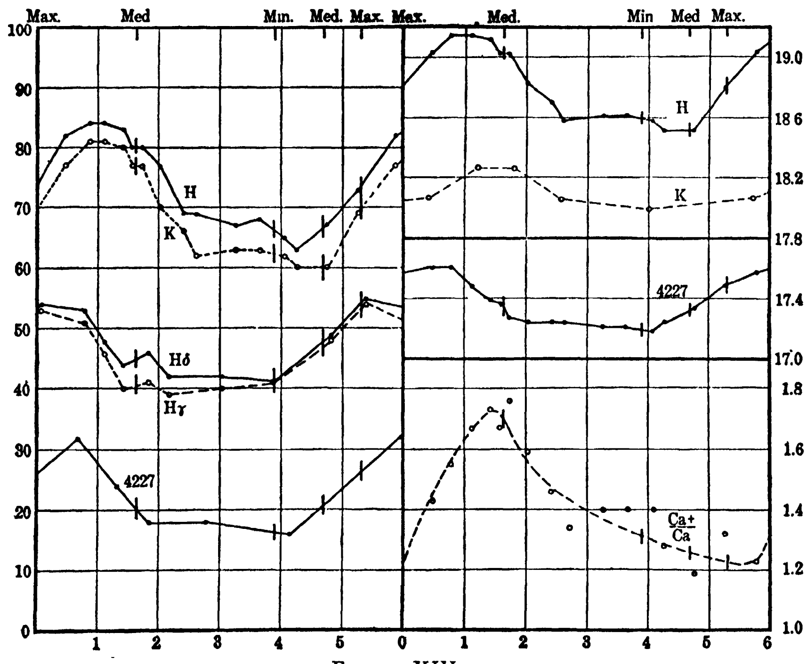

FIGURE XIV, 3.

Variation of measured line depth for δ Cephei. Left side: changes in the depths of H, K, Hδ, Hγ, and 4227 with phase. Ordinate and abscissa are percentage light losses and phases in days. Each point is the unweighted mean of three successive observations. Short vertical lines mark maximum, mean, and the medians. Right side, above: change with phase of numbers of atoms of ionized calcium, from the depths of the ultimate lines (dots and full line), and from the total absorption (circles and broken line; zero point arbitrary). Ordinate and abscissa are log *NH* and phase in days. Right side, middle: change of numbers of neutral calcium atoms with phase. Right side, below: change of ionization of calcium with phase; ordinates are log (ionized calcium/neutral calcium), from line depth data. No importance is attached to the irregularities of this curve, and the broken line indicates the most definite conclusion that seems justified.

TABLE XIV, XI.—ANALYSIS OF THE SPECTRAL VARIATIONS OF η AQUILAE

Line	Maximum	Minimum	Difference	Median Decreasing	Median Increasing	Difference
Mean H and K (per cent loss)	95	76	+19	96	75	+21
Mean Hγ and Hδ (per cent loss)	57	37	+20	42	45	− 3
Hγ (see note)	165	67	+98	100	62	+38
Ti + 4534 (see note)	85	52	+33	60	65	− 5
Fe 4325 (see note)	83	62	+21	71	65	+ 6

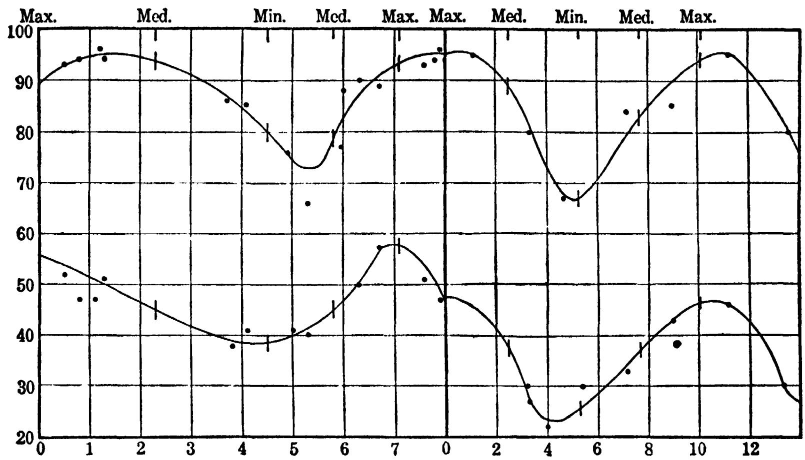

FIGURE XIV, 4.

Variation of line depth for η Aquilae (left) and ζ Geminorum. Ordinate and abscissa are percentage light loss and phase in days. The upper curves represent the mean of H and K; the lower curves, the mean of Hγ and Hδ. Maximum, minimum, and the medians are marked with short vertical lines.

pragmatically sanctioned by their empirical use for absolute magnitudes, are of too complex connotation to consider here.

The data referred to the note are taken from the investigation of Henroteau and Miss Douglas.[54] Their scale is arbitrary.

The lines discussed may be grouped as follows:

Neutral subordinate, at temperature below maximum (hydrogen).

[54] Henroteau and Miss Douglas, Pub. Dom. Obs., 9, 163, 1929.

Neutral penultimate, at temperature above [nonexistent] maximum (iron).

Ionized subordinate, near maximum temperature (ionized titanium).

Ionized ultimate, at temperature above maximum (ionized calcium and strontium).

Type of Line	Effect of Raised Temperature (Surface Gravity Constant)	Effect of Lowered Gravity (Temperature Constant)
Neutral subordinate below its maximum (H)	Strengthened	Slightly weakened
Neutral penultimate (Fe)	Strengthened	Slightly strengthened
Ionized subordinate near maximum (Ti+)	Strengthened	Weakened
Ionized ultimate above maximum		
(Ca+) contour	Narrowed	Widened
depth	Increased	Increased
(Sr+) depth	No effect	Weakened
Molecule (CH)	Weakened	Strengthened

The behavior of hydrogen and the G band in the spectral sequence would lead us to expect that in the changing spectrum of a Cepheid variable they would behave as they actually do with changing temperature. Hydrogen has its greatest strength at A0 (far hotter than the variable ever becomes), and the G band, at about Class K2 (a later spectral class than the variable attains). Raised temperature should therefore move hydrogen toward maximum, and the G band away from it; in accordance with expectation, hydrogen is strengthened, the G band weakened.

The temperature at which ionized titanium is strongest is uncertainly known, and the present data may be used to settle the question, making use of the general observation that the ionized lines of metals tend to behave much as hydrogen does. As the lines are strengthened with raised temperature, we may infer that, like hydrogen, they are thereby brought nearer to their maximum, which we may guess to be at about 8500°.

The strengthening of the neutral iron lines with raised temperature and the deepening of the H and K lines are less comprehensible. But the former rests on slender data, as will be seen from the large scatter of the points from which the strengthening was inferred (Figure XIV, 5).

The behavior of the H and K lines is only half abnormal, for the narrowing with raised temperature would be expected—the lines are moving away from their maximum. The appearance of deepening toward maximum is really the effect of the blurring of the calcium lines at minimum, an effect already long known in connection with the expected broadening[55] of all lines at maximum and minimum. The blurring actually occurs only at minimum; it is well marked, conspicuous, and of evident importance. It has no obvious interpretation.

The effects of surface gravity at approximately the same temperature (third column) are of similar interest. All the strengthening effects are normal. The slight weakening of hydrogen is unexpected, but I believe that the observations indicate it. We recall that for classes earlier than F5 the hydrogen of supergiants is weaker than for fainter stars;[56] as the spectral class of the stars in question is F2 at maximun, we may here have another indication of high luminosity. The reality of the weakening for ionized titanium is doubtful. The observations for Sr+ are not very definite, but they are in line with the well-known fact that the Sr+ lines do not strengthen from giant to supergiant among the G and K stars. The two discrepancies in the third column are so small, however, that they would be removed if the median magnitude were slightly displaced, or if the assumption that the temperature is the same at rising and falling median were shown to be unfounded. Hopmann's[57] data and those quoted by ten Bruggencate[58] suggest that it is lower for decreasing light, which would cause the observed weakening.

[55] Eddington, The Internal Constitution of the Stars, 206, 1926. The observed effect is much larger than the predicted one here referred to.

[56] See p. 269.

[57] A. N., **222,** 6, 1924.

[58] Ann. Bosscha Obs., **2,** C23, 1928.

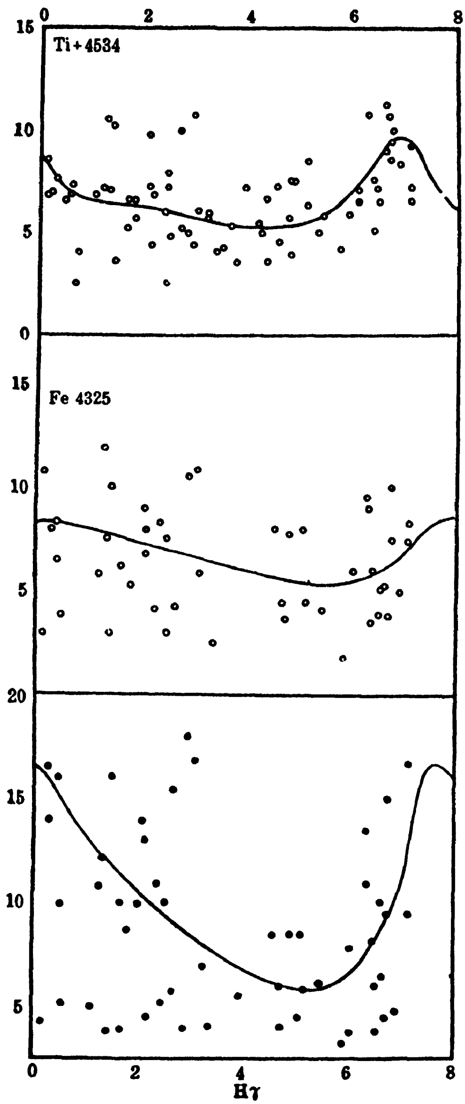

FIGURE XIV, 5.

Variation of the lines 4532 (Ti+), 4326 (Fe), and Hγ, from the data of Henroteau and Miss Douglas for η Aquilae. Their individual estimates are plotted.

The deepening of the H and K lines at maximum as compared with minimum is the only outstanding puzzle in this rough analysis of the spectral changes of the Cepheid variable, and will probably receive an explanation when the problem of the reversing layer and (possibly) photosphere in radial motion is analyzed—an aspect of the matter beyond our present scope.

The above discussion of the spectral variations of the Cepheid explains the observation, repeatedly made,[59] that the hydrogen lines change relatively more than the other lines. The hydrogen lines are the only prominent ones concerned that reached maximum in the spectral sequence at a temperature well above the range covered by the variable. They have the benefit of the full temperature effect, and the surface gravity effect also goes to increase their range. For all other lines (such as those of Ca+) surface gravity and temperature work in opposite directions, so that their range does not appear to be so large. There is nothing abnormal in the behavior of the hydrogen lines.

The treatment of this section is in many ways similar to that of Pannekoek[60] and Reesinck[61]; their p coordinate represents temperature, and their q, surface gravity. The diagram given by Reesinck compares strikingly with mine.

g. Theoretical Considerations. (1) The Period-luminosity Curve.—It can easily be shown that any dynamical theory of Cepheid variability may be made to account for the form of the period-luminosity-spectrum relation, but not for its zero point. Although the period-luminosity curve had long been established for Cepheids in other systems,[62] and credibly assumed for galactic Cepheids,[63] the calculation of a theoretical period-luminosity law for galactic Cepheids by means of the mean period-spectrum relation and an assumed gravitational regulation of the period, was unexpectedly beautiful, and precisely

[59] Cf., for instance, Adams and Shapley, P. N. A. S., **2**, 136, 1926.
[60] Pannekoek and Reesinck, B. A. N., **3**, 47, 1926.
[61] Reesinck, B. A. N., **4**, 41, 1927.
[62] Shapley, Yamamoto, and Miss Wilson, H. C. 280, 1924.
[63] Shapley, Mt. W. Contr. 153, 1918.

fitted the relation otherwise determined.[64] It may be regarded as turning the tables on some of the previous paragraphs, which attempted to evaluate spectroscopically the brightness of the Cepheid variable; it enables us to use the Cepheid variable of known period to calibrate the very luminous stars of similar spectral class and inaccessible parallax.

(2) The Relation of Period to Mean Density.—The extent to which the individual stars satisfy the relation $P^2 \propto 1/\rho$ is of some importance. Shapley has shown that the *mean* period-spectrum relation does so.[65] But as the individual stars fall with a considerable scatter round the mean period-spectrum curve it is not at once apparent how nearly they follow the period-density relation.

The data entering the period-spectrum relation were considered as follows: the period-luminosity law was assumed exact, and the photographic absolute magnitudes for individual stars were read from the photographic period-luminosity curve;[66] they were then reduced to absolute visual magnitudes by King's color indices,[67] and to absolute bolometric magnitudes by Eddington's table.[68] The masses were then read from the mass-luminosity curve (replotted from Eddington's data,[69] and the curve drawn in freehand). The mass thus deduced was combined with a temperature inferred from Shapley's adopted median spectral classes[70] to obtain the mean density with the aid of the formula[71]

$$L = \pi acR^2T_e^4$$

where L is the luminosity, a the Stefan constant, c the velocity of light, R the radius, and T the effective temperature. The

[64] Shapley, H. C. 314, 1927.
[65] *Ibid.*
[66] Shapley, Yamamoto, and Miss Wilson, H. C. 280, 1924.
[67] H. A., **85**, 10, 1928.
[68] The Internal Constitution of the Stars, 138, 1926.
[69] The Internal Constitution of the Stars, 153, 1926.
[70] Shapley and Miss Walton, H. C. 313, 1927.
[71] Eddington, The Internal Constitution of the Stars, 145, 1926.

logarithms of resulting mean densities are plotted in Figure XIV, 6 against period; it is evident that the points fall very nearly on a line. The straight line in the diagram represents a linear relation between period and square root of mean density —the relation that the pulsation theory, or indeed any gravitational theory, requires. There is no indication that the points deviate systematically from it; Dr. Shapley points out that absence of a systematic deviation indicates that the value of Γ (the ratio of the specific heats for the matter within the star[72])

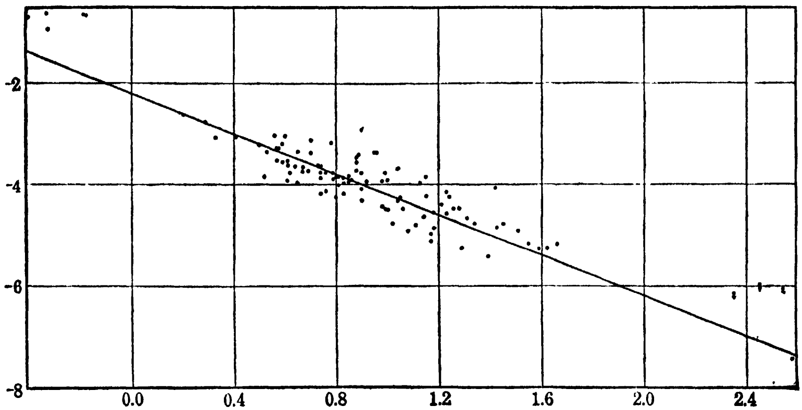

FIGURE XIV, 6.

Relation between period and mean density. Ordinate and abscissa are logarithm of period and logarithm of mean density. The straight line satisfies the relation $P^2 \propto 1/\rho_m$.

does not differ appreciably throughout the sequence of variable stars, an observation important in the study of the stellar interior. The scatter of the points about the line is greater than the errors of observation, and its significance is worth analyzing. Evidently one, two, or all of the three correlations used in obtaining the mean densities (the period-luminosity law, the period-spectrum relation, and the mass-luminosity law[73]) are inexact.

[72] Eddington, The Internal Constitution of the Stars, 190, 1926.

[73] The objections suggested by McLaughlin (A. J., 38, 21, 1927) are dependent on the adopted temperatures of the stars which he discusses.

The period-spectrum relation has undoubtedly a real scatter; reclassification has confirmed the spectral classes of Harvard Circular 313, and the dispersion is also shown by the spectra of stars more recently classified.[74] A little consideration will show that a scatter in the period-luminosity relation is an inevitable consequence, if the period of a Cepheid variable is governed by pulsation or similar dynamical cause.[75] A pulsation theory leads to a direct relation between period and *mean density;* the relation between period and luminosity is indirect.

If we assume that the period-density relation is exact, all stars of given period (the quantity most accurately known) must have the same mean density.[76] If, moreover, all stars of a given period have the same absolute bolometric magnitude (that is, if there is an exact period-luminosity relation), they will all have the same effective temperature, because effective temperature is prescribed by the radius for stars of the same mass. But the wide scatter of the period-spectrum relation shows that stars of one period have a large (and real) range of spectral class. Therefore, with an exact period-luminosity relation, either the temperature of a Cepheid variable is a function *only* of period, or else the period-luminosity law is inexact. The assumption that all Cepheids of given period are of the same effective temperature, whatever their spectral class, is very unlikely; and the existing data on the temperatures of Cepheid variables do not support it.[77] The bolometric period-luminosity curve is therefore probably inexact. A crucial observation would be a determination of the colors of the stars that contribute to the period-luminosity curves in the Magellanic Clouds.[78]

[74] Miss Cannon and Miss Walton, H. B. 874, 1930.

[75] H. B. 876, 1930.

[76] Unless they are built on different models, with different relations of mean to central density—the suggestion made by Shapley to account for the cluster type anomaly.

[77] There are indications that the energy temperatures of Cepheid variables are systematically low; but their values might be expected to show a correlation with period, if the true effective temperatures were functions of period only.

[78] The data should aim to show whether the dispersion in color (if found) were as large as the dispersion in spectral class for galactic Cepheids at the corre-

(3) The Problem of the Cluster Type Variable.—Shapley[79] has recently emphasized and analyzed the dilemma in which we are placed when we recognize that the cluster type variables, with periods ranging from 3 to 18 hours, show no corresponding progression in mean color, spectrum, or, most important of all, brightness. (They show changes of spectrum and variations—sometimes erratic[80]—of color.) If therefore the variations are governed gravitationally, than which no more acceptable suggestion has been offered, either the masses or the sizes of cluster type variables must differ widely. The absence of a period-spectrum relation rules out differences of size, and differences of mass (great enough to satisfy requirements—at least fivefold) seem improbable.

We cannot find a good reason, then, for supposing that the cluster type variables are not similar in size and color. Wherein then do they differ? Unless they are built on different models their mean and central densities must be similar, and also their central temperatures, and the observed similarity of energy output for the stars (individually, and also volume for volume) might be expected under such conditions. Shapley's "developing nucleus," "which leaves the total mass, the *mean* density, and the size and brightness of the surface unaffected," implies a central density not simply proportional to the mean density. It is possible to imagine a pair of stars so arranged that they differed merely in the relation of mean density to central density; but whatever else that implied it would imply some dissimilarity (perhaps an inconspicuous one) in the very conditions that are found to be correlated with the occurrence of variability. The dilemma still confronts us.

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sponding period. Of course if there were an exact bolometric period-luminosity curve (the most likely kind, if there is any direct correlation of period and luminosity), the *photographic* period-luminosity curve should show a dispersion, if the dispersion in the period-spectrum relation is real. The dispersion of the photographic period-luminosity curve in this event would be smaller than it would be if the period were directly correlated with the mean density.

[79] H. C. 315, 1927.

[80] Shapley and M. B. Shapley, Ap. J., **42**, 148, 1915.

Perhaps more detailed spectroscopic studies would enable us to find differences in surface conditions between long and short period cluster type variables, thus leading below the surface to differences of constitution.[81]

General Discussion.—A theory of the Cepheid variable must take account of the formidable network of observations sketched in the present section; the great brightness of the stars; their peculiarities of light variation, so closely connected with radial velocity changes; their parallel variations of temperature and spectrum; and their relationships to other kinds of variables. A pulsation theory such as Shapley[82] outlined and Eddington[83] developed takes account of several of these matters: the dependence of the period on the mean density (from the period-luminosity curve); the radial variations; the changes of temperature and spectrum; and the affiliation with other intrinsic variables. But even here there are serious difficulties: the relation of period to mean density seems to break down within the cluster type; the *detailed* relation of light and velocity curve is not explained;[84] the "lag" in the outflow of the radiation, which arrives about a quarter of a period later than simple expectation suggests,[85] presents serious difficulty, though it has been qualitatively interpreted by Rosseland;[86] and the shallowing and blurring of the spectral lines at minimum, without conspicuous narrowing, is uninterpreted.[87] The sinuous form of the light curve for many Cepheids, and its relation to period, remains an empirical observation; on the other hand the theory

[81] It is worth noting that this is one of the few places in which we have any basis for direct guesses on homology in stars; the appearances here are against the stars being homologous. It is of course very likely that the interpretation quoted (and therefore its implications about homology) is not the right one. Also it is not possible to draw from variables conclusions about the homology of invariable stars; the difference between them might lie in this very point.

[82] Mt. W. Contr. 92, 1915.

[83] Eddington, The Internal Constitution of the Stars, 180, 1926.

[84] Eddington, The Internal Constitution of the Stars, 205, 1926.

[85] *Ibid.*

[86] Trans. Norwegian Acad., Oslo, Math.-Naturv. Kl. No. 6, 1929.

[87] See p. 27.

has not excluded the possibility of higher harmonics than the one contributing to the observed period[88] (but the corresponding incommensurable periods do not seem to occur). Neither have the high luminosities of Cepheid variables been explicitly discussed.

No reference has been made in the preceding paragraphs to numerical predictions and verifications, because the data are at present too inaccurate to apply at the crucial points. For example, Baade[89] points out that the velocity curve can be integrated, and the change of dimensions thus determined directly; also by a knowledge of the effective temperatures at various phases and the bolometric range, the ratio in diameters can be computed. The former gives $r - r_0$ (the radii at two specified phases), and the latter r/r_0, so that r and r_0 are determinate. Baade shows that by comparing the absolute magnitude (derived from r and the effective temperature) with the luminosity from the period-luminosity curve it should be possible to test the pulsation theory crucially.

A second important numerical point arises out of the implication of the pulsation theory that period is directly related to mean density. On this theory (unless specific internal rearrangements occurred) changes of period should be accompanied by changes of mean brightness and of spectrum. The effect would be very small, but is worth looking for; we recall that changes of period, secular,[90] periodic,[91] and discontinuous,[92] have been recorded. On the other hand, the change in mean brightness, amplitude, and color, found for Y Sagittarii by ten Bruggencate,[93] was unaccompanied by any change of period.

The data presented have been looked at from the point of view of the pulsation theory, because that theory seems on the whole to be the one in accordance with the largest number

[88] Eddington, The Internal Constitution of the Stars, 203, 1926.

[89] Hamburg Mitt., **6**, No. 29, 1928.

[90] For instance, Hertzsprung, A. N., **210**, 17, 1919.

[91] For instance, Ludendorff, Handbuch der Astrophysik, **6**, 198, 1928.

[92] For instance, Hellerich, A. N., **227**, 133, 1926.

[93] H. C. 351, 1930.

of observations. Pulsations, however, have very definite difficulties in producing some of the observed effects. Other theories have difficulties in various points—and in points that seem at present more essential than those in which the pulsation theory is inadequate. Little space is here devoted to the discussion of the numerous theories of Cepheid variability because no truly satisfactory theory has ever been worked out in detail, and observations are therefore of the greater importance.

70. The Long Period Variable. *a. Stellar Data.*—That the long period variables are very bright had long been realized when Merrill and Strömberg[94] derived from proper motions a mean absolute visual magnitude at maximum of +0.1 for the available stars of Class Me. Using the same material, but rejecting proper motions with probable errors greater than $0''.010$, Oort[95] derived a value $-2^{M}0$.

A more definite and satisfactory method for estimating their luminosities is given by their presence in systems of otherwise determinate distance. That the long period variables in galactic star clouds must be very luminous was early pointed out by Shapley,[96] and the presence of long period variables in the globular cluster 47 Tucanae[97] was another definite indication of their great brightness. Shapley,[98] in comparing the photographic magnitudes of the numerous long period and cluster type variables discovered by Miss Swope[99] in the direction of the galactic center, has more recently shown quite definitely that the long period variables so far observed in that region have a mean absolute visual magnitude of −2.5, and thus are definitely supergiant stars.

If the absolute bolometric magnitude is obtained by applying to the mean absolute visual magnitude the bolometric correc-

[94] Mt. W. Contr. 267, 1924.
[95] B. A. N. 120, 132, 1927.
[96] H. B. 804, 1924.
[97] Shapley, H. B. 783, 1923.
[98] H. Repr. 53, 1928.
[99] H. B. 863, 1928.

tion appropriate to a temperature of 3000°, the result is about −4.2. But the absolute visual magnitude is probably cut down by several magnitudes by the strong band absorption that affects the spectra of all long period variables,[100] so that the absolute bolometric magnitudes are probably of the order of −6.

A period-luminosity curve for the long period variables has been sought by several investigators. Gerasimovič,[101] discussing all the material available at the time (considerably

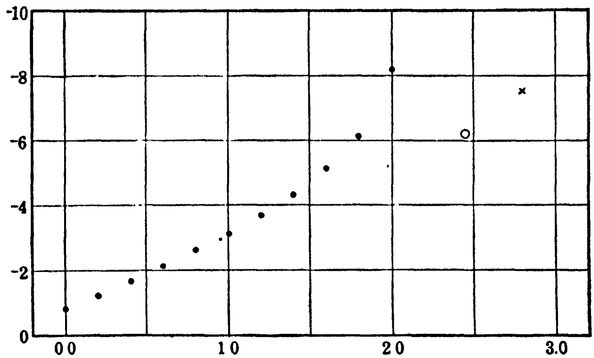

FIGURE XIV, 7.

The bolometric period-luminosity curve. Dots represent mean points for Cepheids; circles and crosses represent M and N stars.

more than used by Oort), obtained absolute magnitudes of −2.3 for periods 100 to 150 days; −1.1 for periods 250 to 340 days; and +0.3 for periods greater than 340 days. Another period-luminosity curve for long period variables, this time for minimum light, has been evaluated by Gyllenberg.[102] Neither Gyllenberg nor Gerasimovič found any increase in brightness with period, as is found for Cepheids. Incidentally the large proper motions of the long period variables of shorter period, noted by Merrill, suggest low luminosity.

[100] Payne and ten Bruggencate, H. B. 876, 1930.
[101] H. Repr. 54, 1928.
[102] Lund Medd., Series 2, 53, 54, 1929.

Long period is correlated, as Campbell and Miss Cannon[103] have shown, with late spectral class in the mean; and the effect of the band spectrum in decreasing the observed absolute magnitude becomes progressively greater the later the spectral class.[104] Probably between M1 and M9 the cutting down of the visual light of the variable by band absorption amounts to three or four magnitudes, so that the corrected period-luminosity curve for the M stars would show greater brightness for long periods. In fact, the bolometric period-luminosity curve for the M stars that are long period variables seems to fit continuously on to the bolometric period-luminosity curve for Cepheids.

b. Variability of Light.—The long period variables are a rather unhomogeneous collection of stars, classed together because of similar variability. The Harvard Catalogue of Long Period Variables[105] adopted the following criteria for the stars that were included:

1. A variation with a period of fifty days or more, provided the star had not been assigned to another class of variables.
2. A range of variation of at least two and a half magnitudes, except where published observations prevent the inclusion of the star among the long period variables.
3. A variation of fairly large range, for which observers have stated that the period is probably long.
4. A spectrum of Class Me or Se, generally indicative of long period variation.
5. A light curve of the R Coronae Borealis, U Geminorum, or RV Tauri type, especially if the variation exceeds one magnitude.

Stars in the last group are considered separately in the present monograph in Sections 71, 75, and 74.

Requirements 1 and 2 define a natural group of stars of similar properties. The spectra may be of Classes M, S, N, or R, but despite superficial differences, the stars all exhibit variability similar in its general features. As pointed out in Sec-

[103] H. B. 862, 1928.

[104] Payne and ten Bruggencate, H. B. 876, 1930.

[105] Townley, Miss Cannon, and Campbell, H. A., **79**, Part 3, 1928.

tion 68, stars of these spectral classes are not necessarily long period variables, but typical long period variables occur in all of them.

The periods of stars in this natural group are between 80 and 700 days, with a median period near 300 days.[106] The range in both visual and photographic brightness is generally between two and a half[107] and six magnitudes; much of this change is occasioned by a considerable change of temperature,[108] coupled with the concomitant spectroscopic changes. That the changes of total energy output are far smaller than the changes in photographic and visual brightness is shown by the fact that the radiometric ranges are not much greater than the visual (and radiometric) ranges of Cepheid variables. Data obtained at Mount Wilson[109] for nine stars are summarized below. The mean radiometric range for the nine stars tabulated is $0^m.71$.

Star	Spectrum	Radiometric Range	Star	Spectrum	Radiometric Range
		m			m
o Cet	M5e-M8	0.8	R Aqr	(M5e-M7e) + P	0.5
X Oph	K0-M7e	0.5	R LMi	M7e-M8e	0 7
χ Cyg	M5e-M8e	0.9	R Leo	M6e-M8e	0.6
R Cnc	M6e-M8e	0.7	R Hya	M6e-M8e	0.9
R Aql	M5e-8e	0 8			

Like the Cepheids, long period variables display great diversity of light curves; classifications of light curves according to their shape have been made by Campbell,[110] by Phillips,[110a]

[106] Miss Cannon and Campbell, H. B. 862, 1928.

[107] That this is the lower limit of range for long period variables is partly the result of the criteria applied in selecting stars for the catalogue; but the criteria themselves were based on the observed properties of known variables, so the restriction may not be so artificial as would at first appear.

[108] That the variation is chiefly the result of temperature change is not incompatible with the observed fact that long period variables of Classes M and S are no redder at minimum than at maximum; for these spectral classes temperature and band absorption compensate one another in their effect on color and work together in their effect on brightness. See Section 70*f*. No data are available as to the radiometric ranges of N and R stars; possibly the statement as to the effect of change of temperature on light curve should be modified for these classes.

[109] Pettit and Nicholson, Mt. W. Contr. 369, 1927.

[110] H. Repr. 21, 1920.

[110a] J. B. A. A., **27**, 2, 1916.

and by Ludendorff.[111] Some few stars show secondary maxima, and halts on the rise are common. The forms of the light curves have not as yet been satisfactorily explained.

All long period variables differ in brightness at different maxima, and also at different minima. The uncertainty of constant maximum or minimum brightness is greater for some stars than for others, and notably large for stars of Class N. Maxima that are abnormally faint visually tend also to be abnormally faint photographically;[112] this shows that the abnormality is one of brightness rather than of color, but as the lowering of the surface temperature of an M star (as evinced by advancing spectral class) is unaccompanied by a reddening, we cannot be sure that the abnormal brightness is not a result of abnormal temperature. It seems probable[113] that abnormal temperature is in fact the cause of the irregularity, and that it operates by changing the very sensitive intensity of band absorption. The regularity of the radiometric light curve would test crucially the question whether abnormally bright maxima were the result of abnormal energy output or of unusual surface temperature, but the radiometric range of M stars is so small that the observations would be very difficult, and the question will probably be answered for some time to come in terms of probabilities.

Future analyses may elicit some interpretation of the differences displayed by successive maxima and minima of the same variable; perhaps it will be found that high maxima or low minima occur in cycles, or that they are associated with uncommonly long or short intervals between successive maxima. The available data are few and inexplicit. For one star periodic changes in the depth of minimum have been lately noted,[114] and very possibly the phenomenon is more widespread than is now apparent. Such an effect might be of great value

[111] Handbuch der Astrophysik, 6, 127, 1928.
[112] Campbell and Payne, H. B. 872, 1930.
[113] Payne and ten Bruggencate, H. B. 876, 1930.
[114] Campbell and Payne, H. B. 872, 1930.

for the guidance of theory, and is worth looking for in the variations of other stars.

c. Variations of Period.—Long period variables proper are notorious for their unpunctuality in coming to maximum; for example, predictions for 1928 on the basis of well-known elements for the stars on the visual list summarized annually at Harvard were within 5 days for only 40 per cent of the stars, within 10 days for 62 per cent, within 15 days for 82 per cent, and within 25 days for 93 per cent.[115] These numbers represent almost entirely the inherent unpunctuality of the stars.

Type of Change	Example	Reference
1. No change over observed interval	S Tucanae	Campbell, H. B. 847
	RR Scorpii	Campbell, H. B. 846
2. Steady change (*a*) linear	X Camelopardalis	
	(R Scuti)	Gerasimovič, H. C. 333*
(*b*) sinuous	R Aquilae†	
	R Centauri	Campbell, H. B. 836
3. Periodic fluctuation	R Virginis	
	R Chamaeleontis	Campbell, H. B. 842
	T Columbae	Campbell, H. B. 837
4. Abrupt change of period	U Tucanae	Campbell, H. B. 840
	RT Eridani	Payne, H. B. 868
	(R Sagittae)	Gerasimovič and Hufnagel, H. C. 340‡
5. Abrupt shift of epoch	R Lupi	Ludendorff, Handbuch der Astrophysik, **6**, 127, 1928
	Z Aquarii	Mrs. Shapley, H. B. 868

* Semiregular variable; cf. Gerasimovič, H. B. 865, where a sine term is suggested for the star.

† The period has also decreased from 350 days to 310 days during the interval studied.

‡ Semiregular variable.

The observed variations of period are of many kinds:[116] there are steady changes of period, periodic fluctuations, abrupt changes of period, and abrupt shifts of epoch. There are also a few stars for which the period seems to be quite steady. Probably steady sinuous changes of period are all but universal for long period variables, and there does not seem to be conclusive evidence that delays in coming to maximum are cumu-

[115] Campbell, H. C. 321, 1928.

[116] Payne and Campbell, H. B. 875, 1930.

lative.[117] The average deviation of the observed from the predicted date of maximum is about four per cent of the period, being numerically largest for the longest periods.

d. Variations in Velocity.—Like the Cepheid variable, the long period variable shows changes of radial velocity, different for absorption and emission lines.[118] But the relation of the radial velocity curve to the light curve differs in phase from that displayed by the Cepheid—maximum velocity of recession coincides with light maximum for absorption lines. We also note that other red stars (not long period variables) show similar changes of radial velocity.[119]

e. Spectroscopic Data.—The variable spectra of the long period variables, especially the conspicuous bright lines of hydrogen and other substances, have been extensively and perhaps too minutely studied. A summary of their classification is given by Merrill.[120] The salient points are:

(1) There are three main types of spectrum in the long period group: the M spectrum, the S spectrum, and the R-N spectrum; they are characterized respectively by absorption bands of titanium oxide, zirconium oxide, and carbon.

(2) The M and S types have some features in common; titanium oxide bands occur in the spectra of many S stars and in some are so strong that the classification of the spectrum is in doubt. A few stars, with the other earmarks of Class S, seem to lack the bands of zirconium oxide.

(3) A similar substratum of metallic lines underlies all the spectra; but relative intensities differ greatly; for instance 4227 (Ca) is immensely strong for Me stars, 4554 (Ba+) and 4607 (Sr) for Class S, and the first also in Class N; all these are ultimate lines.

(4) The variable bright lines are the most striking feature of the long period variable. As yet they have not been success-

[117] See below, p. 236.
[118] Merrill, Pub. Mich. Obs., **2**, 45, 1916.
[119] See Section 72.
[120] Pop. Astr., **37**, 444, 1929.

fully interpreted. Merrill and Joy[121] find them weaker in Me stars of short period.

From point (2) we may guess that the band spectra are not the most essential and characteristic indices of the variables. Possibly the physical knowledge that is so urgently needed in the interpretation of the spectrum of the long period variable will spring from a study of the metallic absorption spectrum, rather than the spectra of compounds, or the puzzling bright lines.

The long period variables, in the mean, show a period-spectrum relation that is continuous with that shown by the Cepheids,[122] the two being joined by thes emiregular variables. Within the long period variables, also, there is a definite period-spectrum relation, stars of long period having statistically late spectra.[123]

From many points of view the possession of banded spectra is the most important feature of the long period variables. In the various subsections of Section 70 I have pointed out the features of the variability that can be attributed to the absorption bands. In brief, as summarized by the writer and ten Bruggencate,[124] the band spectra can be held responsible for "the ranges, visual, photographic, and radiometric, of the long period variables; the changes of color of variable stars of Classes M and N; the relation of the luminosities of M (and N) stars to their periods; the general small-range variability of late-type stars." Each of these aspects of the band spectrum is mentioned in the appropriate section. I take the opportunity of emphasizing here that the band spectrum undoubtedly plays a major rôle in controlling the observable behavior of all red variables; but other features of the spectrum are probably of deeper physical import, and the chief thing to remember is that the variation of brightness for the late-type variable has probably a spurious conspicuousness.

121 Mt. W. Contr. 382, 1929.
122 Shapley, H. B. 861, 1928.
123 Miss Cannon and Campbell, H. B. 862, 1928.
124 Payne and ten Bruggencate, H. B. 876, 1930.

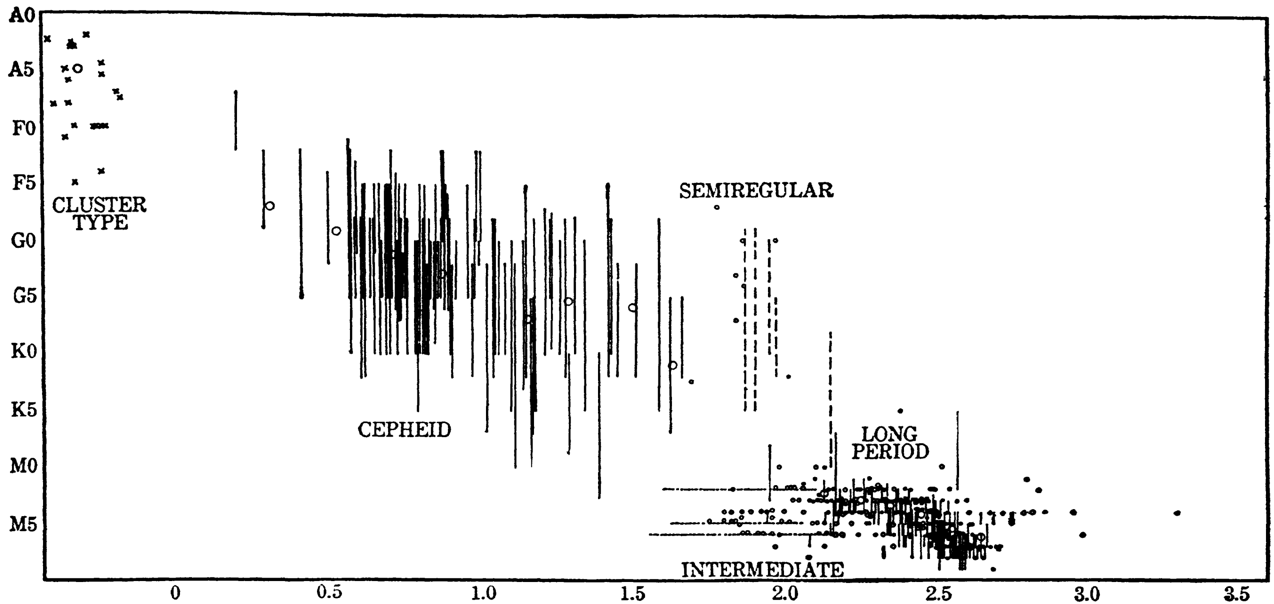

FIGURE XIV, 8.

The period-spectrum relation. Ordinate and abscissa are spectral class and logarithm of period. Crosses denote cluster-type variables. The vertical lines represent the spectral ranges of Cepheids (on the left) and long-period variables. Broken vertical lines represent the spectral ranges of semiregular variables. Small dots represent single spectral observations of long-period variables (it should be remembered that these observations probably refer to maximum light, or near it; because of the large visual and photographic ranges of long-period variables the observed spectral ranges also probably refer to the part of the light curve around maximum). Small circles represent intermediate and semiregular variables; all semiregular variables are plotted with the *whole period*, and both long and short periods of intermediate variables are represented. Horizontal dotted lines represent the range of cycles of some semiregular variables. Large circles denote the mean period-spectrum relation as given in Harvard Bulletin 861.

f. Changes of Temperature.—As might be inferred from the radiometric range, the large part of the visual and photographic fluctuation is occasioned by an effective change of temperature. Such data as are available are contained in Table XIV, XII. The temperatures are of doubtful significance, because of the disturbing effect of absorption bands; but those tabulated are very probably of the right order.[125]

TABLE XIV, XII.—TEMPERATURES OF LONG PERIOD VARIABLES

Star	Maximum	Minimum	Reference	Spectrum
	°	°		
Mira	2540	2020	1	M6e
Mira	3730	<2000	2	M6e
X Ophiuchi	2260	1890	1	M6e
X Cygni	2260	1580	1	M6e
X Cygni	2230	1200	3	M6e
R Cancri	2450	1890	1	M7e
R Aquilae	2360	1890	1	M7e
R Aquarii	2180	1760	1	M7e
R Leonis Minoris	2230	1860	1	M8e
R Leonis	2260	1760	1	M8e
R Hydrae	2360	1950	1	M8e
R Trianguli	...	1950	1	....
U Hydrae	2360	...	1	N2
X Cancri	2260	.	1	N3
VX Andromedae	2010	. .	1	N7

REFERENCES

1 Pettit and Nicholson, Mt. W. Contr. 369, 1928.
2 Hopmann, A. N., **226**, 1, 1925.
3 Hopmann, A. N., **222**, 237, 1924.

Besides the more direct information from radiometric measures, it is possible to draw some conclusions from color indices measured in the ordinary way by comparing visual and photographic magnitudes. The method has indeed advantages of its own, as leading to the evaluation of the complete color curves of variables for which reliable visual and photographic mean light curves have been obtained.

[125] See p. 235.

The most extensive data yet published on the color indices of long period variables at maximum were given by Gerasimovič and Shapley[126] for 24 M stars, 7 S stars, 1 R star, and 6 N stars. It appeared that all the M and S stars were of approximately the same color irrespective of period or of spectral subdivision; the mean color index for stars of Class M is 1.35, and for stars of Class S, 1.99. Among the N stars, on the other hand, the stars of longest period and latest spectrum were by far the reddest, the range of color index within Class N being from less than 3.0 to more than 5.0. The star of Class R was comparable in color with an early N star.

The same authors gave data for six M stars at minimum, and showed that they were in the mean of about the same color as at maximum.

The same subject has been extended to a comparison of visual and photographic light curves, and the derivation of color curves, by Miss Cannon,[127] and by Campbell and the writer.[128] The nine stars discussed, all of Class M, showed no conspicuous change of color in going from maximum to minimum, nor any specific type of color curve. The effect of the observed spectrum changes upon the apparent color would be exactly the observed one: the stars actually become cooler at minimum (cf. Table XIV, XII) and the bands of titanium oxide, increasing in strength, cut down the red portion of the spectrum so that the effect of the fall of temperature on the color of the star is all but neutralized. The two effects (fall of temperature and rise of band absorption) do not neutralize one another exactly, and the residual effects appear as the erratic and dissimilar color curves reproduced by Campbell and the writer in Harvard Bulletin 872.

This interpretation accounts not only for the lack of conspicuous variations of color but also for the failure of the M and S stars to progress in color with advancing spectrum and

[126] H. B. 872, 1930.
[127] Miss Cannon, H. B. 872, 1930.
[128] H. B. 872, 1930.

long period; spectrum and period are rather closely correlated,[129] and thus similar effects will be expected for both. The longer period stars show by their late spectral classes that their temperatures are lower than those of stars of earlier spectrum; and the increased band absorption that accompanies the lower temperatures cuts down intensity at the red end of the spectra and leaves the color about the same as that of hotter stars of earlier spectral class and shorter period.

Band absorption in the M stars cuts down the red and raises the apparent temperature. The bands in the N stars, however, cut down the violet (the strong cyanogen bands, for instance, are all to the violet of 4215), and their progression in redness with spectral class and period, shown by the work of Gerasimovič and Shapley, is the natural result. Band absorption and temperature work together in affecting the colors of N stars; they work against each other for M stars.

TABLE XIV, XIII.—COLOR INDICES OF LONG PERIOD VARIABLES

Class	Color Index		Deduced Temperature	
	Maximum	Minimum	Maximum	Minimum
	m	m	°	°
M	1.35	(1.13)	3600	4100
S	1.99	....	2700	....
N3	3 00	....	2000	....
N8	4 50	. .	1400	....
(N3) (RV Cen)	2.54	4.67	2200	1300

On the same basis N stars should be redder at minimum than at maximum, and recent analyses of RV Centauri by Campbell and Miss Hoffleit[130] have shown this to be the case: the color index is 2.54 at maximum and 4.67 at minimum—an additional effect of the weakening of the violet end of the spectrum by the bands that strengthen with falling temperature.

[129] Miss Cannon and Campbell, H. B. 862, 1928.

[130] H. B. 875, 1930.

It may be of interest to tabulate the mean color indices and the directly deduced temperatures for the several classes of long period variables. Comparison with Table XIV, XII shows that the M stars appear from their color indices to be hotter than they actually are, and the N stars cooler, as was to be expected from the discussion of their spectra. In both cases the discrepancy is largest at minimum, where the band spectrum that causes it is the strongest.

g. Theoretical.—Interpretations of long period variability are even less certain than those of Cepheid variability. Three main types of mechanism may be envisaged: (1) periodic outbursts of a continuously luminous star; (2) pulsations; and (3) periodic occultations by opaque clouds. Apparently the first mechanism is of the type contemplated by Eddington and Plakidis,[131] in their analysis of the changes of period of long period variables, as leading to cumulative delays in the time of maximum.[132] The third mechanism is represented by Merrill's "veil theory,"[133] and has much in common with the first. The idea of pulsation, which commends itself because of the obvious affiliations of the long period variable with the Cepheid[134] shown by the period-spectrum relation, the type of light curve, and other features requires a period-luminosity curve within the long period variables, and also one connecting them with the Cepheids.

To determine a period-luminosity curve it is necessary to select a fundamental luminosity; median brightness was chosen for the Cepheid, but maximum brightness has generally been used for long period variables. Recently Gyllenberg[135] has advocated minimum brightness as more representative for the long period variable. Evidently the three mechanisms of long period variability just enumerated consider respectively minimum, median, and maximum brightness as normal.

[131] Eddington and Plakidis, M. N. R. A. S., **90**, 65, 1930.

[132] See Section 70*c*, p. 229.

[133] Merrill, Publ. Obs. Mich., **2**, 70, 1916.

[134] Shapley, Preface to H. A., **79**, Part 3, 1928; H. B. 861, 1928.

[135] Lund Medd., Series 2, 53, 54, 1929.

Much of the difficulty is removed by contemplating radiometric magnitudes, which approach more nearly than any other system to the bolometric magnitudes to which a period-luminosity curve must ultimately refer. As the radiometric ranges of long period variables are less than one magnitude, and as the disturbing effect of band spectra on the color and brightness of the stars is greatest at minimum, it seems clear that the observed visual maximum magnitude is in practice nearer to the "normal" brightness of the star than the minimum magnitude, even if (as seems rather improbable) bolometric minimum is actually the "normal" brightness of the star. As Gyllenberg's conclusion as to the normality of minimum brightness was based on observed visual minimum it cannot have an obvious relation to the theoretically significant bolometric minimum. Whether maximum, median, or minimum brightness is normal for long period variables must follow from a discrimination of the three types of theory of the variability.[136]

The existence of a bolometric period-luminosity curve, continuous from Cepheids to long period variables, was adumbrated in Section 70*a*. The curve is illustrated in Figure XIV, 7.

On the whole it seems that a pulsation (or similar) theory fits the long period variable as well as the Cepheid. The period-spectrum relation, and the period-luminosity curve, as well as the period-density relation (Figure XIV, 6) all point

[136] The "normal" stage of a variable star might be defined as the stage in which it had the greatest tendency to stay. Two tests suggest themselves: the brightness at which the star spends the largest part of its time might be its normal brightness; or the brightness that tended to be the most stable might be normal. Long period variables usually have wider minima (i.e., of longer duration) than maxima; and the dispersion in minimum brightness seems to be rather smaller than in maximum brightness. Both tests suggest minimum as normal. But if the star is in pulsation, or if the variability has some analogous *dynamical* cause (unlike the periodic relief of strain or the regular clouding up), the criterion of normal brightness as the brightness that lasts longest is not relevant; in this case the "normal" state is the middle of the swing and is of shorter duration than the two extremes. The criterion of normality may well differ for variables of different types (e.g., for Cepheid and R Coronae stars).

to a similar mechanism of variation.[137] The detailed differences between the two classes of stars seem to be losing their weight. The general picture is summed by the writer and ten Bruggencate:[138] The variability of stars that show a considerable range in light variation (Cepheids and long period variables) may be regarded as due to a pulsation of the star. The mean density of the star will fix its period, and the amplitude of the pulsation will fix the range in bolometric brightness, and the range in temperature. Besides these regular changes, there may be irregular temperature changes at the surface of the star, affecting the spectrum of the variable. Such changes will affect the visual and photographic brightness only of the long period variables, because they alone have band spectra. We have here the explanation of the irregularities in maximum brightness mentioned in Subsection 70*b*.

71. The RV Tauri Variable.—In period and in spectrum the Cepheid and the long period variable are linked by the RV Tauri star.[139] Variables can be assigned to this class only after intensive study, and Gerasimovič, a first authority in the matter, enumerates but 12 authentic members.[140] There are many suspects, but probably the number of these stars is really small.

The characteristic of the variability is the alternation of deep and shallow minima, and the occasional interchange of minima of the two kinds. Minima for these stars are less punctual than for Cepheid variables, and the periods of many of them show considerable harmonic inequalities, but no true secular changes of period have been recorded.

The RV Tauri variables conform in the mean to the period-spectrum relation when the *interval between successive minima* (not between similar minima) is considered to be the period.

[137] Shapley, Preface to H. A., **79**, Part 3, 1928.
[138] Payne and ten Bruggencate, H. B. 876, 1930.
[139] Shapley, H. B. 861, 1928.
[140] H. C. 341, 1929.

The spectra show the c-character,[141] and some[142] have emission lines, so that the stars are presumably bright, as is also suggested by the high galactic concentration.[143]

The velocity curves, so far as they are known, appear to mirror the light curves, with phase displacements such as are observed for Cepheids, but larger. A fairly good mean velocity curve may be obtained by using the *interval between similar minima* as the period.

The relationship between the RV Tauri star and the Cepheid has been summarized by Gerasimovič in the paper just quoted: the supergiant spectrum, the period-spectrum relation, the changes in temperatures and spectra, the ranges in light and radial velocity, and the form and relationships of the light and radial velocity curves, all confirm it.

The RV Tauri star cannot at present play a large part in the discussion of high luminosity stars, for our knowledge of its luminosity is indirect and qualitative. Its importance to theories of stellar variability is obvious: so closely is it related to the Cepheid that a satisfactory theory of Cepheid variability must include it as a possibility. Evidently a crucial point is that the "half period" is associated with the spectrum,[144] and the "whole period" with the changes of radial velocity. The "half period," it seems, is fundamentally connected with the variability, and the whole period with the free vibration of the star (and thus presumably with the mean density).

Gerasimovič discusses the theory of RV Tauri variability[145] in terms of a viscous rotating star with the rotation period picked out by resonance of the outer envelope. His discussion

[141] Adams and Joy, Pop. Astr., **28**, 513, 1920; Mt. W. Rep., p. 234, 1922.

[142] TT Ophiuchi (F7e); AC Herculis (F8e-K5), and R Scuti (G8e-M) have emission lines; U Monocerotis and V Vulpeculae have their hydrogen lines weakened, apparently by incipient emission.

[143] Gerasimovič, H. C. 341, 1929; the mean galactic latitude is 12°.

[144] The scatter of the RV Tauri stars on the period-spectrum relation is very large, and is not much more serious when the "whole period" is used than for the "half period."

[145] H. C. 341, 1929.

essentially describes a way of providing two periods in a ratio 1/2, 1/3, or some other simple relation, and is admittedly too simple. Gerasimovič bases his discussion on Jeans' mechanism,[146] but as Rosseland[147] has pointed out: "Jeans' theory of Cepheid variability misses the point by identifying the period of variability with the period of rotation, instead of with the 'activity period' of the internal circulation."

72. The Intermediate Group.—Between the maxima of period frequency representing the Cepheids and the long period variables fall the stars with periods between 50 and 100 days, designated by Gerasimovič[148] the "intermediate group," and including RV Tauri stars and also stars, not of the RV Tauri group, that are statistically not members of the principal sequence (Cepheids and long period variables).

The "intermediate stars" do not conform closely to the period-spectrum relation; with two exceptions (UU Herculis, G0, and SV Ursae Majoris, K2) their spectra are of Class M, without recorded emission lines. They show changes of period so sharp and so large that Gerasimovič did not regard their variations as produced by simple gravitational pulsations, because this would imply large changes of mean density and brightness.

In Figure XIV, 8 are five points referring to the "intermediate" stars μ Cephei, UZ Persei, TW Pegasi, AF Cygni, and SV Ursae Majoris. The periods used in placing these five stars in the diagram are not the comparatively short periods of variation, but the long periods that govern the changes of variability. As Gerasimovič has remarked: "Perhaps cyclical variables satisfy the period-spectrum relation at its extreme end with the μ Cephei variables, but unfortunately the physical nature of these stars is entirely unknown."

[146] M. N. R. A. S., **85**, 797, 1925; Astronomy and Cosmogony, Chapter 10, 1928.

[147] Trans. Norwegian Acad., Oslo, Math.-Naturv. Kl., No. 6, 1929.

[148] H. C. 342, 1929.

The temperatures of the intermediate variables seem to be higher than those of long period variables of similar spectral class.[149] M stars that show no emission lines are less luminous, and, spectrum for spectrum, therefore probably hotter, than Me stars.[150]

The recent investigation of the radial velocity of R Lyrae[151] is of great importance in linking this star, quoted by Gerasimovič as a typical regularly cyclic variable, with the Cepheid, in which a similar relationship appears between light and radial velocity. The orbital interpretations involved in Plassmann's summary of observations for this star[152] lead to very low values of mass function and are presumably not real.

There can be no doubt that the erratic changes of period shown by stars of the groups under discussion present very great difficulties to a pulsation hypothesis. It seems best, as Gerasimovič has suggested, to look on the associated *long* period as the one related to a gravitationally governed pulsation rather than the short period. In addition to the stars plotted in the figure that have long periods with a comparatively short period oscillation superimposed, it seems probable that many stars of Class N are of similar character; probably their radiometric changes are small and regular, the visual and photographic irregularity being superficial. A plausible case for such variability is presented by ten Bruggencate[153] for SX Scorpii, and (with slighter data) for RX Scuti, both of Class Nb. W Pictoris (Nb) is probably a star of similar behavior. Also it has been the common experience in the photographic

[149] Pettit and Nicholson give for R Lyrae a temperature of 2400°, and for the long period variable, as measured by the same method, mean temperatures of 1990°, 1550°, and 1350°, at maximum, median, and minimum.

[150] The absolute magnitude of the non-emission M star is rather lower than that of the Me star. In confirmation of this I note that the spectrum of R Lyrae is not that of a very luminous star, such as Betelgeuse.

[151] Mrazek, A. N., **236**, 281, 1929.

[152] Himmelswelt, **40**, 70, 1930.

[153] Ann. Bosscha Obs., **2**, C33, C40, 1928.

study of long period variables[154] that stars of Class N show larger residuals from the mean light curve than the Me stars. Their periods are longer in the mean than those of M stars, in harmony with their great brightness and presumably low temperature and mean density.

The suggestion that the short periods of these variables are caused by rotation of a star with variable dark or bright markings is made by Gerasimovič in his discussion of the light changes.[155]

It is perhaps to under-represent the variable stars of Class M if we discuss only those with periods less than 100 days. All the M variables without emission lines should probably be discussed together.[156] The variable stars[157] with spectra of Class M, 78 in number, have a mean period of 216 days, and a median period of 165 days; only 15 of them have periods of less than 100 days, and 4 have periods longer than for any known Me star. Their ranges are smaller than those of most Me stars, and their variability less regular. They present an important and little touched subject. The shortness of their median period suggests that they are indeed related to the long period variable, the bright lines being commonly weaker for Mira stars of short period,[158] though obviously this represents a statistical tendency only.

73. The Irregular Variable, Class K.—Fifty-four variable K stars are enumerated in Prager's catalogue, and nineteen of them are definitely styled irregular. Though perhaps for some small-range K variables semiregularity may be established by intensive study, there are undoubtedly some like V Pyxidis,

[154] H. B. 860, 861, 1928.

[155] H. C. 342, 1929.

[156] Cf. Chapter XIII.

[157] Miss Cannon and Campbell, H. B. 862, 1928. Probably many of the stars classed M have actually very weak emission lines; see Merrill and Joy, Mt. Wilson Contr. 382, 1929, for the relation of period to strength of emission lines. We note that most if not all N stars have emission spectra, though the bright lines are not generally apparent with short dispersion.

[158] Merrill and Joy, Mt. W. Contr. 382, 1929.

which has been shown by Jordan[159] and the writer[160] to be really irregular.

Spectroscopically these small-range K stars resemble the normal K star; they are not supergiants, and there is nothing to link them spectroscopically with the N stars that have similar variations. The titanium oxide bands are in evidence (a feature that never occurs for N stars) and the G band is conspicuous.

We do not know at present what place in the general complex of variable stars is occupied by the small-range K. It is most probably connected with the M stars of the previous section. The study of its spectrum is recommended. Very possibly its variability is entirely the result of variations of the band spectrum.[161]

The variable stars so far enumerated have some very definite points in common, and even though we may exclude some from the principal sequence, they are related to it more or less distinctly.

The types of variables now to be mentioned differ radically in spectrum, in spectral changes, and in light, not only from the principal sequence but from all other stars as well.

74. The SS Cygni Type.—Of a very different class are those stars to be discussed in the four sections below. There can be no question of their belonging to the principal sequence and but little hope of attributing their behavior to pulsation. The features of their spectra show certain similarities but have never been interpreted.

The typical variability of the SS Cygni star need not be described. The impression is received that the normal magnitude is at minimum; the outbursts are cyclical, with a rather small dispersion about the mean cycle.

[159] H. B. 831, 1926.
[160] H. B. 868, 1929.
[161] Payne and ten Bruggencate, H. B. 876, 1930.

The three stars with known spectra are described below:

Star	Description	Reference
SS Aurigae	Maximum: almost continuous, with narrow dark lines of hydrogen and helium. Color white	Adams and Joy[162]
U Geminorum	Maximum: continuous on best spectrograms	Cannon[163]
	Class F, narrow hydrogen lines, with H and K lines	Cannon[164]
SS Cygni	"Maximum: Spectrum is continuous with faint dark bands of hydrogen and helium 20 Angstroms wide"	Adams and Joy[165]
	"Minimum: Strong bright bands of hydrogen and helium about 20 Angstroms wide but not displaced. Possibly a few faint absorption lines. The spectrum of this star bears considerable resemblance to that of novae"	

These spectroscopic features are significant in comparison with those described in the two following sections.

75. The R Coronae Borealis Star.—A rather small class of variable stars, although a considerable number have been assigned to it tentatively that are not genuine members, the R Coronae Borealis group is of unusual interest and possibilities. The accepted members of the group, with their galactic latitudes and spectral classes, are enumerated below:

Star	Galactic Latitude °	Spectrum	Star	Galactic Latitude °	Spectrum
T Tauri	20	Gpe	R Coronae Borealis	50	cGo
SU Tauri	3	G	XX Ophiuchi	11	Pec.
Z Canis Majoris	1	Pec.	RY Sagittarii	21	Gop
UW Centauri	9	K	R Coronae Austrinae	20	Gpe
S Apodis	12	R3	RS Telescopii	15	R8
			Y Muscae	3	..

R Coronae Borealis has a very striking spectrum at maximum; the lines show the c-character to a remarkable

162 Mt. W. Rep., p. 234, 1922.
163 Henry Draper Catalogue, H. A., 93.
164 H. A., 56, 210, 1912.
165 Pop. Astr., 30, 103, 1922.

extent, and there are numerous unusual lines present.[166] At minimum the absorption spectrum is unchanged; but in addition sharp bright lines of ionized titanium, and sharp bright H and K, have been observed. As the minimum passes, the bright lines weaken and disappear.[167] We may take it that the spectra of T Tauri, SU Tauri, and R Coronae Austrinae are fairly similar.

That two spectra of Class R should occur in this uncommon type of variable star cannot be a coincidence; temperature, luminosity, and therefore mean density may well be the same for Classes R and Gp. Thus we have strong reason to suspect that the R Coronae star is an intrinsic variable, since eight out of the ten known members have spectra that point to similar density and size, and nothing can be asserted of the other two in this respect. The low galactic latitudes and the c-character suggest great brightness for these stars. We recall the suggested affiliation between the R Coronae star, by way of S Apodis, and ϵ Aurigae.[168]

The abrupt changes of brightness, and the spectrum at minimum, recall the SS Cygni stars. The R Coronae star, with its far longer period (if such it can be called—perhaps cycle would be the better term) has a later type spectrum than the SS Cygni star.

76. The Nova.—Our justification for mentioning the nova is its high luminosity at maximum, and an obvious affiliation with the two preceding classes. The novae for which we have spectroscopic details have all shown an absorption spectrum with a strong c-character just before maximum,[169] ranging from cB8 in Nova Persei 1901 to cF5 for Nova Pictoris. The fall to minimum is always accompanied by the appearance and growth of the complex bright bands of hydrogen and the metals, with the ultimate development of the nebular lines. The

[166] Frost, Ap. J., **22**, 215, 1905; Ludendorff, A. N., **201**, 439, 1915.
[167] Joy and Humason, P. A. S. P., **35**, 325, 1923.
[168] H. B. 868, 1929.
[169] Adams, P. N. A. S., **4**, 254, 1918.

minimum spectrum of SS Cygni is recalled, and also the bright lines at minimum for R Coronae Borealis.

The novae, of course, qualify at maximum as high luminosity stars. Data on individual maximum luminosities are enumerated by Stratton,[170] and also discussed by Lundmark,[171] who derives a mean absolute magnitude -7.1 at maximum. The mean absolute magnitude -5.2 and small spread in luminosity of the novae found by Hubble[172] in Messier 31 is of interest also in this regard.

Because the nova is in so unstable a condition, I do not think that discussion of its spectrum would be profitable in a book devoted for the most part to static stars. But I cannot refrain from calling attention to the general similarity of the energy spectra of Wolf-Rayet stars reproduced in Chapter VI and the energy distribution in the spectra of Nova Geminorum No. 2 and Nova Pictoris, as reproduced [173,174] by Furuhjelm and by Davidovich.[175]

One further comment is relevant. Two of the most interesting stars in the sky—P Cygni and η Carinae—are ex-novae; whether true novae or of the recurrent T Pyxidis type we cannot of course determine, but the distinction may have little meaning. The spectra of these two stars are similar in having the c-character, bright lines, and in being variable, but in nothing else. They again recall the spectra of SS Cygni and R Coronae Borealis at minimum, and their spectra provide a unique chance for physically studying a complex problem. Neither P Cygni nor η Carinae is the only star of its type; in particular there are eight P Cygni stars in the Large Magellanic Cloud,[176] a place where long and diligent search has revealed only one

[170] Handbuch der Astrophysik, **6**, 262, 1928.

[171] P. A. S. P., **35**, 106, 1923.

[172] Ap. J., **63**, 258, 1926.

[173] Publ. Pots. Obs., No. 68, 1913.

[174] H. B. 838, 1926.

[175] The analogy between the nova and the Wolf-Rayet star has since been elaborated by Beals. See Chapter VI, and H. B. 874, 1930.

[176] Miss Cannon, H. B. 801, 1924.

nova.[177] Are all hese stars ex-novae also? Or is the P Cygni type of spectrum one that may appear after more than one life history? Probably the latter is more nearly true, and, if so, we must use caution in assigning absolute magnitudes on the basis of the P Cygni type of spectrum. We note that the mean absolute magnitude of the Magellanic P Cygni stars is −5.3.

77. The Peculiar Variable.—Reference to a few specially interesting stars will develop the idea that in SS Cygni, R Coronae, and Nova variability we see a number of phenomena that have features in common. T Pyxidis[178] has a spectrum like that of typical nova, and RY Scuti[179] seems also to fall with the nova-like variables; both have predominantly bright-line spectra. RT Serpentis[180] has a bright-line spectrum that strengthens as the star fades.[181] XX Ophiuchi has shown erratic changes[182] in its bright-line spectrum. The bright-line spectrum of Z Andromedae,[183] associated with a nebula, furnishes further material. We note that the radial velocities of different lines are very different for RY Scuti and that the radial velocity of RT Serpentis is not constant; whether these are the signs of intrinsic variability or not is at present undetermined.

78. The Variable Star in Its Affiliations. *a. Vestigial Variability.*—The interconnections of the classes of variable stars have been long recognized, and the present view of the matter is well summarized by the diagrams and tables of Ludendorff[184] and of Gerasimovič.[185] A class of star imperfectly

[177] Luyten, H. B. 847, 1928.

[178] Adams and Joy, Pop. Astr., **28**, 514, 1920.

[179] Merrill, Mt. W. Contr. 349, 1928.

[180] Shapley, P. A. S. P., **31**, 226, 1917.

[181] Adams and Joy, P. A. S. P., **60**, 252, 1928; the same phenomenon is shown by the spectrum of a declining nova; cf. H. B. 874, on the spectrum of Nova Pictoris.

[182] Merrill, P. A. S. P., **36**, 225, 1924; **38**, 26, 1926.

[183] H. H. Plaskett, Pub. Dom. Ap. Obs., **4**, No. 10, 1928.

[184] Festschrift für Hugo v. Seeliger, p. 80, 1924.

[185] H. C. 342, 1929.

represented there is the irregular variable of small range, of whose real numbers and relationships we have at present little idea.

The most important observation on this matter is Shapley's[186] demonstration that there are a number of variables of small range within the same interval of brightness and spectrum as the true cluster type variables in the cluster Messier 3; stars of that magnitude and color seem to be prone to variability. The question naturally arises as to the tendency to variability of stars of similar brightness and color to the cluster type variables, Cepheids, semiregular variables, and long period variables of the galactic system.

There is no evidence that galactic stars near to the cluster type variables in mass and brightness show vestigial variations to any great extent.[187] Struve would place a number of short period spectroscopic binaries of Class A in this category, and the stars of Classes A3 and A5 in the Revised Harvard Photometry (visual) have residuals rather larger on the average than those for stars of Classes G5 and K0, with a few exceptionally large residuals. It seems probable that some of the A stars with variable radial velocity are either cluster type variables of small range or similar stars with vestigial variation.[188]

There are about 200 known galactic cluster type variables and over 40,000 A stars, so that the chance of an A star being a cluster type variable is very much less than one half of one per cent, as the variable stars are known to much fainter magnitudes than the spectra. This is out of all proportion to the difference in dispersion in absolute magnitude between normal A stars and cluster type variables;[189] unless there are large numbers of cluster type variables of undetectably small range we must regard the occurrence of variation of this type as very exceptional.

[186] Mt. W. Contr. 176, 1920.

[187] Ap. J., **60**, 167, 1924.

[188] The stars 12 Lacertae (Stebbins, Publ. Washburn Obs., 15, Part 1, 1928) and γ Bootis (Frl. Güssow, A. N., **229**, 199, 1924) furnish examples of regular and irregular variations.

[189] Cf. Section 49.

The parallel to the Cepheids is furnished by the supergiants of the second type ("Pseudocepheids"), which have been found to be more liable to variability, both of light and velocity, than the normal giant of similar class.[190] They probably furnish an excellent example of "vestigial variability" to parallel the Cepheids, and we note that they are more nearly comparable to the Cepheids themselves than the normal giants, in spectrum and in luminosity.

The long period variable seems to be at the most critical point of the stellar sequence as far as variability is concerned. Stebbins and Huffer[191] have shown that all M stars are more or less variable (they examined no dwarfs) the stars of greatest luminosity and latest spectrum tending to vary the most. Stars of Class N and Class S, also probably Class R stars, all tend to variability, and the tendency extends to some (giant) K stars, though out of 164 K stars (probably mainly dwarfs) used by Stebbins and Huffer as standards of brightness not one turned out to vary.

The vestigial variables in clusters, and the galactic Pseudocepheids, are known to be at very nearly the same temperature and brightness as the true variables. The irregular M stars of small range cannot be asserted to be necessarily connected in the same way with the long period variables. Gerasimovič places[192] them in the "intermediate group," arguing from their large and abrupt changes of period that their mean density does not govern the variation so intimately as that of a Cepheid. Wilson[193] found statistically that the M stars without emission lines are rather fainter than long period variables at maximum, but they are undoubtedly giants. Hoffmeister's suggestion that they are absolutely faint,[194] on the basis of their galactic distribution, is probably a result of the incomplete knowledge

[190] Cf. Section 58.
[191] P. N. A. S., **14**, 491, 1928.
[192] H. C. 342, 1929.
[193] A. J., **35**, 35, 1923.
[194] Himmelswelt, **40**, 37, 1930.

of irregular M stars, many of which will on further study be found to be periodic or semiregular.

Data on individual variables are meager, but Mrazek[195] finds for R Lyrae, a typical "intermediate" variable quoted by Gerasimovič, a spectrum, a form of light variation, and a relation between light- and velocity-variation that simulate the Cepheid very closely. The Cepheids and R Lyrae may be in a similar stage where variability is easily acquired, though the variability when it arises is not necessarily similarly governed in all cases. As far as physical conditions are concerned, the "intermediate variable" lies in the same district as the long period and RV Tauri variable. Further spectroscopic data are much to be desired.

b. The Physical Condition of Variable Stars.—The relationships of the variable stars are apt to be obscured if we attempt to analyze them in spectroscopic terms. The spectrum may be an index of internal conditions, but from Table XIV, I, we may be sure that it does not govern them—spectral class and type of variability are not uniquely connected. Evidently the variability is a deep-seated affair, and if connected with any one thing it is related to internal conditions such as temperature and density. Therefore in Figure XIV, 9 we examine the relation of central density and central temperature for variable stars.

The observed facts used for compiling the figure are essentially the period-luminosity law and the period-spectrum relation. The former gives (subject to a temperature correction supplied by the latter) the absolute magnitude and mass of the variable; the latter the surface temperature and (with the mass) the mean density. The central density and central temperature (p_c and T_c) are given by the two expressions[196]

$$\rho_c = 12.98\rho_m;\ T_c = 0.856\frac{G}{\mathrm{R}}\frac{\mu\beta M}{R}$$

[195] Mrazek, A. N., **236**, 281, 1929.

[196] Eddington, The Internal Constitution of the Stars, 136, 1926.

It seems that the conditions for variable stars lie upon a fairly smooth curve, which is expressed very roughly by

$$\rho_c = T_c^4$$

The smoothness of the curve is not less interesting than the blank parts of the diagram, filled with normal non-variable

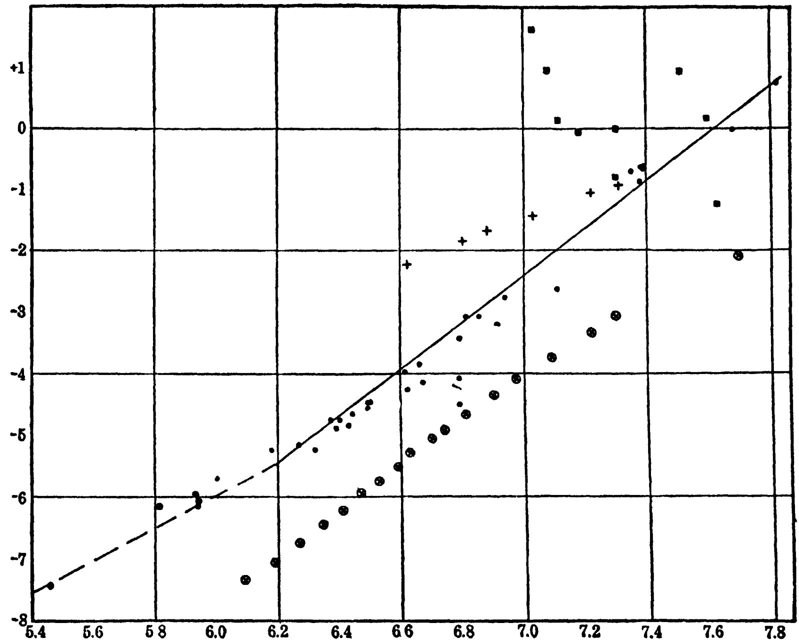

FIGURE XIV, 9.

Relation between logarithm of central density (abscissa) and logarithm of central temperature (ordinate). Dots represent variable stars; circled crosses, giant stars from Messier 22; crosses, mean values for the Pleiades; squares, individual stars, from data given by Eddington. References: Messier 22, Shapley, H. B. 874; Pleiades, Hertzsprung, George Darwin lecture, 1929; individual stars, Eddington, The Internal Constitution of the Stars. The diagram is explained in the text.

stars—for instance the section where the normal B and O stars fall. It is well known that there are hardly any variable B stars (the companion of Mira falls near our curve, however) and that the O stars are remarkable for steadiness of light. In the same way the white dwarfs are far from the curve con-

necting the variable stars. For any central temperature there seems to be a particular mean density appropriate to variability. At some point the propitious concatenation of central density and central temperature occurs, and while that condition prevails the star can (but not necessarily must) be a variable of the principal sequence type. One type of variable does not develop into another.

Another aspect of the idea is seen if the spectrum-luminosity curve (derived from the period-luminosity curve and the period-spectrum relation) is inserted in the Russell diagram. We are impelled to the idea that the variable does not move down the spectrum-luminosity curve, but across it. We need only point to the essential similarity of the absolute magnitudes of the brightest stars in all the spectral classes to feel the unlikelihood of a development parallel to the spectrum-luminosity curve.

For the development of the individual Cepheid, RV Tauri, and long period variables there have been guesses based on observed changes of period.[197] But many changes of period have turned out to be fluctuating and not secular; and of the apparently secular ones there is no evident tendency either to shortening or to lengthening.[198] On the basis of changes of period little can be asserted as to the development of variable stars; moreover the necessary changes of mean brightness in the stars with large period change do not seem to occur, as they would if the changes of period indicated real changes of mean density. The phenomena are probably of a more superficial nature.[199]

c. Summary of Variable Stars.—The foregoing chapter treats the variable star empirically, laying special stress on the Cepheid. It appears that the spectroscopic features of the

[197] Gerasimovič, H. C. 321, 1927; 323, 333, 1928.

[198] See Sections 69*a*(6) and 70*c*.

[199] The form of Figure XIV, 9, depends on the equations used; but the implied difference between variable and normal stars resides in the observables—brightness, color, and spectrum, not in equations. A similar observation was made by Shapley about the colors and magnitudes of galactic Cepheids and giant stars in clusters (Mt. W. Contr. 154, p. 7, 1918).

Cepheid point to high luminosity; furthermore the observed changes in light, spectrum, and temperature, so far as known, are not incompatible with a pulsation theory; but no theory hitherto proposed covers the facts adequately and in detail.

The Cepheid furnishes the most detailed case of the general problem of intrinsic variability, and after summarizing similar data for other classes of variable very briefly I have assembled the data empirically in order to examine what common feature of all these stars could be associated with their variability. We might expect that the relation, if it depends on anything we can evaluate, would depend on the conditions of pressure and temperature near the stellar center.

There are three fundamental conditions for periodic stellar variability—those that govern the source, the regulation, and the maintenance of the variation. The three must obviously be closely linked, but variables differ so much in surface conditions, size, mean density, brightness, and mass that we may be sure that exactly the same conditions do not govern all three.

IV

ANALYSIS OF STELLAR ATMOSPHERES

CHAPTER XV

AT THE SURFACE OF THE NORMAL STAR

DATA on line intensities and contours have been incorporated in the chapters dealing with the various spectral classes. The material may be used collectively in a variety of ways: to examine the properties of matter at high temperature; to determine the physical state of the atmospheres of the stars; and to deduce (empirically, in the present state of knowledge) the brightness, mass, density, stability, and history of the stars themselves—each use depending on approaching the data with some particular assumption or generalization. The first two of these aspects of the data of stellar spectroscopy are the subject of the present chapter; the third may be traced throughout the preceding nine chapters.

The predictions of ionization theory, and their test by comparison with observation, may be divided into three main sections:

a. General verification of the predictions by the qualitative data of the spectral sequence.

b. Quantitative test of ionization theory on the basis of the atomic abundance in atmospheres in different spectral classes.

c. Application of the generalized Saha equations to the analysis of stellar atmospheres.

Each of these sections involves two steps: (1) test of the theory by the condition that it consistently satisfy the available data and (2) the use of the theory thus accredited in building a physical picture of the atmospheres of individual stars, since the general model suggested is acceptable. The history of the analysis of stellar atmospheres from the physical standpoint warns us to be wary in making the transition from (1) to (2), and it is hoped that the assumptions and steps involved, and the uncertainties implied, are clearly brought out in what follows.

79. General Data of the Spectral Sequence.—The well-known progression of line intensity along the spectral sequence has been satisfactorily interpreted in its main outlines by the Saha theory,[1] especially as expanded and expounded by Fowler and Milne.[2] Presentation of the astrophysical data, on a qualitative scale, was made by Miss Maury,[3] Miss Cannon,[4] Rufus,[5] Menzel,[6] and the writer.[7] In summary, these early and inexact methods permitted the conclusions that thermal ionization could account for the observed spectral sequence, at least in its main features and for normal stars: and, granting this, that the atmospheres of all stars are both qualitatively and quantitatively similar. Pressures of the order of 10^{-4} atmospheres were suggested, and in the main substantiated, by these researches, and their recognition was perhaps the most important of the immediate results of the work.

Though the qualitative data of the stellar sequence were of a low order of accuracy, they sufficed not only to substantiate the Saha theory in its main features but also to point to several unsolved problems. Absolute magnitude effects presented several anomalies—the intensities of the metallic lines in the spectra of c-stars of Class F were too great for the crude theory, as could be seen by comparing them with the lines for normal stars, even before their intensities had been measured. The lines of hydrogen presented another anomaly, being strengthened in bright cool stars and greatly weakened in bright hot stars. Particularly striking was the observed strengthening of the Sr^{+} lines in bright second-type stars, whatever the temperature, whereas the early forms of the theory led to the expectation that they should be weakened in the spectra of stars hotter than the maximum at K2.[8]

[1] Proc. Roy. Soc., **99A**, 136, 1921; Milne, Obs., **44**, 264, 1921.

[2] M. N. R. A. S., **83**, 403, 1923; **84**, 499, 1924.

[3] H. A., **28**, 58, 1897.

[4] See, among others, the Preface to the Henry Draper Catalogue, H. A., **91**, 1915.

[5] Pub. Obs. Mich., **3**, 258, 1923.

[6] H. C. 258, 1924.

[7] H. C. 252, 256, 1924; H. Mon. No. 1, 1925.

[8] H. Mon. No. 1, 148, 1925.

The theory of Fowler and Milne in its original form did not satisfy the observational data just mentioned, but Milne's generalization of the Saha equations meets the problem of absolute magnitude effects in showing that on certain assumptions the strengthening of the metallic and hydrogen lines in the spectra of bright stars can be predicted. The observations therefore constitute a strong case for the appropriateness of the assumptions, provided it can be shown that the same assumptions do not also lead to consequences that are at variance with observation. The qualitative data, with the exception of the absolute magnitude effect for hydrogen in the hotter stars, may in fact be said to harmonize, with slight modifications, with the theory now available.

Besides absolute magnitude effects we may note one or two observations that are as yet theoretically untouched, confining ourselves to the relatively tractable problems involving absorption lines: the silicon stars; the strontium stars; the division at the cooler end of the sequence; and the mysterious cases of unusual abundance of a substance, such as that of ionized manganese in the spectrum of α Andromedae.[9]

The original treatment of thermal ionization by Fowler and Milne is proved acceptable by representing qualitatively the main data of the spectral sequence, and many (though not all) of the particulars in which it failed are remedied in Milne's later generalization. The most important feature of the physical picture involved in these treatments is the low pressure in the atmosphere. The observations for which the theory has no place appear to be special cases—probably involving stars in some way abnormal.[10]

The predictions of the Fowler-Milne theory in its original form are compared with qualitative observation in an earlier publication;[11] the general correspondence was excellent, considering the crudeness of the estimates and the preliminary nature of the theory.

[9] Lockyer and Baxandall, Proc. Roy. Soc., 77A, 550, 1906.

[10] Cf. Miss Williams, H. C. 348, 1929, and Chapter XIII.

[11] H. Mon. No. 1, 1925.

80. Quantitative Data and the Test of Theory.—In testing the predictions of ionization theory, the quantitative measures that are the subject of the present chapter can perform three offices: those of a further test of the ionization equations; a measure of the inadequacies noted in the last section; and a discriminant for the modifications and refinements of

TABLE XV, I.—CONTOURS OF HYDROGEN LINES IN THE SPECTRAL SEQUENCE

Class	Observer	No. of Spectra	Hγ r = 0.96	0.83	0.69	dl	Hδ 0.96	0.83	0.69	dl	Hε 0.96	0.83	0.69	dl
O8.5	P	3	4.3	1.8	1.0	29	4.6	2.3	1.2	31	4 5	2.2	1.4	31
B2	P	1	14.0	6.0	....	..	10.2	2.7	0.6	.	16.2	6.7	3.2	..
B3	P	2	11.9	6.6	4.4	..	11.2	5.2	2.3	..	13.2	6.0	3.3	..
B5	P	5	18.1	8.2	4.5	49	16.0	7.6	4.2	56	18.2	9.4	5.7	57
	W	5	17.7	9.1	5.4	56	13.7	7.8	4.9	60	....	....	....	..
	Mean	10	17.9	8.6	5.0	52	14.8	7.7	4.6	58	....	....	....	..
B8	W	9	19.9	11.2	6.9	59	16.9	9.7	6.4	63	....	....	....	..
B9	P	2	21.0	12.0	7.9	71	21.0	12.2	8.0	74	22.8	14.2	10.3	75
	W	1	22.3	13.9	8.6	66	20.1	11.5	7.5	68	....	....	....	..
	Mean	3	20.8	12.6	8.1	69	20.7	12.0	7.8	72	....	....	....	..
A0	W	22	29.4	16.8	11.2	71	24.5	14.5	9.7	73	....	....	....	..
	D	11	27 0	15.1	9 1	..	25.5	12 9	9.4	..	22.4	13.3	9.4	..
	Mean	33	28 6	16.2	10.5	71	24.8	14.6	9.6	73	...	....	....	.
A2	W	12	31.7	18.5	12.0	70	26.9	16.2	10.1	68	....	....	....	..
	D	2	28 2	14.4	9.2	..	26.8	13.6	9.2	.	24.4	14.1	9.2	
	Mean	14	31.2	17.9	11.7	70	26.9	15.8	10.0	68	....	....	....	.
A3	W	3	34 0	20.0	12.3	70	28.4	16.7	10.2	72	....	....	....	..
A5	P	4	33.7	14.4	8.4	55	34.4	15.3	8.8	62	33.4	17.6	11.3	75
	W	7	33.9	19.1	11.2	67	28 9	15.5	9.3	70	....	...	....	..
	D	1	30.6	14.4	8.2	..	33.0	14.0	7.8	..	32.0	15.1	8 2	
	Mean	12	33.6	17.1	10.0	63	31.1	15.3	9.0	67	33.1	17.1	10.7	75
F0	P	4	17.0	8.8	5.4	57	16.7	9.5	6.2	74	17.9	10.8	7.6	76
	W	8	23.5	13.1	7.3	64	19.0	10.8	6.6	65	....	....	....	.
	Mean	12	21.3	11.7	6.7	62	18.2	10.4	6.5	68	....	....	....	..
F2	P	1	13.8	6.5	3.5	59	13.9	7.3	4.2	57	28.0	16.6	7.8	88
	W	1	18.1	8.3	4.2	53	12.5	7.3	3.1	56	....	....	....	..
	Mean	2	16.0	7.4	3.8	56	13.2	7.3	3.6	56	....	....	....	..
F5	P	2	11.6	4.6	2.2	45	13.0	6.0	2.5	48	28.0	16.7	10.1	81
F8	P	1	12.8	7.8	4.5	42	11.0	6.4	4.2	45	20.4	11.6	8.4	79
G5	P	1	4.5	2.5	2.0	33	4.4	2.4	1.3	32	....	....	....	78
K0	P	1	4.3	2.5	1.5	29	....	....	....	32	28.2	12 9	9.8	81
K2	P	3	....	....	....	..	....	....	....	..	27.7	15.6	10 5	..
	H	2	....	....	....	..	..	....	....	..	25.8	12.5	9 2	.
	Mean	5	....	....	....	.	...	....	....	..	26.9	14 4	10.0	..
K5	P	1	4.5	3.2	2.5	31	5 5	3.5	2.6	31	...	.....	.	.
dK5	P	1	2.5	0.8	....	..	4.8	2.6	1.3	..	....	....	...	.

theory required to meet the problem. The first step is to make a condensed presentation of the measures.

Table XV, I contains data on the contours of hydrogen lines throughout the spectral sequence. Results from three investigators are given separately, and then combined into weighted means, weights being proportional to numbers of

TABLE XV, II.—CONTOURS OF CALCIUM LINES IN THE SPECTRAL SEQUENCE

Class	No. of Stars	H					K					4227		
		r = 0.96	0.83	0.69	0.58	0.48	0.96	0.83	0.69	0.58	0.48	0.96	0.83	0 69
Ao	1	....	.....	.	.	..	2.5	0 8	0.5	....	..	....	....	...
A3	1	.. .	.		...		6.7	4.0	2.5	....	...	....	...	...
A5	5	....		.	. .	..	10.4	6.2	4.1	2.3	1.8	...	..	.
A7	1	. ..	. ..	..		...	9.3	5.6	4.4	3.8	3.2	....	.	. .
A8	1	....	....	....	. .	..	11.5	7.0	5.3	4.2	3 1	.. .	....	...
Fo	4	17.9	10 8	7.6	6.1	4.9	11.5	7.0	5.7	4.5	3.4	2 9	....	...
F2	1	28.0	16.6	7.8	5 8	4 8	17.5	8.6	6.8	6.3	5.3	....	....	...
F5	2	28.0	16.7	10.1	7.4	6.3	25.0	17.2	10.9	7.9	5.8	3.9	2.6	0.6
F8	2	12.6	8.4	6.8	5.4	...	16.4	10.7	7.2	5.7	4.6	5.5	2.1	1 0
G5	4	15.0	10.6	7.9	6.2	...	21.2	13.6	9.6	7.4	...	3.9	2.5	1.5
G5	3	29 5	15.4	10.1	7.3	...	23.1	15 0	10.5	8.0	...	....	....	...
Ko	5	28 2	12.9	9.8	7.8	6 2	23 8	16.4	12.2	10.0	8.3	6.7	4.4	2.9
K2	3	27.7	15.6	10.5	8.2	6.5	29 2	20.5	14.8	11.3	8.8	7.0	....	...
	2*	28.8	12.5	9.2	7.4	5.8	28.0	16.1	11.8	9.4	7.1	8.4	4.5	2.8
	Mean	26 9	14.4	10.0	7.9	6 2	28.7	18 7	13.6	10.5	8.1	7.7	4.5	2.8
K5	1*	....	....	....	...	...	19 4	14 2	9.7	6.4	4.2	14.0	7.6	4.1
Mo	6*	19.0	10 2	6.3	4.8	3 6	22 2	15.8	10.9	8.2	6.2	15 5	10.3	6.5
M1	1*	20.5	9.5	4.5	3.3	2 5	27.2	18.0	10.0	6.8	4 8	16.0	9.8	5 0
M2	1*	17.8	7 5	5.0	3.8	2.8	22.3	15.6	10.6	7.9	6.1	17.4	9.6	4.7
M3	2*	16 1	7 9	5.2	3.6	2 5	22 2	10 8	7 0	5.0	4 0	16.4	9.8	6 4
M5	2*	11.9	6.6	4.1	3.2	2.4	23 2	15.8	10.2	7.9	5.1	....	12.2	9.0

* Measured by Hogg, H. B. 859, 1928.

spectra. Table XV, II gives similar data for the H and K lines, and for 4227. The investigators who measured the contours are designated as follows: D = Dunham;[12] H = Hogg;[13] W = Williams;[14] and P = Payne. The tabulated values of *dl* refer to the same spectra as the contour measures; hence they differ slightly from those of Table XV, V.

[12] H. B. 858, 1927.
[13] H. B. 859, 1928.
[14] H. C. 348, 1929.

Table XV, III contains the mean values of log NH, derived, according to the method of Chapter III, from the contours of Table XV, I and XV, II. They are expressed as *numbers of effective atoms*, and cannot be taken without correction to

TABLE XV, III.—NUMBERS OF EFFECTIVE ATOMS IN STELLAR ATMOSPHERES FROM MEASURED CONTOURS

Spectral Class	Mean log NH				
	$H\gamma$	$H\delta$	$H + H\epsilon$	K	4227
o8 5	17.56	17.72	17.86	.	.....
B2	18.62	17.94	18.59	.	. ..
B3	18.73	18.30	18.55	. ..	.....
B5	18.90	18.82	18.98		...
B8	19.14	19.04	.....	.... .	....
B9	19.24	19.21	19.38	.....	.....
Ao	19.45	19.37	19.32	17.00:	.....
A2	19.54	19.42	19.34	.....	. ..
A3	19.62	19.46	.....	18.24	....
A5	19.46	19.37	19.48	18.67	.....
A7	.. .	. .	..	18.66	.....
A8	... .	... .	.....	18.82	.. ..
Fo	19.16	19.08	19.16	18.84	17.28
F2	18.72	18.68	19.34	19.01	.....
F5	18.23	18.40	19.45	19.49	17 56
F8	18.81	18.70	19.22	19.13	17.61
Go	17.92	17.75	19.29	19.38	17.82
Ko	17.81	.....	19.32	19.52	18.36
K2	.....	.....	19.38	19.64	18.37
K5	18.14:	18.19:	.....	19.36	18.76
Mo	. ...	. ..	19.06	19.46	19.08
M1	...	..	18.90	19.48	18.96
M2	.. .	... .	18.84	19.44	18.92
M3	.....	...	18.88	19.12	19.08
M5	.....	. .	18.71	19.43	19 28

represent relative numbers of atoms. Table XV, IV contains separately the contours, and the corresponding mean values of log NH, for the few dwarf stars that were accessible.

Table XV, V contains the data derived from the measurement of line depth on uniform dispersion, and in Table XV, VI

TABLE XV, IV.—CONTOURS FOR SELECTED DWARF STARS

Class	Star	Line	r = 0.96	0.83	0.69	0.52	0.48	Mean log NH
dF5	α CMi	Hε	17.2	11 4	8.5	6 8	5.6	19.17
		K	14.4	10.6	7.8	6.5	5.7	19.06
		4227	3 9	2.6	0 6	...	.	17 53
dF8	α For	Hε	13.0	7.3	6.3	5 0	3 9	18.89
		K	20 5	11 7	7.2	5 6	4 5	19.12
		4227	4.5	2 1	.		..	17.67
dGo	α Cen A	Hε	16 0	10.0	7.2	5.4	4.0	18.94
		K	15 2	9.9	7 2	5.7	4 5	18 93
dK5	α Cen B	Hγ	2 5	0.8	...	...	..	17.07
		Hδ	4 8	2 6	1.3	..	...	17.74
		4227	7.2	3 8	2.9	2.2	1.7	18.26

TABLE XV, V.—MEAN PERCENTAGE LIGHT LOSSES FOR LINES THROUGHOUT THE SPECTRAL SEQUENCE

Class	No. of Stars	Hβ	Hγ	Hδ	Hε	K	4026	4227	4215	4077	4046	4326	G
O	4	18	29	31	31	..	..	..	..	..	..	..	..
B5	6	41	49	56	57	..	17	..	..	.	..	..	..
B8	4	64	59	60	63	7	15	..	..	..	..	..	..
B9	4	60	71	74	75	11	..	..	..	..	..	..	..
Ao	7	54	63	67	70	19	..	..	..	..	..	..	..
A2	5	50	68	72	80	38	..	..	..	..	..	..	..
A5	7	54	55	62	75	51	..	..	..	..	.	..	..
Fo	15	49	57	74	76	58	..	22	..	..	..	..	..
F2	1	53	59	57	88	87	..	..	..	.	..	..	.
F5	15	41	45	48	81	78	..	24	12	22	22	16	26
F8	4	36	42	45	79	76		34	10	(27)	28	33	44
Go	10	31	33	34	74	75	..	27	13	22	23	21	34
G5	27	24	33	32	78	73	.	39	29	31	31	31	50
Ko	69	23	29	32	81	78	..	51	37	34	39	36	57
K2	10	16	28	34	82	82	..	51	(56)	36	42	35	50
K5	11	19	31	31	78	73	..	70	37	37	48	41	58
Mo	5	12	.	..	83	80	..	72	40	37	47	44	57
M1	2	.	.	.	73	77	..	61	..	..	..	..	..
M2	1		..	..	86	80	..	67	..	..	..	..	.
M3	2	.	..	.	75	78	..	62	..	.	..	..	..
M5	1		..	.	74	77	..	70	..	..	..	.	..

TABLE XV, VI.—LOGARITHMS OF NUMBERS OF EFFECTIVE ATOMS, DEDUCED FROM LINE DEPTHS

Class	Hβ	Hγ	Hδ	Hε	K	4227	4215	4077	4046	4326	G
F0	18.18	18.42	18.94	19.00	18.45	17.09	.....	.....	.....	.....	.....
F2	18 30	18 50	18.42	19.37	19 35	.....	.....	.....	.....	.....	.....
F5	17.93	18.06	18.15	19.12	19.07	17 20	17.07	17.41	17.41	17.18	(17.30)
F8	17.78	17.94	18.06	19.10	19.00	17.69	17.00	17.51	17.54	17.69	(18.02)
G0	17.63	17.69	17.72	18.95	18.97	17.51	17.09	17.41	17.38	17.33	(17.72)
G5	17.21	17.69	17.66	19.07	18 92	17.87	17.36	17.33	17.33	17.33	(18 21)
K0	17.38	17.56	17.66	19.16	19.07	18.24	17.81	17.72	17.87	17.78	(18 42)
K2	17 18	17.54	17.72	19.34	19.37	18.40	17.84	17.77	17.96	17.74	(18.21)
K5	17 27	17.62	17.62	19.10	18.91	18.48	17.81	17.81	18.15	17.93	(18.45)
M	17.06	.....	.....	.....	.....	18.76	17.90	17.81	18.12	18.02	18.42

TABLE XV, VII.—INTENSITIES OF LINES FROM TWO-PRISM PLATES

Atom	A5 β Pav	F0 η Sco	F2 ε Sco	F5 δ Vel	G0 A. G. C. 10215	G0 μ Vel.	dG0 α Cen A	K0 ε Sco	dK5 α Cen B
Fe (I. P. 1.54)	14	14	14	36	41	68	53	60	46
Fe (I. P. 2.84)	6	18	16	21	39	35	11	34	19
Fe+	10	..	17	43	41	41	16	40	18
Ca 4227	11	24	..	56	62	73	42	78	59
Hγ	64	59	..	.	60	66	30	59	23
Ti+	6	14	..	39	39	34	..	30	..

TABLE XV, VIII.—APPROXIMATE LOG *NH* FOR GIANTS

Atom	A5 β Pav	F0 η Sco	F5 δ Vel	G0 A. G. C. 10215	G5 μ Vel	K0 ε Sco
Mean Fe	16.34	16.51	17.09	17.23	17.58	17.52
Mean Fe+	16.36	....	17.23	17.18	17.18	17.15
Mean Ti+	16.26	....	17.12	17.12	16.95	16.98
Ca	16.39	....	....	16.56	16.69	16.80

TABLE XV, IX.—APPROXIMATE LOG *NH* FOR DWARFS

Atom	dF5 α CMi	dG0 α Cen A	dK5 α Cen B
Mean Fe	17.24	17.00	17.12
Mean Fe+	17.06	16.51	16.58
Mean Ti+	17.09	.....	16.39
Hγ	18.12	16.88	16.69
4227	17.21	17.20	17.64

the entries are converted into log *NH*, using the method and table of Section 16.[15]

The total absorptions of a few strong lines, measured as described in Section 14,[16] are given in Table III, II; they are

TABLE XV, X.—IONIZATION OF IRON IN GIANTS AND DWARFS

Class	Log Fe/Fe+	
	Giants	Dwarfs
A5	−0 02	. ..
F5	−0.14	+0 18
G0	+0.05	+0 49
G5	+0.40	.....
K0	+0 37	. ...
K5	.....	+0.64

shown in Figure XV, 1, together with the data on mean contours.

A few stars photographed with large dispersion were measured in greater detail, and the results are given in Table XV, VII

TABLE XV, XI.—NUMBERS OF IRON ATOMS, GIANTS AND DWARFS

Class	Log *NH*, Fe and Fe+	
	Giants	Dwarfs
A5	16.64	.. .
F5	17 47	17 46
G0	17.51	17.12
G5	17.73	.....
K0	17.67	.....
K5	.....	17.25

to XV, XII; Table XV, VII gives the mean measured percentage light losses for a series of lines (six of iron, three of ionized iron, and three of ionized titanium). In Table XV, VIII and XV, IX the measures are converted into log *NH*, whose values

[15] P. 40.

[16] P. 33.

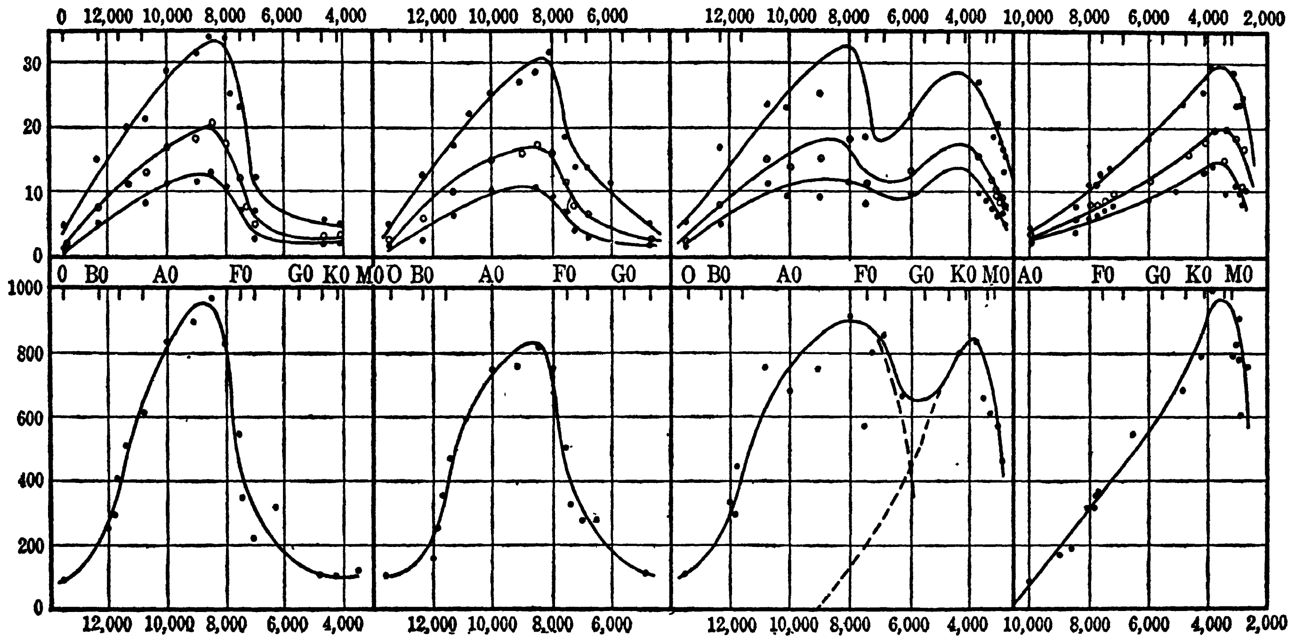

FIGURE XV, 1.

Above: mean contours for $H\gamma$, $H\delta$, $H + H\epsilon$, and K. Ordinates are half breadths in Angstroms; abscissae are spectral classes on an approximate temperature scale. The three sets of points for each line represent the half breadth at $r = 0.96$, 0.83, and 0.69, respectively. The points are connected by smooth empirical curves. Below: total absorptions deduced from the data in the upper half of the figure.

are affected by a somewhat uncertain correction depending on the dispersion (evaluated empirically from the few available data); they are probably *relatively* correct. The next two tables give the degree of ionization of iron (expressed by log Fe − log Fe+) and the relative numbers of iron atoms (sum of neutral and ionized) in the atmospheres of giants and dwarfs.

81. Test of the Generalized Saha Equations.—The theory of the stellar atmosphere has seemingly been placed on a lasting basis by Milne's comprehensive generalization of the ionization equations. Using the main features of the original theory, he shows that the crucial point lies in the dependence of the general absorption coefficient on the partial electron pressure. I have discussed the relevant observational data, and their bearing on the theory, in another place.[17]

The observed line maxima, which form one basis of the test, are summarized in Table XV, XII.

TABLE XV, XII.—OBSERVED MAXIMA

Atom	I. P.	E. P.	Maximum	Temperature	Evidence
				°	
H	13.54	10 15	A0	10000	Contour
			F5*	6500	Contour
He	24.41	20.81	B1.5	12500	Estimate
Mg+	14.97	8.83	A3	9000	Estimate
Ca+	11.82	0.0	K2	3800	Contour
			G5*	5000	Contour
Ti+	13 6	1.16	F5	7000	Line depth
Fe+	16.5	2.82	F5	7000	Line depth
Sr+	10.98	0.0	K2	3800	Line depth
Ba+	9.96	0.0	M0?	3000	Line depth

* Supergiants.

The observed absolute magnitude effects constitute another observational test. They are summarized below, together with the predicted absolute magnitude effects on the two assumptions: (1) $\kappa = \bar{\kappa}$; (2) $\kappa \propto P_e$.

[17] H. B. 867, 1929.

TABLE XV, XIII.—OBSERVED AND PREDICTED ABSOLUTE MAGNITUDE EFFECTS

Line	Maximum	Observed Effect			Predicted Effect			
					$\kappa = \bar{\kappa}$		$\kappa \propto P_e$	
		Above	Below	At*	Above	Below	Above	Below
H	Ao	−	+	−	−	+	o	+
He	B1.5	+	+	+	−	+	o	+
O+	B3	+	+	+	−	+	o	+
Ca+	K2	o	+	+	−	+	o	+
Sr+	K2	+(o?)	+	+	−	+	o	+
Ti+, Fe+	F5	+	+?	+	−	+	o	+
N+	B5	+	+	+	−	+	o	+
C+	B5	+	+	+	−	+	o	+

* Referring to temperature of maximum.

The fitting of the theory to the observations will inevitably be very tedious, and certainly cannot be attempted at present. When a quantitative test of the two basic assumptions is made by comparing the observed and predicted maxima, it appears that neither assumption satisfies the observations in detail, but the variable absorption coefficient is somewhat better than the constant one. Professor Milne points out to me that it is better to predict the maxima from plausible values of κ and α rather than to deduce κ and α from observed maxima as a test of the theory. I believe that in either case, however, the discrepancy between theory and observation is definite.[17a]

The test by means of the observed absolute magnitude effects is no less complicated. Evidently the variable coefficient of absorption is the more satisfactory; but there is an almost universal residual strengthening with absolute magnitude far above maximum for which it does not provide, and the behavior of hydrogen in the hotter stars is also beyond it.[18]

[17a] I used the former line of attack in Harvard Bulletin 867. The latter, which I have worked out in some detail, leads to no more explicit results, and I do not reproduce the material; the statement in the text sufficiently represents the conclusions.

[18] It may be suggested that if the Stark effect were eliminated from the line contours within Class B, the weakening of hydrogen with absolute magnitude

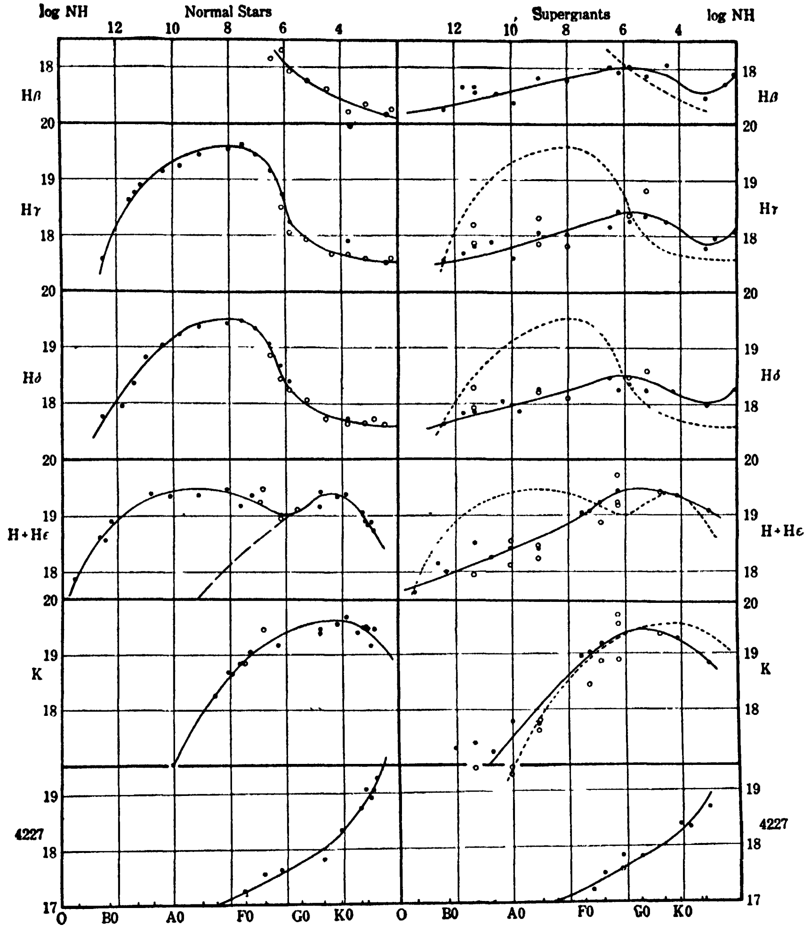

FIGURE XV, 2.

Logarithms of numbers of effective atoms (ordinates) in their relation to spectral class (approximate temperatures indicated above, in thousands of degrees). The left column refers to normal stars. The supergiants (data from Chapter XVI) are represented on the right, with broken lines indicating the curve for normal stars. The broken line in the "normal" figure for H + Hε represents the probable course of H.

would disappear; but the weakening involves also the total absorption of the lines. The influence of Stark effect on total absorption has at present been theoretically discussed only for a much simplified case (Miss Anger H. C. 352, 1930).

A third, more quantitative test, is available. The relative numbers of atoms in giant and dwarf spectra are available for a few spectral classes (Table XV, XI, and Figure XV, 3). It seems that from Go to Ko there are between two and three times as many iron atoms in the atmospheres of giants as of dwarfs. The prediction of theory[19] for such lines is that

$$N_0 \propto g^{-1/6}$$

The difference in surface gravity at Go would lead us to expect from this relation that there should be about twice as many

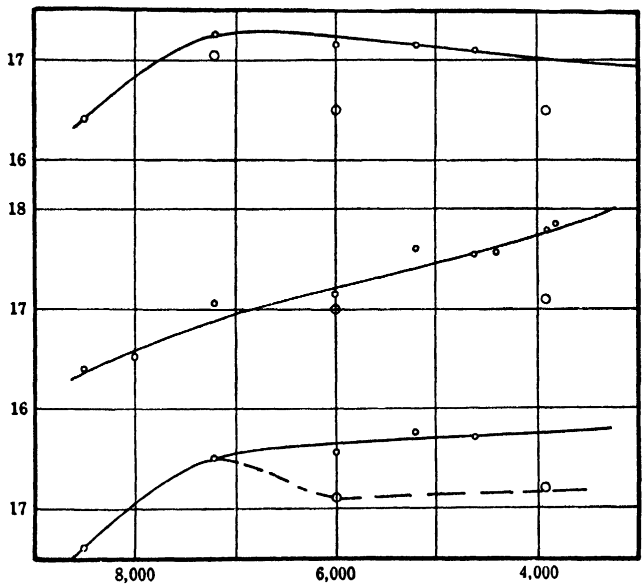

FIGURE XV, 3.

Number of ionized iron atoms (above) and neutral iron atoms, and their sum (lowest figure) for spectra of giants (small circles) and dwarfs (large circles). The continuous and broken lines in the lowest figure indicate the relative amounts of iron atmosphere for giant and dwarf.

atoms in the giant as in the dwarf atmosphere; and the corresponding number at Ko is just over two; the agreement is satisfactory.

[19] Milne, M. N .R. A. S ., 89, 167, 1928.

The data for the hydrogen lines in the cooler stars shows that the ratios of numbers of atoms from giant to dwarf in Classes F5, G0, and K5 are about 1, 6, and 9. On the basis of Milne's prediction[20]

$$N_0 \propto g^{-4/6}$$

we should expect ratios of about 2, 5, and 12; again the numbers are of the same order. But we note that his prediction for hydrogen near maximum[21]

$$N_0 \propto g^{-0.185}$$

is not satisfactory; the sign is contrary to observation.

Another quantitative test is very instructive; it is contained in Figure XV, 4, which shows the observed mean values of log NH for $H\gamma$ and $H\delta$, and the curve representing the predicted number of effective hydrogen atoms, adjusted for a pressure of electrons of 10^{-5} atmospheres (thus making the maxima coincide). The curves coincide from the maximum at A0 to about F5; higher than the maximum the predicted number of atoms is greater than is observed; at low temperatures it is much less.

The temperature scale adopted is that derived from measures of the continuous spectrum, summarized in Chapters VI, VIII, and X. Reasons have been shown for thinking the temperatures of the early B and O stars actually higher.[22] If the observed points fell on the predicted curve in Figure XV, 4 the temperatures would be as follows:

	°
O	20000
B0	16000
B2	15500
B5	14000
B8	13000
B9	12000

[20] M. N. R. A. S., **89**, 166, 1928.
[21] M. N. R. A. S., **89**, 167, 1928.
[22] See p. 113.

Such temperatures are not unreasonable; there is a possibility that the observed and theoretical curves can be reconciled for the early classes.

For the classes from G5 onward the problem is less simple. Here there are far more observed atoms than predicted—over

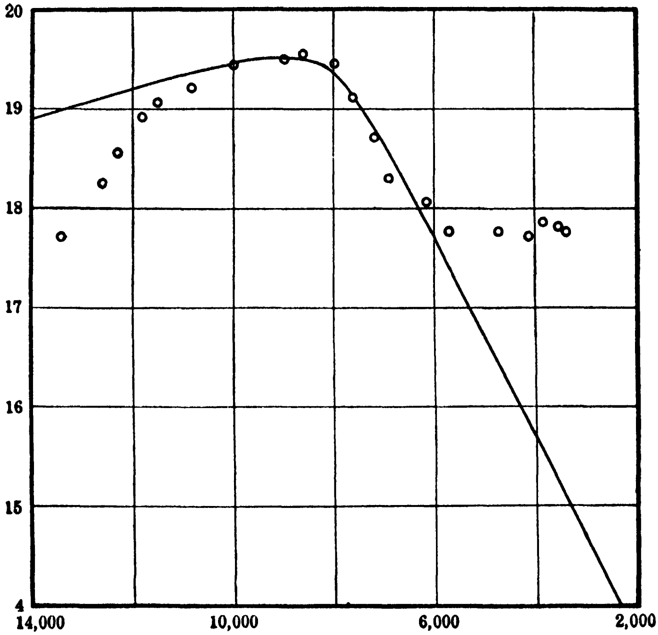

FIGURE XV, 4.

Numbers of effective hydrogen atoms (mean of $H\gamma$ and $H\delta$) in the spectral sequence. The line represents the number predicted on the basis of a fit at the maximum by the Boltzmann factor. It is seen that no possible adjustment of temperatures could bring the observed points onto the curve.

a thousand times more at Class M—and this time no reasonable adjustment of temperatures can bring theory into harmony with observation. This phenomenon is the one pointed out by Adams and Russell[23] in their analysis of the spectra of bright stars of late type, and attributed by them to a deviation from thermodynamic equilibrium. The deviation from the curve

[23] Mt. W. Contr. 359, 1928.

given by the simple theory sets in at Class Go, and is of course even more pronounced for the M supergiant than for the normal stars illustrated in Figure XV, 4. The various attempts to explain the deviation made by Adams and Russell, Eddington,[24] Gerasimovič,[25] Minnaert,[26] again by Russell, [27]and by Unsöld[28] need not be discussed at length; we merely note that a successful explanation must take account of the whole phenomenon of the strength of the hydrogen lines in the cool stars; their great intensity in the supergiants is in truth less surprising than the fact that they appear at all in Classes M and K. The predicted numbers for these classes are far below the threshold of visibility for one-prism Harvard plates, but the presence of the lines is in no doubt at all.

In this connection we note that Babcock has observed Paschen δ in the solar spectrum to be about as strong and wide as Hα. Disregarding a possible Stark effect (which should be large for this line), we should expect from the Boltzmann factor that the Paschen lines would be very weak. The deviation from thermodynamic equilibrium is again evinced.

To summarize the present status of the observational test of theory:

The observed maxima of the spectral sequence are in general accordant with the form of Milne's generalization of the Saha theory that regards the coefficient of general absorption to vary as the partial electron pressure. Far greater refinement of observation is necessary before the test can be made with any precision.

In the main the observed absolute magnitude effects are more consonant with a variable coefficient of absorption than with a constant one. The effects for hydrogen are anomalous, and difficult to interpret on either view.

[24] Letter quoted by Russell, Mt. W. Contr. 359, 1928.
[25] M. N. R. A. S., **89**, 272, 1929.
[26] Obs., **51**, 347, 1928.
[27] Mt. W. Contr. 383, 1929.
[28] Mt. W. Contr. 379, 1929.

The relative amounts of iron atmosphere for giant and dwarf stars are in good general agreement with the numerical predictions of the theory.

The predicted numbers of hydrogen atoms for the cooler stars are in glaring disagreement with observation. Giant M stars have thousands more observed than predicted atoms; supergiants, tens of thousands. This point is in urgent need of theoretical elucidation.

We conclude that the evidence seems at present to point to a variable absorption coefficient rather than a constant one, but that more refined observations are essential.[29]

82. The Central Intensities of Lines.—There is no doubt that the observed wide and strong absorption lines are shallower than would be expected from the theory now current for the stellar reversing layer, and also that they differ considerably from star to star.

By comparing the central intensities of the H and K lines given in Table XV,V for the normal star and in XV for the supergiant, it is clearly seen that the most luminous stars have the blackest lines. Unsöld[30] has discussed the theory of line depth and, interpreting central intensities in terms of collisions, predicts that: (1) bright stars should have blacker lines than faint ones; (2) central intensities should be the same for all components of the same multiplet. The first condition is fulfilled for the H and K lines; on the second he presents a few measures that are in agreement with the prediction. The case seems to be clear observationally that no metallic lines are black to

[29] Professor Rosseland points out to me that a variable absorption coefficient does not necessarily imply one that varies as the first power of the pressure. Theoretically any power of the pressure is possible, and the power may vary, for instance, with temperature or ionization. The phenomena of the B stars might be better reconciled with some power of the pressure between 0 and 1; this would permit of a small absolute magnitude effect instead of Milne's "null effect," and almost all the observed intensities could be satisfied by a suitably changing power of the pressure.

[30] Festschrift für Sommerfeld, 95, 1928.

the center, and the components of multiplets tend not to differ greatly in depth, but more measures are to be desired.

A very different type of problem confronts us in the hydrogen lines, whose mean central intensity (expressed in percentage light loss) is given in Table XV, XIV for a number of spectral classes. The depth seems greatest at about Hζ, falling off on either side of it. I have discussed the depth of the Balmer lines elsewhere; for the members of lowest frequency it is connected with a tendency to central emission.

TABLE XV, XIV.—RELATIVE DEPTHS OF THE BALMER LINES

Class	Hβ	Hγ	Hδ	Hϵ	Hζ
B5	41	49	56	57	59
B8	63	59	60	(64)	53
B9	60	71	74	(75)	67
A0	59	66	71	(75)	72
A2	48	78	84	(86)	71
A5	54	55	62	(75)	71
F0	..	49	57	(74)	71
F5	41	45	48	(81)	56

Emission lines of course take many forms;[31] some at least of the hydrogen lines in early stars have a broad absorption line with a central emission, divided centrally by a narrow central reversal. Possibly all central emission in hydrogen lines is of this basic form, perhaps smeared by the same cause that widens and flattens all early hydrogen lines. The same type of line—a broad absorption, with central emission and superimposed central absorption—is found in the H and K lines of the sun and, according to recent work,[32] of several giant stars of late type. The emission and central absorption in the solar H and K lines are not very strong; Unsöld[33] finds that "the maxima of the H_2 and K_2 lines rise only about 6 per cent above the brightness of the surrounding minima . . . The intensity in the middle

[31] Cf. Scheiner and Graff, Astrophysik 354, 1922.

[32] Adams and Joy, P. A. S. P., 41, 311, 1929.

[33] Mt. W. Contr. 378, 8, 1929.

of the H_3 and K_3 lines came out about 23 per cent smaller than the intensity of the H_2 and K_2 maxima . . . " Such effects, blurred by finite resolving power,[34] would of course shallow a line somewhat, and as they are not considered in our theories of line contour it is rather idle to discuss the central intensities of lines at present. The observations are at present considered empirically.

It seems best to leave the tables and diagrams of the present chapter without more discussion. We may feel confident that they give the right order of the relative numbers of atoms of any one substance along the spectral sequence. But that they do not give actual numbers of atoms is certain, and it is wise to guard ourselves from overdiscussion. For instance, they do not assist us to amplify the conclusion on the quantitative composition of stellar atmospheres, made four years ago on the basis of estimates of marginal appearance.[35] At that time we had to assume that all lines at marginal appearance were given by the same number of atoms; now we must assume that lines of the same contour are given by the same number of atoms. The two assumptions are strictly equivalent, and the data now available are of no more value in this connection than the original estimates. An advance in the quantitative study of the atomic population of the universe must rest at least in part on a study of multiplet intensities, and their relation to the probability of atomic states under stellar conditions. Measures of stellar absorption lines must also be made with individual refinement, rather than roughly for statistical purposes, before a detailed discussion is justified.

[34] The Mount Wilson observations show bright reversals for H and K in α Bootis, α Tauri, α Scorpii, α Orionis, and α Herculis; none of these stars shows any reversals on Harvard objective prism plates.

[35] H. Mon. No. 1, 177, 1925.

CHAPTER XVI

THE ATMOSPHERES OF VERY LUMINOUS STARS

Quantitative data on the atmospheres of supergiants are scattered throughout the preceding chapters and collected in this one. The discussion of the data is to some extent disappointing, for current theory has not as yet interpreted many of the main outlines of the results, and we are therefore discouraged from an attempt to apply it in interpreting the details.

83. The Spectral Sequence for Supergiants.—In Chapters IV and V the high luminosity stars were discussed in general terms; we seem to be justified in regarding them as running parallel to the giant stars, on the average about two magnitudes brighter, and pervading to a comparable extent

Table XVI, I.—Mean Percentage Light Losses for Supergiants

Class	Hβ	Hγ	Hδ	Hε	Hζ	K	4026	4227	4215	4077	4046	4326	G
O	18	29	31	31	..	(18)	..	..	..	..	..	..	..
B0	32	33	38	48	47	(18)	16	..	..	..	..	..	..
B5	28	37	39	44	45	21	18	..	..	..	..	..	..
B8	28	40	45	60	45	24	..	..	..	..	..	..	..
B9	23	30	39	52	45	19	..	..	..	..	..	..	..
A0	37	46	52	57	54	28	..	..	..	..	..	..	..
A2	36	44	48	57	41	36	9	..	..	..	..	..	..
F0	44	48	59	78	..	76:	0	..	..	..	..	..	..
F2	41	58	52	78	57	78	0	17	..	..	..	..	..
F5	44	52	54	84	56	83	0	28	24	32	20	..	33
F8	38	56	51	90	..	89	0	38	27	44	29	26	38
G0	46	52	51	78	..	76	0	38	33	52	32	34	52
K0	26	36	43	88	..	86	0	57	45	46	52	36	62
K2	34	43	..	..	..	..	0	56	46	..	..	40	60
M1	40	46	52	..	..	..	0	67	50	50	45	34	56
M2	..	..	..	78	..	71	0	..	..	..	..	..	..

TABLE XVI, II.—MEAN NUMBERS OF EFFECTIVE ATOMS, SUPERGIANTS, FROM PERCENTAGE LIGHT LOSSES

Class	Hβ	Hγ	Hδ	Hε	Hζ	K	4026	4227	4215	4077	4046	4326
O	17.24	17.57	17.63	17.63	.....	(17.24)	....	.....	.....	.....	.....	.....
Bo	17.66	17 69	17 84	18 15	18.12	(17.24)	17 18	.. ..	.....	.....	.....	.....
B5	17 54	17 81	17 87	18 02	18.05	17.33	17.24	..	.. ..	.....	.....	.....
B8	17.54	17.90	18 05	18.52	18 05	17.42	... ..	. ..	. .	.. ..	.....	.....
B9	17.39	17 60	17.87	18.27	18.05	17.27	....	... .	.. ..	.. ..	.....	.....
Ao	17.81	18 08	18.27	18.42	18.33	18.76	....	....	.....	.....	.....	.....
A2	17.78	18 02	18.15	18.42	17.93	17.78	16 97	.....	.....	.....	.....	.....
Fo	18 02	18.15	18.48	19.06	.....	19.00	<16 5	. ..	.....	.....	.....	.....
F2	17.93	18 45	18.27	19.06	18.42	19.06	....	17 21	.....	.....	.....	.....
F5	18 02	18.27	18 33	19.24	18.39	19.21	....	17.54	17.42	17.66	17.30	.....
F8	17.84	18.39	18 24	19.42	.....	19 39	....	17.84	17 51	18.02	17.57	17.48
Go	18.08	18 27	18.24	19.06	.....	19.00	....	17.84	17.69	18.27	17.66	17 72
Ko	17.48	17.78	17.99	19 36	.....	19.30	. . ..	18.42	18.05	18.08	18 28	17 78
K2	17.72	17.99	.....	. ...	.....	. ...	.. ....	18.30	18 08	.....	.....	17 90
M1	17.90	18.08	18.27	.....	.....	..... .	...	18.73	18 21	18.21	18.05	17.72
M2	.....	.....	.....	19.07	.....	18.86	. . ..	.....	.....	.....	.....	.....

TABLE XVI, III.—DATA ON CONTOURS FOR SUPERGIANT STARS

Class or Star	Hγ			Hδ			Hε			K		
	0.96	r = 0.83	0.69	0.96	0.83	0.69	0.96	0.83	0.69	0.96	0.83	0.69
cB5(6)	10.5	3.8	1.1	10.7	5.5	1.0	14.1	5.8	3.0	....	....	....
η CMa												
cB8	....	...	...	....	...	...	8.0	3.0	1.9	2.4	0.7	....
β Ori												
cAo												
σ Cyg	6.4	2.4	0.3	11.3	3.8	1.0	13.0	6.0	3.4	2.2	0.7	....
a Vel	6.3	4.0	2.9	....	...	...	4.8	3.4	2.4	2.0	0.6	....
cA2												
α Cyg	....	...	...	....	...	...	7.0	3 8	2.6	4 4	2.0	0 8
A. G. C. 93737	6.0	2.2	0.2	8.2	3.4	1.0	10.8	6.0	2.8	5 8	2.0	1 0
cF2												
π Sgr	....	...	..	...	...	...	11.5	5 8	3 4	7.2	4.6	3.6
cF5 (5)	7.8	4.6	2.9	8 5	5.1	3.4	17.0	8 0	5.1	13.6	7.9	5.5
cF8												
δ CMa	....	...	...	...	...	...	13 5	10.4	8 5	22.0	16.5	12 6
ρ Pup	12 8	7.8	4.5	11.0	6.4	4.2	20 4	11.6	8 4	20.4	7.8	6.1
x Car	....	...	...	....	...	..	33 8	21.7	14.0	...	22 8	20.0
cG5												
ζ Cap	....	...	...	....	...	...	32.3	14.6	10.5	26.8	17.2	12 0

all spectral classes from B5 to M. It is therefore admissible to combine the data for supergiant stars into a spectral sequence parallel to that discussed in the last chapter, although the supergiant sequence is undoubtedly less homogeneous, and the scatter within a definite spectral class is larger; moreover there are fewer available stars, and mean results are therefore of less weight.

The mean percentage light losses for a number of lines in the spectra of the c-stars of various spectral classes are contained in Table XVI, I. The corresponding numbers of effective atoms are shown in Table XVI, II; they are probably somewhat too great.

TABLE XVI, IV.—MEAN VALUES OF LOG *NH* FROM CONTOURS

Class or Star	Hγ	Hδ	Hε	K	4227
cB5 (6)	18.21	18.30	18.40	.....	.....
η CMa	17.85	17.96	.....	.....	.....
cB8					
β Ori	.....	.....	17.99	17.0:	.....
cA0					
σ Cyg	17.86	17.74	18.56	17.0:	.....
a Vel	18.32	.....	18.14	16.9:	.....
cA2					
α Cyg	.....	.....	18.24	17.67	.....
A. G. C. 93737	17.80	18.13	18.47	17.81	.....
cF2	.....	.....	18.55	18.48	.....
π Sgr					
cF5 (5)	18.38	18.49	18.87	18.90	.....
cF8					
δ CMa	.....	.....	19.18	19.55	17.60
ρ Pup	18.82	18.60	19.22	18.92	.....
x Car	.....	.....	19.70	19.71	.....
cG5					
ζ Cap	.....	.....	19.41	19.38	.....
M7e	.....	.....	.....	.....	19.25

The next two tables summarize the data on the contours of the lines of supergiants along the sequence and the resulting values of *NH*; they corroborate the data of Table XVI, II.

The data of Tables XVI, II and XVI, IV are embodied in Figure XV, 2, which also indicates the intensities of the same lines for normal stars.

The striking feature of the spectral sequence for supergiants is the occurrence of the hydrogen, helium, and (possibly) calcium maxima at temperatures different from those for the normal spectral sequence. The maxima for these and other lines are described in Table XVI, V.

TABLE XVI, V.—COMPARISON OF THE NORMAL AND SUPERGIANT SEQUENCES

Line	Remarks
Hydrogen	Normal maximum at A0:, supergiant maximum at F5
Helium	Normal maximum at B 1.5, supergiant maximum at B5
Calcium	Deeper in supergiant spectra; no maximum
Ionized calcium	Deeper in supergiant spectra; normal maximum at K2, supergiant maximum at G5?
Iron	Deeper in supergiants; maximum absent or ill defined
Ionized strontium	Deeper in supergiants; supergiant maximum or ill defined

The maximum for hydrogen and at a lower temperature for the supergiant points to lower pressure, and the effect must be even greater than appears at first, since the temperatures of supergiants are probably lower, in all classes, than those of giant stars of similar spectral class.[1] From the displacement of the hydrogen maximum we infer that the ratio in P_e, giant/supergiant, is about $10^{2.5}$. Incidentally the ratio in surface gravity at this point is also about $10^{2.5}$, so that the partial electron pressure seems for A and F stars to be simply proportional to the gravity at the surface.

The curves relating spectral class to number of hydrogen and helium atoms for supergiants are exactly similar to those relating the same quantities for normal stars, but they are displaced in the direction of lower temperatures and lower pressures (the right and downward in Figure XV, 2), in qualitative but not quantitative agreement with the early Fowler-Milne theory.[2] But the lines of iron, ionized iron, calcium,

[1] See, for instance, the data in Table VIII, I (p. 103), Table XII, I (p. 165), and Table XIV, IX (p. 209).

[2] M. N. R. A. S., **83**, 403, 1923; **84**, 499, 1924.

ionized calcium, ionized strontium are always stronger in the supergiant than the corresponding lines in the normal star, even though most of the observations are made on the hot side of their maximum. The reconcilement of these observations with theory is briefly mentioned in the previous chapter.

84. The Composition of Supergiant Atmospheres.—The tables contained in the earlier section of this chapter summarize the measured number of atoms corresponding to a *mean line* given by various atoms observed in the spectrum, and they enable us to form an estimate of the total extent (in terms of numbers of atoms) of the atmospheres of supergiant stars, compared with normal stars. Leaving hydrogen out of account, and assuming that the same percentage of calcium enters the composition of the atmospheres of both types of stars, we obtain for the relative numbers of atoms the numbers contained in Chapters VI to XIII and summarized in Table XVI, VIII.

An alternative method of reaching the same conclusion is provided by the depth measures of the calcium lines, though because these lines tend to be deepest for a given width in supergiant spectra these measures lead to rather an underestimate of the ratio. The relative depths of the H and K lines for groups of stars are summarized in the next table.

TABLE XVI, VI.—RELATIVE PERCENTAGE LIGHT LOSSES FOR CALCIUM LINES

Class	Normal		Supergiant	
	H	K	H	K
F5	81	78	85	85
F8	79	76	89	89
Ko	81	78	88	86

The corresponding relative numbers of atoms, derived from Table III, II, are given in the next table.

TABLE XVI, VII.—CALCIUM ATMOSPHERES FOR GIANTS AND SUPERGIANTS

Class	Normal		Supergiant		Difference	
	H	K	H	K	H	K
F5	19.15	19.06	19.27	19.27	0.12	0.21
F8	19.09	19.00	19.39	19.39	0.30	0.39
K0	19.15	19.06	19.36	19.30	0.21	0 24
Mean	.....	...	.....	.....	0 21	0.28

The relative amounts of calcium in the atmospheres of supergiant and normal stars is therefore in a ratio not much greater than two to one. If we express our results in terms of the number in the atmosphere of the sun we obtain something of the order of ten to one. Comparisons for iron and other metals are not accurately obtainable from the present material, but the order of the relative number of atoms is probably the same.

It is of interest to compare the numbers of atoms derived by the writer with those tabulated by Adams and Russell.[3] The numbers given by them and the corresponding measures from our data are compiled in the next table.

TABLE XVI, VIII.—RELATIVE NUMBERS OF ATOMS, DETERMINED BY ADAMS AND RUSSELL AND THE WRITER

Class	Iron		Ionized Iron		Calcium	
	A and R	P	A and R	P	A and R	P
cM1	2.11	17.89	......	....	2.04	18 73
gK0	1.39	17.82	1.25	18 24	0.81	17 90
cF8	0.27	17.52	.... .	.	0.33	17 84
cF5	−0.81	17.30	2 24	17 24	−0 91	17.50
dF5	−0.92	17.29	−0.53	17.12:	−1.05	17 20
A0	−2.74	<16.5	−1.45	17.00	−2.76	17.00

It is evident that the logarithms of the numbers of atoms derived from the two sources are linearly related, but the scale adopted by Adams and Russell is somewhat more extended. The

[3] Mt. W. Contr. 359, 1928.

difference probably springs from the methods used by them in calibrating line intensities.

The inherent probabilities seem to be in favor of the less extended scale. The scale of Adams and Russell, for instance, gives a ratio of 10^6 between Classes Ao and cM1 for iron, titanium, calcium, manganese, chromium, and vanadium. If the Unsöld formula[3a] gives a relation of the right order between number of atoms and form of line, a less number of atoms than 10^{16} will not give a visible line with the Harvard one-prism dispersion.[4] If the ratio for an atom—say of iron—between Ao and M1 is 10^5, and if it had the value of $NH = 10^{16}$ at marginal appearance (*i.e.*, when just visible), there would be 10^{21} atoms of iron in the atmospheres of stars of the latter type (for each line observed), a far larger quantity than is associated with any known line, by the accepted Unsöld formula, and implying an enormous amount of iron altogether. The conclusion is fairly definite, for the breadths of lines increase very rapidly when the number of atoms is as great as this, and the largest number of atoms ever reported (Ca+ in certain K stars) is $10^{19.8}$; lines corresponding to larger numbers of atoms on the same scale are probably never found. Therefore, unless the Unsöld formula gives results of the wrong order, the relative numbers of atoms derived by Adams and Russell are difficult to reconcile with the observed strengths of spectral lines.

The predictions of the theory of Fowler and Milne incidentally lead us to expect smaller ratios than 10^6 between Ao and M except for hydrogen, but this is of small weight in establishing the plausibility of one or the other scale, since, if the Adams-Russell scale is accepted, it is necessary to assume large deviations from thermodynamical equilibrium, and the Fowler-Milne theory then becomes inapplicable.

[3a] See p. 29.

[4] The line would have a half breadth of about an Angstrom at the extreme wing (1 per cent light loss); It is probable that faint lines do not follow the theoretical contour, being widened and flattened, which would raise the limiting number for detectability yet more.

85. Hydrogen in Stellar Atmospheres.—A comparison of atmospheric content for hydrogen is also of interest. Adams and Russell give a table of the equivalent Rowland intensities of the hydrogen lines which may readily be translated into relative numbers of atoms. The first line of Table XVI, IX gives the equivalent Rowland intensity; the next, the corresponding logarithm of the number of effective atoms; then, the computed fractional concentration ($P_e = 10^{-4}$ atm); the difference, giving the total relative number of hydrogen atoms; the values of log NH, referring to *total hydrogen*, from my measures, for supergiants of the same class; and the difference between Adams and Russell's results and mine.

TABLE XVI, IX.—DATA FROM ADAMS AND RUSSELL FOR $H\gamma$

	α Ori	α Sco	α Boo	γ Cyg	α Per	α CMi
Intensity	25	25	20	25	25	30
Log N	3.91	3.91	3.76	3.91	3.91	4.02
n_r (cf. tables)	−15.00	−15.00	−13.00	− 9.20	− 7 70	− 8.40
Difference	18.91	18.91	16.76	13.11	11.61	12 42
Log NH (total)	33.33	33.33	30 60	27 65	26.15	26 48
(A, R-P)	14 42	14.42	13.84	14.54	14.54	14.06

The constant difference (last line of Table XVI, IX) shows that the numerical results agree closely, and since the hydrogen lines have a range of about 10, and the values of n_r a range of about 10^7, the agreement is striking and satisfactory. The conclusion of Adams and Russell is fully substantiated: "There appears . . . to be much more hydrogen, in the excited states, in the atmospheres of the red giants than in the sun's atmosphere. With the smallest plausible allowance for the effect of the excitation potential . . . the abundance of normal hydrogen must be thousands of times greater in Antares or Betelgeuse than in the sun."

Their next comment is worthy of further analysis: "The great intensity of the hydrogen lines in the cooler giant stars,

especially in the c-stars, and its conspicuous dependence on absolute magnitude have long been a puzzle. Hydrogen, being light, is particularily subject to radiation pressure; and the greater strength of the latter, in comparison with gravity, in stars of great absolute brightness, may afford the explanation. As this, however, would lead us to anticipate stronger hydrogen lines in γ Cygni and α Persei than are observed, other factors may also be at work." The interpretation of the behavior of hydrogen in the supergiant sequence contained in the earlier part of the present chapter accounts for the relative weakness of the hydrogen lines of the F (and A) stars and leaves without serious drawbacks an interpretation based on radiation pressure. It is borne out by the increase of hydrogen content from G5 onward, as well as by the sharp drop in ionization pressure from about the same place.[5] Several lines of argument suggest that radiation pressure plays a larger part in the arrangement of stellar atmospheres than has hitherto been credited to it.

The hydrogen content of the atmosphere of the cool giant is the subject of Section 81. We must be prepared, in accepting the enormous amount of hydrogen in the atmospheres of red stars, to contemplate large deviations from thermodynamic equilibrium, so that quantitative conclusions can no longer be drawn from the generalized Saha equations. The dilemma in which we are left discourages us from further discussion of the data. Their first office must be to lay down the requirements for a satisfactory theoretical treatment; the present state of theory will not serve.

The chief positive results derived in the present chapter are:

Difference of spectral class of maximum between giant and supergiant for lines of hydrogen, helium, ionized calcium.

Consequent ratio of about $10^{2.5}$ in P_e (of the same order as the ratio in surface gravity) for giant/supergiant in Class A and F.

About twice the amount of calcium atmosphere for second-type supergiants as for normal stars.

Enormous hydrogen content of the supergiant atmosphere.

[5] Payne and Hogg, H. C. 334, 1928.

Probability that there are deviations from thermodynamic equilibrium in the atmospheres of red stars.

At best these data can be used to direct the growth of theory; at worst they can be applied empirically. They have called attention to several serious questions—not the least of these being the problem of hydrogen in stellar atmospheres.

APPENDIX A

Catalogue of c-Stars

H. D.	Star	Mag.	Spectrum	Remarks
38771	κ Ori	2.20	Bo	Lines well defined, somewhat narrow
47129	+6 1309	6.06	Bop	Hβ appears slightly bright. Lines narrow, 4200 strong. Plaskett's massive star (Journ. R. A. S. C., **16**, 284, 1922). See Chapter VI
52382		6.36	Bo	Lines appear narrow
68450	L 3183	6.31	Bo	Lines appear narrow
91651		9.1	Bo	Lines narrow
93030	θ Car	3.03	Bo	Lines somewhat narrow; 4686 as strong as in class Oe5; 4649 very faint, less strong than 4642. Radial velocity variable (L. O. B., **8**, 125, 1915), range 70 kilometers
145846		8.7	Bo	Lines very narrow; Hβ not seen distinctly, suspected bright
157973		8.0	Bo	Lines appear narrow
163522		8 6	Bop	Lines narrow
167264	15 Sgr	5.42	Bo	Lines somewhat narrow. Radial velocity variable (Adams, Joy, and Sanford, P. A. S. P., **36**, 137, 1924), range 68 kilometers
186994		8.1	Bo	Lines narrow
213087		5.66	Bo	Lines rather narrow; I place in Class O. Temperature seems low

Catalogue of c-Stars.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
				from spectrum. K line not strong
224151		6 05	Bo	Lines may be narrow; K line strong for class. 4686 bright; Pickering lines not seen. N++ stronger than in ε Orionis; helium weak
13841	+56° 470	7.21	B1	Lines probably narrow
13854	Pi 22	6.42	B1p	Lines narrow, and spectrum as θ Arae. K line strong, He seems weak
14956		7.32	B1	Lines probably narrow
52089	ε CMa	1.63	B1	Lines narrow
95707		7.4	B1p	Lines narrow
108002		7.3	B1p	Lines narrow and spectrum as θ Arae
111123	β Cru	1.50	B1	Lines somewhat narrow; several appear slightly bright on the edge of greater wave length. Radial velocity variable, range 19 kilometers (L. O. B., 5, 177, 1910).
152236	ζ^1 Sco	4 88	B1p	Lines narrow; Hβ and Hγ bright
154090	κ Sco	4.87	B1p	Lines somewhat narrow; as ζ^1 Scorp, but hydrogen lines not bright
165024	θ Ara	3.90	B1p	Lines narrow
13866	+56° 475	7 7	B2p	Lines narrow
14818	10 Per	6.24	B2	Lines probably narrow; K strong
41117	χ^2 Ori	4.71	B2p	Lines narrow. Variable radial velocity (Merrill, L. O. B., 6, 144, 1911), range 14 kilometers
113606		9.0	B2	Lines probably narrow; K line strong
151985	μ_2 Sco	3.64	B2	Lines narrow

Catalogue of c-Stars.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
187879		5 62	B2	Lines rather narrow. K line sharp, equals K in γ Ori, as does 4026. Oxygen lines barely seen
206165	9 Cep	4 87	B2p	Lines very narrow; Hβ may be bright to the red. Color equivalent 2.05 (Hertzsprung, Leiden Annals, 14); temperature low, from spectrum. Oxygen, carbon, magnesium strong; hydrogen and helium weak
4841		7 06	B3	Lines narrow; K line strong for class
13267	5 Per	6.36	B3p	Lines narrow
56211		7.4	B3p	Lines narrow
72787		6.38	B3	Helium lines very narrow, hydrogen moderately broad
92964		5.44	B3p	Lines narrow
166418		8 7	B3	Lines probably narrow
197637		6 78	B3	Lines somewhat narrow
7902		7.9	B5	Lines narrow; Hβ suspected bright
12301	53 Cas	5.62	B5p	Lines very narrow; K line strong for class; appears sharper than the others. H is bright to red; 4922 has bright edges. Sensibly cooler than 52 Cas (A2)
13590		8.0	B5p	K line strong as in A2. Lines narrow
14010		7.05	B5p	Lines narrow; K line strong as in A2
31894		8.4	B5	Lines probably narrow
51480		6.97	B5p	Lines narrow; K line strong as in A2. Hβ bright, bright lines and

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
				spaces, not due to hydrogen, also seen
53138	o^2 CMa	3 12	B5p	Lines very narrow. Ionized oxygen unusually strong. Emission edges to red of Hβ, 4922, 5015. Radial velocity variable, Mitchell (Ap. J., **30**, 242, 1909), apparent period 24.27 days. Cf. Curtis, L. O. B., **6**, 57, 1910
58350	η CMa	2 43	B5p	Lines narrow. Spectrum very like that of H. D. 53138
60098		6 46	B5p	Lines very narrow
61071		6 80	B5	Lines appear narrow
61851		10.2	B5	Lines appear narrow
72127	34 Vel	6.66	B5	Lines narrow. Radial velocity variable (L. O. B., **8**, 125, 1914)
74371		5.23	B5	Lines narrow, not previously announced
75149		5.54	B5p	Lines narrow, and the spectrum resembles those of o^2 and η Canis Majoris
75241		7.0	B5	Lines may be narrow. K line strong for class
92854		8.9	B5	Lines narrow
93911		8.9	B5	Lines narrow
125823	a Cen	4.55	B5	Lines may be narrow
164353	67 Oph	3.92	B5p	Lines narrow. Color equivalent 1.54
164384		8 5	B5	Perhaps lines narrow. K line strong
167409		10.1	B5	Lines probably narrow
169014		9.4	B5	Lines probably narrow
187311		10.3	B5	Lines probably narrow
191139		8.1	B5	Lines narrow
205637	ε Cap	4.72	B5p	Spectrum very peculiar. Hβ is bright and vari-

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
				able. Helium lines very wide, hydrogen lines narrow
216534		8.0	B5	Lines narrow
217675, 6	*o* And	3.63	B5	Spectrum composite; lines in spectrum of fainter component narrow. Color equivalent = 1.47
4976		.....	B	Lines narrow; Class B5?
7099		11.5	B	Lines faint, and appear narrow
7103		8.6	B	Hγ, Hδ very narrow
7583		10.07	B	Lines narrow
17145		8.0	B	Hydrogen lines very narrow, others faint
167791		9 7	B	Lines appear narrow
172488		7.9	Bp	Hydrogen lines narrow
6226		6 70	B8p	Lines narrow; K line strong as Class A2
12709		8 0	B8	Lines somewhat narrow
13890	+56° 478	8.9	B8	Lines appear narrow
14542	Br 328	6.95	B8p	Lines narrow
28868		9 4	B8	Hydrogen narrow, other lines very indistinct
31407		7.5	B8	Lines probably narrow. K line as strong as 4026.3
38402		7 7	B8	Lines somewhat narrow
40589		6.08	B8p	Lines very narrow
46210		9.1	B8p	Lines narrow
48549		9.0	B8	K line strong for class, perhaps lines narrow
50850		9.1	B8p	Lines narrow. Spectrum like β Orionis
64109		8.3	B8	Lines may be narrow
66713		7.7	B8	Lines somewhat narrow
77852		8.8	B8	Lines narrow
80558		5.87	B8p	Lines narrow; spectrum as β Orionis
92451		8.8	B8	Lines narrow
92704		8.6	B8	Lines appear narrow

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
92910		9 0	B8	Lines narrow
94054		8 4	B8	Lines narrow
94367		5 57	B8p	Lines narrow; spectrum as β Orionis
95880		7.08	B8	Lines appear narrow
102334		11.3	B8	K line strong; lines probably narrow
105071		6.48	B8	K strong as in Class Ao. Lines probably narrow
166937		4.01	B8p	Lines narrow; spectrum as β Orionis. Radial velocity variable. Ca+ strong; H and Si+ rather strong
168673		9 4	B8p	Lines narrow
168798		9 7	B8	Lines probably narrow
168833		9 9	B8	Lines appear narrow
171876		8.7	B8p	Lines narrow
174512		7 97	B8	Brightest star in N. G. C. 6705; lines appear narrow
174638	β Lyr	var.	B8p	Very peculiar
199478	Br 2720	5 76	B8p	Lines narrow, spectrum as β Orionis
212593	4 Lac	4 64	B8p	Lines narrow, spectrum as β Orionis. H, Ca+ and Si+ strong; He rather weak. Coolness evident from the spectrum.
14322	+55° 588	6 84	B9p	Lines narrow
16778		7 71	B9p	Lines very narrow
21291	Pi 51	4 42	B9p	Lines narrow. Large color equivalent (Hertzsprung, Leiden An., 14). K very strong; Si+, Mg+, strong. He weak. Temperature evidently low
35600	Pulk 844	5.72	B9	Lines appear narrow
50003		8.5	B9	Lines somewhat narrow
50320		8.3	B9	Lines somewhat narrow

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
68474		7.23	B9	Lines appear narrow
86441		7.4	B9	Lines narrow
86557		7.6	B9p	Lines narrow
93504		8.9	B9	Lines narrow
111904		5.84	B9p	Lines narrow, 4128, 4131 strong
112366		8 4	B2p	Lines very narrow
120473		8 9	B9	Lines narrow
131856		10.4	B9	Lines appear narrow
162089		9 5	B9	Lines narrow
165246		8.1	B9	Lines narrow; K line strong as in Class A3
167313		9 7	B9p	Lines very narrow
4725		10.9	A	Hγ and Hδ very narrow
44074		8.9	A	Hydrogen very narrow; perhaps Class B8
44351		8.5	A	Lines narrow; may be Class A2
48455		10.1	A	Lines appear somewhat narrow; may be Class B8
48663		8.4	Ap	Hydrogen very narrow. Perhaps Class B8
68025		8.2	A	Lines appear somewhat narrow
70376		9.9	A	Lines appear somewhat narrow
73617		9.6	A	Hydrogen lines appear narrow
75219		9.2	A	Lines narrow; class very uncertain
79646		10.5	A	Lines narrow
96040		10.0	A	Lines appear narrow
112406		9 0	A	Lines appear narrow
123447		9 1	A	Lines narrow; may be B8
130921		8 8	A	Hydrogen lines narrow
140225		9.8	A	Lines narrow; perhaps Class B8
185336		8.6	A	Lines appear narrow, perhaps Class B8

Catalogue of c-Stars.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
13476		6.50	Aop	Lines narrow; probably like η Leonis
13744	+57° 526	7.8	Aop	Lines narrow
14535		7.46	Ao	Lines may be narrow
14899		7.42	Ao	Lines somewhat narrow
15963		7.98	Aop	Lines somewhat narrow. Numerous peculiar lines as in α Cygni may be due to fainter component
18294		9.2	Ao	Lines somewhat narrow
20041	Groombr 627	5 92	Aop	Lines narrow; like η Leonis
21389	Pi 54	4.76	Aop	Lines narrow; like η Leonis. Radial velocity variable, range 10 kilometers, period nine days (Young, L. O. B., 6, 143, 1911). Large color equivalent (Hertzsprung, Leiden Ann., 14). He seen; Si+, Mg+ strong. Many fine lines
31205		8 6	Ao	Lines sharply defined; 4077, 4128, 4131 well marked
32185		9 2	Ao	Lines somewhat narrow
34452	Br 737	5.39	Aop	Lines narrow and sharply defined; 4128, 4131 strong
41069		7.57	Ao	Lines somewhat narrow
46300	Br 958	4.50	Aop	Lines very narrow and sharply defined; as η Leonis
48716		8 3	Aop	Lines narrow
51723		9 2	Ao	Lines narrow
55424		9 5	Ao	Lines somewhat narrow
63320		9 1	Ao	Lines narrow
75485		8.0	Ao	Lines rather narrow
87737	η Leo	3.58	Aop	Lines very narrow and sharply defined

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
91958		9.7	Ao	Lines narrow
92252		9.44	Ao	Lines narrow
93503		8.9	Ao	Lines narrow
93619		7.2	Ao	Lines narrow; 4077 strong
93844		8.4	Ao	Lines narrow; 4077 strong
93923		8.9	Ao	Lines narrow; 4077 strong
100826		6.2	Aop	Lines narrow; as η Leonis
117585		9.7	Ao	Lines appear narrow
129511		8.2	Ao	Lines appear narrow
133202		9.1	Ao	Lines narrow
137939		10.3	Ao	Lines appear narrow
142282		6.53	Ao	Lines probably narrow
144059		8.8	Ao	Lines probably narrow
151604		8.2	Aop	Hydrogen narrow; many well-marked metallic lines, and the G band, present. Probably composite
168388		9.1	Ao	Lines somewhat narrow
168814		7.26	Aop	Lines narrow
172324		8.0	Ap	Lines very narrow; helium suspected
184883		7.9	Ao	Lines somewhat narrow
202850	σ Cyg	4.28	Aop (B)	Lines narrow; as η Leonis. He faintly seen. Si+ very strong, Mg+ strong; bright edge to 4922
207673		6 49	Ao	Lines narrow; appears to resemble η Leonis
2885	β^2 Tuc	4 48	A2	Lines narrow, probably as in α Cygni. Spectroscopic binary, orbit
8065		6.10	A2	Lines narrow. H and K as in α Cygni; metallic lines much weaker; Si+ equal; no He

Catalogue of c-Stars.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
8159		8.2	A2	Lines narrow
12953	Br. 283	5.90	A2p	Lines narrow; spectrum as α Cygni. Very similar to H. D. 8065
13412		8.2	A2	Lines probably somewhat narrow
14433	Br. 323	6 54	A2p	Lines narrow
14489	Br 326	5.22	A2p	Lines narrow; spectrum closely resembles α Cygni
15316		7.30	A2p	Lines narrow; spectrum closely resembles α Cygni
20210		6 42	A2	Lines narrow; 4077, 4128, 4131 strong
23193		5 57	A2	Lines narrow, spectrum as α Cygni
24993		8.8	A2	Lines narrow; spectrum may be as α Cygni
31892		9.2	A2	Lines appear narrow
34018	Br 711	7.38	A2	Metallic lines strong; 4077, 4128, 4131 strong
39866		6.42	A2	Lines somewhat narrow
40062		6.48	A2	Metallic lines strong; and somewhat narrow
62623	l Puppis	4.10	A2p	Lines narrow; spectrum as α Cygni. Radial velocity variable (Olivier L. O. B., 6, 145, 1911), range 10 kilometers
65339	Br 1135	6.00	A2p	Hydrogen lines wider than in α Cygni; metallic lines appear narrow and as in α Cygni, 4077 4128, 4131 strong
91054		7.9	A2	Lines narrow
91533		6.19	A2p	Lines narrow; spectrum as α Cygni
92207		5.57	A2p	Lines narrow; spectrum as α Cygni

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
93737		6.12	A2p	Lines narrow; spectrum as α Cygni
100198		6.36	A2p	Lines narrow; spectrum as α Cygni
100262	o^2 Cen	5.26	A2p	Lines narrow; spectrum as α Cygni
102878	L 4908	5.65	A2p	Lines narrow; spectrum as α Cygni
103516		6.05	A2p	Lines narrow; spectrum as α Cygni
104035	L 4963	5.66	A2p	Lines narrow; spectrum as α Cygni
110500		6.94	A2	Lines somewhat narrow
111613		5 94	A2p	Lines very narrow; spectrum as α Cygni
111885	A. G. C. 19432	9.0	A2	Lines narrow
125835	L 5890	5.71	A2p	Lines narrow; spectrum as α Cygni
129772		8.4	A2	Lines appear narrow
130335		7.6	A2p	Lines narrow
135539		9.9	A2	Lines appear narrow
136357		10.3	A2	Lines narrow
148972		9.5	A2	Lines narrow
161912	ι^2 Sco	4 88	A2p	Lines narrow; spectrum as α Cygni
165784		6.58	A2p	Lines narrow; spectrum as α Cygni
172546	26 Sgr		A2	Not previously announced
197345	α Cyg	1.33	A2p	Lines very narrow. Radial velocity varies. Possibly associated with nebulosity
207260	ν Cep	4.46	A2p	Spectrum has narrow lines; very like α Cygni
210010		8.5	A2	Lines appear narrow
213470, 1		6.73	A2p	Lines very narrow; as α Cygni. A trace of G band interpreted as fainter star
216852		8.6	A2	Lines appear narrow
13929	$+57^{\circ}$ 533	8.0	A3	Lines probably narrow

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
17093	Br 386	5.16	A3	Lines appear narrow
27628	60 Tauri Br 589	5.76	A3	Metallic lines strong. Radial velocity variable (Young, Pub. Dom. Ap. Obs., 1, 163, 1920), range 46 kilometers
59612	97 G Pup	4 80	A3	Hydrogen lines somewhat wide. Solar lines narrow, as in α Cygni
84123		6.82	A3	Lines appear narrow
95390		10 0	A3	Lines narrow
125335		7.13	A3	Lines may be narrow. 4077, 4172, 4173, strong
160129		10.6	A3	Lines narrow
167656		10.1	A3	Lines appear narrow
167719		var.	A3	Lines appear narrow
170582		10.3	A3	Lines probably narrow
170603		8.86	A3	Lines appear narrow
197381		9 6	A3	Lines appear narrow
210221		4 6	A3p	Lines narrow and sharply defined
40536	2 Mon	5.10	A5	Lines somewhat narrow. Radial velocity variable (Barrett, Pop. Astr., 22, 234, 1914)
34578	Br 739	5.16	A5p	Metallic lines very sharp, resembles ε Aur
142846		8.07	A5	The H lines are narrow; the spectrum may be composite
148743		6.39	A5	Lines somewhat narrow; metallic lines strong
	RT Ser	var.	cA8	Radial velocity +44 to +125 (Adams, Joy, Sanford, P. A. S. P., 36, 139, 1924).
167689		9.5	A5	Lines appear to be narrow
188097			A5	Not previously published

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
218393		6.85	cA5	Lines very narrow; intensities as in α Cygni
571	22 And	5.08	Fo	Metallic lines narrow. Typical, many lines
1778		8.0	Fo	Lines narrow, probably as α Cygni
24550		7 65	Fo	Lines appear narrow; strong lines, somewhat as in δ CMa
33054	Br 711	5 47	Fop	Radial velocity varies, range 14 kilometers (Young, Pub. Dom. Ap. Obs., **1**, 287, 1921)
40535	Br 872	6.28	Fo	Lines somewhat narrow
45348	α Car	0.86	Fo	Lines narrow—but no c-character. Mount Wilson, $-3^M.0$
70761	298 G Pup	5.86	Fo	Lines somewhat narrow, intensities as in δ CMa. Mount Wilson, $-2^M.0$
70825		7.27	Fo	Lines somewhat narrow
75276		5 83	cFo	Lines narrow and sharp; resembles ϵ Aurigae
76227		9 8	Fo	Lines narrow
81471		6.14	cFo	Lines narrow
86023		9.5	F	Lines narrow; class uncertain
105702		5 74	Fo	Lines somewhat narrow; strong lines present with intensities as δ CMa
109241		6.78	Fo	Lines rather narrow; K line faint
110628		6.71	Fo	Lines somewhat narrow
116108		9 1	Fo	Lines appear to be narrow. Class not well defined
137052	ϵ Lib	5 08	Fo	Lines probably narrow. Radial velocity variable, range 28 kilometers (Campbell, Ap. J., **10**, 178, 1900)

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
143584		5.90	F0	Lines narrow
149748		7.22	F0	Lines probably narrow; 4172, 4177 strong
164136	ν Her	4.48	F0	Lines somewhat narrow. Mount Wilson, $-0^M.2$
194943	ρ Cap	5 1	cA9 (F0)	Mount Wilson, $-1^M.0$
201638		8 72	Fp	Lines narrow; slight contrast with the continuous spectrum. Only hydrogen and K seen
201700		8 1	F	Hydrogen lines narrow
214470		5 22	F0	Lines probably narrow
13824		8 60	F2	Lines narrow
43382		6.62	F2	Hydrogen lines narrow
75292	a Vel	8.8	F2	
82554	ι Cha	5.44	cF2	Lines narrow and sharp; as ε Aurigae
90089	Pi 22	5.34	cF2	Lines narrow; lines of peculiar intensity probably present
102942, 3		6.14	cF2	Brighter component has narrow lines
130818	RY Boo	var.	F2	Mount Wilson, $+0^M.5$. See Chapter XIV
178524	π Sgr	3 02	F2	Lines narrow. Mount Wilson, $-0^M.8$
181615	υ Sgr	4.58	cF2	Lines very narrow. See p. 146
187258		7 6	F2	Lines narrow
207826	Gr 3591	6.51	F2	Lines probably narrow
7927	φ Cas	5.25	F5p	Lines very narrow; as δ CMa. Radial velocity variable, range 10 kilometers (Adams, Joy, Sanford, P. A. S. P., **36**, 137, 1924); enhanced lines strong and displaced from arc lines. Typical cF5; resembles y Carinae
20902	α Per	1 90	F5	Lines somewhat narrow. Mount Wilson $-1^M.3$

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
24546	A Per	5.47	F5p	Lines narrow. Spectroscopic binary. Not a supergiant (see Gerasimovič and Payne, H. B., 866, 1929)
	SV Per*	var.	cF5	Radial velocity varies from −28 to −12 (Adams, Joy, Sanford, P.A.S.P., 36, 139, 1924)
31964	ε Aur	var.	cF5	Lines very narrow. See p. 170
	δ Lep		cF4	Mount Wilson, $-0^M.7$
37350	β Dor*	var.	cF5	Lines narrow; spectrum as δ CMa. See Chapter XIV
50058		7.66	F5	Lines appear narrow; several narrow lines of well-marked intensity
57623	δ Vol	4.02	cF5	Lines somewhat narrow; intensities of some as in δ CMa
61715		5 65	cF5	Lines narrow; as in δ CMa
67523	ξ Pup	2.9	F5	Mount Wilson, $-2^M.1$
74180	b Vel	4.06	cF5	Lines very narrow; as ε Aur
83808, 9	o Leo	3.8	cF5	Spectroscopic binary. Mount Wilson, $-1^M.4$
90772		4.94	cF5	Lines narrow; several very intense
97534	y Car	4 73	cF5	Lines very narrow; as ε Aur
. . .	SW Dra*	10.0	cF4	Mount Wilson, $+0^M.6$
161471	ι_1 Sco	3.14	cF5	Lines very narrow; enhanced lines strong; spectrum as ε Aur
163506	Br 2249	5 48	cF5	Lines somewhat narrow
174694	κ Pav*	var.	cF5	See Chapter XIV. Lines narrow
195295	41 Cyg	4.1	cF5	Lines narrow. Mount Wilson, $-1^M.1$. Typical cF5

CATALOGUE OF C-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
196524	β Del	3.72	F5	Lines somewhat narrow; hotter than ϵ Aur
8890	α U Mi*	2.12	F8	Cepheid variable; $-3^M.0$, Mount Wilson. See Chapter XIV
	RX Aur*	var.	cF9	See Chapter XIV
	RS Ori*	var.	cF8	See Chapter XIV
	W Gem*	var.	cF8	See Chapter XIV
54605	δ CMa	1.98	F8p	The lines are very narrow. Radial velocity variable, Wright, L. O. B., **177**, 1910; range 3 kilometers, period ¾ year. Lunt, Ap. J., **48**, 271, 1918, queries binary character. Mount Wilson, $-2^M.9$
65228	j Pup	4.4	F8	Cepheid spectrum, Adams and Joy
68808	3213	5 68	F8p	Lines narrow and sharply defined; intensities as in δ CMa
96918	x Car	4.02	F8p	Lines very narrow; spectrum as δ CMa. Radial velocity varies, range 14 km. (Wright, L. O. B., **4**, 161, 1907)
97082	ER Car*	var.	F8	Lines somewhat narrow; intensities in some respects as δ Canis Majoris. See Chapter XIV
100261	o^1 Cen	4.96	F8p	Lines narrow; spectrum as δ Canis Majoris. Radial velocity varies, range 13 kilometers (P. A. S. P., **34**, 168, 1922)
101947	L 4868	5.18	F8p	Lines narrow; spectrum as δ Canis Majoris
108968		5.44	F8p	Lines narrow; spectrum as δ Canis Majoris.

Catalogue of c-Stars.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
				On B43211 the hydrogen lines appear very strong and the spectrum was classified F2
128027		7.6	F8	Lines narrow
133683	A. G. C. 20470 L 6197	5.80	F8p	Lines narrow; intensities as in δ Canis Majoris
137170		9.3	F8	Lines narrow; 4227 strong for class
161796		7.27	F8p	Lines narrow; enhanced lines strong, as for Cepheid variables
	W Ser*	var.	cF9	See Chapter XIV
171620		7 8	F8p	Lines probably narrow
171635	d 45 Dra	5.0	F8p	Lines narrow, as for δ Canis Majoris. Cepheid spectrum (Adams and Joy)
172052		6.79	F8p	Lines narrow; as δ Canis Majoris
180028		7.24	F8p	Lines narrow; as δ Canis Majoris
194093	γ Cyg	2.32	F8p	Lines narrow, as δ Canis Majoris. Mount Wilson, $-3^{M}.0$
	VZ Cyg*	var.	cF8	See Chapter XIV
224014	ρ Cas	4 9	F8p	Lines narrow and sharply defined. Spectrum may be variable; it is of Class Ko near Hβ. Star is variable with small range. Large color equivalent
4852		10.2	G	Hγ and Hδ narrow; spectrum may resemble δ Canis Majoris
14662		6.5	Gop	Lines narrow; as δ Canis Majoris, 4077 strong. Cepheid spectrum, Adams and Joy
26673–4	f Per	4.9	Go, A5	Spectrum composite

Catalogue of c-Stars.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
29094, 5	58 Per	4.5	G	Cepheid spectrum; composite (Adams and Joy, P. A. S. P., 31, 185). Mount Wilson, $-4^M.2$
30353		7.7	Gop	Narrow lines, very little absorption at the G band, resembling H. D. 18474 (cG5) and R Coronae Borealis
31910	ϕ Cam	4.22	Gop	Lines narrow; strong lines as in δ Canis Majoris. Mount Wilson, $-2^M.9$
52973	ζ Gem*	var.	Gop	Lines narrow. Mount Wilson, $-3^M.4$. See Chapter XIV
57146, 154	Boss 1908	5 4	Gop	Cepheid spectrum, Adams and Joy
62058	R Pup	6.64	Gop	Narrow lines, lines as in δ Canis Majoris, G band unusually weak. Probably not variable
63700	ξ Pup	3.5	G	Spectrum presents a peculiar combination of hydrogen lines and characteristics of Class Ko. Mount Wilson, $-4^M.2$
84810	l Car*	var.	G	See Chapter XIV
95109	U Car*	var.	Go	See Chapter XIV. The lines are narrow
97334		6.32	Go	Lines somewhat narrow; strong metallic lines present. Hδ strong for Go
108134		7.41	Go	Hydrogen lines sharply defined
116802	W Vir*	var.		See Chapter XIV
146323	S Hor*	var.	Gop	See Chapter XIV
159181	β Dra	3.0	Go	Mount Wilson, $-3^M.5$

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
166126	W Ser*	9.0	F9p	See Chapter XIV. Hδ and K are narrow. Mount Wilson, $-0^M.6$
167660	WZ Sgr*	var.	G5 / cGo	See Chapter XIV
170764	U Sgr*	var.	F8 / cGo	See Chapter XIV
	RZ Oph*	var.	cGo	See Chapter XIV
187203		6.38	Gop	Lines narrow; 4077 strong
174089	YZ Sgr*	var.	cG1 / G5p	See Chapter XIV
177441	SZ Aql*	var.	cG1 / K2	See Chapter XIV
	TT Aql*	var.	cG2	See Chapter XIV
	U Vul*	var.	cGo	See Chapter XIV
	R Sge	var.	cG1	See Chapter XIV
192876	α^1 Cap	4.55	Gop	Lines narrow; 4077, 4215 very strong. Mount Wilson, $-3^M.9$
209750	α Aqr	var.	Go	Lines narrow. Resembles Ko in distribution of light. Cepheid spectrum, Adams and Joy. Cooler than α Aur
	Z Lac*	var.	cGo	See Chapter XIV
217476	Boss 5931	5 48	Gop	Lines narrow, intensities as δ Canis Majoris, Cepheid spectrum, Adams and Joy.
222574	Boss 6083	5 0	Go	Cepheid spectrum, Adams and Joy. Hydrogen strong, G band strong, 4077 very strong
16901	14 Per	5 6	G5	Mount Wilson, $-2^M.6$
18474	Groomb 590	5 61	G5p	Spectrum very peculiar, G band not continuous (Cf. H. D. 30353, cGo). See later remarks. Class?

CATALOGUE OF c-STARS.—(*continued*)

H. D.	Star	Mag.	Spectrum	Remarks
25056		7.4	G5p	Hydrogen strong as in F8; other lines probably as δ Canis Majoris. Cepheid spectrum, Adams and Joy
67594	ζ 29 Mon	4.4	G0	Mount Wilson, $-4^{M}.2$
86728	Br. 1397	5.60	G5	Lines somewhat narrow; strong lines present; Hδ strong
101501	Br. 1593	5.46	G5	Lines narrow; 4227 and others strong
119796		6.23	G5p	Lines narrow; intensities peculiar. G band weak as in R Coronae Borealis. Hδ and 4120 form a well-marked pair
180262		5.69	G5	Spectrum resembles Cepheid variables in such lines as 4172
	V Vul*	var.	cG7	See Chapter XIV
44537	ψ^1 Aur	5.1	K2	Cepheid spectrum, Adams and Joy. Radial velocity variable. Mount Wilson, $-3^{M}.6$
52497	Boss 1806 ω Gem	5.2	K0	Cepheid spectrum, Adams and Joy
25878		7.1	K0	Cepheid spectrum, Adams and Joy; narrow bright hydrogen lines
50877	Boss 1785 o_1 CMa	4.1	K2p	Combines characteristics of G5 and K2. Cepheid spectrum, Adams and Joy
78004	c Vel	3.69	K0	All lines sharply defined; bright spaces conspicuous. 4227 very strong
106690	Br. 1640	5.80	K5	Lines somewhat narrow; 4227 strong
184398		6.52	K2	Lines narrow; spectrum composite

CATALOGUE OF C-STARS.—(*continued*)

H. D	Star	Mag.	Spectrum	Remarks
192410		7 76	K5	Lines narrow; Hδ appears bright
192909, 10	o^2 Cyg	4.0	K	Spectrum composite; Cepheid spectrum, Adams and Joy
19299, 10	Boss 5200	4.2	K0, A3	Like ξ Cyg; TiO weak; cool; Cepheid spectrum, Adams and Joy.
196725	Boss 5299	6.1	K5	Very cool; TiO weak; like ξ Cyg
200905	ξ Cyg	3 9	K5	Cepheid spectrum, Adams and Joy, very cool; TiO evident. Peculiar, Fe+ bright. Spectrum composite
205114, 15		6.2	K0, A3	Spectrum composite. Very cool, TiO evident. Peculiar
213310, 11	5 Lac	4.6	K5	Spectrum composite. TiO evident. Peculiar. Like ξ Cyg; almost as α Ori but less TiO

APPENDIX B

Catalogue of Cepheid Variables*

Star		Designation	Period	Med. Pg. Mag.	Spectrum	Star		Designation	Period	Med. Pg. Mag.	Spectrum
			d						d		
CG	Sgr	185335	64 1	14.2	......	UZ	Sct	182512	14.74	13.48	.. ..
U	Mon	072609	46.13	7 38	G5-K2	TX	Cyg	205642	14 71	10.37	K2-K7
RS	Pup	080934	41.34	8 4	G5-K7	W	Ser	180415	14 15	9 0	cF9
U	Car	105359	38 74	8.55	F8-K5	SV	Vel	104055	14.10	9 3	F5-K2
l	Car	094262	35 52	4 85	F8	TT	Aql	190301	13.75	8.67	Go-K3
TX	Aql	200103	34 8	11.05	.. ..	SZ	Cas	021959	13.60	10.87	G
RU	Cen	133155	32.37	9 2	G2-K2	FI	Car	104758	13.45	13.2	.. ..
RY	Vel	101654	28.14	9.3	G2-K2	Z	Sct	183705	12.90	10 27	G2-Mo
CD	Sgr	184636	28 0	14.0	......	U	Nor	153454	12.64	9 97	Go-K5
SW	Aur	043231	27 33	13.32	......	XY	Car	105863	12.43	9.8	......
T	Mon	061907	27.00	7 52	F8-Ko	VX	Cas	002561	12.4	11.1	......
X	Pup	072820a	25 96	10 11	F5-K2	RY	Cas	234758	12 14	10 87	G2-K
RY	Peg	220133a	25	10.3	......	RX	Aur	045439	11.63	8 77	Go-Ko
SU	Gem	060727	24.81	11.2	Ko-M3	SV	Per	044242	11.13	10 06	F8-Ko
FK	Car	104959	23 25	13.7	......	TY	Sct	183604	11 05	12 06	F8
WZ	Car	105160	23.01	10 5	.....	BH	Oph	181112	11.04	13.06	......
WZ	Sgr	181119	21.7	9 5	Go-K5	VX	Per	020057	10.90	10.1	......
BI	Sco	164427	21 47	12 5	. . ..	Z	Lac	223656	10.89	9 96	F8-K2
RY	Sco	174433	20 32	9 3	F8-Ko	FR	Car	111059	10.72	10.4	......
VX	Cyg	205339	20 14	11 5	. ..	AP	Her	184515	10.38	11 1	......
RU	Sct	183604	19 70	10 0	Ko-M	FO	Car	105761	10.36	11.3	.....
VY	Car	104057	18 98	8 6	F8-K2	Y	Sct	183208	10.35	9 15	G2-K7
YZ	Aur	050839	18 36	11.8	Go-K	FQ	Car	110660	10.27	13.6	......
YZ	Car	102458	18.16	9.5	......	ζ	Gem	065820	10.15	4.95	F2-Go
CT	Car	103361	16.08	12 4	......	SY	Aur	050542	10.14	10.05	Go-G2
RS	Cet	022200a	17.41	9.79	G2 5	CR	Car	102958	10.0	11 4	......
W	Vir	132002	17.27	10.89	......	β	Dor	053262	9.84	4.9	F2-Go
SZ	Aql	185901	17.14	9.79	.....	AQ	Car	101860	9.77	9.8	......
Y	Oph	174706	17.12	7 7	F8-G5	S	Nor	161057	9.76	7 2	Gop
CD	Cyg	200033	17.08	10.2	Go-K2	S	Mus	120769	9.66	7 84	F8-G5
XZ	Cyg	110061	16.64	9.3	......	YZ	Sgr	184316	9.55	8.0	G2-K2
SX	Cen	121548	16.50	9 4	.....	κ	Pav	184667	9.09	5.53	F5-G5
RW	Cam	034658	16.41	9 89	. ..	RY	Boo	144523	9.01	7.6	F5
X	Cyg	203935	16 39	7.89	F8-Ko	XZ	Gem	075032	8 88	11.11	......
FF	Car	104057	16.33	14.2	....	S	Sge	195116	8.38	6.8	Go-G5
XX	Car	105364	15 72	9.9	.....	RU	Dor	053166	8.35	14 2	......
AV	Sgr	175822	15.39	12.68	......	CC	Sgr	184636	8.03	15 0	......
SV	Mon	061606	15.23	9.6	F8-K5	U	Vul	193220	7.99	8.0	F8-Ko
SZ	Cyg	202946	15.11	10.28	G5-K7	W	Gem	102915	7.91	7.8	F6-G6
VW	Cen	132763	15.04	10 1	......	RX	Cam	035658	7.91	9 0	G2-K2
RW	Cas	013057	14.80	10.57	G5-M	RW	Aql	200715b	7 87	9.8	cFo

* Complete to the beginning of 1929, and retained as the basis of statistical discussions in the text. About twenty stars have since been added to the list.

CATALOGUE OF CEPHEID VARIABLES.—(*continued*)

Star	Designation	Period	Med. Pg. Mag.	Spectrum	Star	Designation	Period	Med. Pg. Mag.	Spectrum
		d					d		
VY Cyg	210039	7.85	10.0	......	UZ Car	*103260*	5.20	9.9	......
ER Car	*110558*	7.72	8.0	F5-F8	AP Sgr	*180623*	5 06	8.03	F5-K0
FM Car	*105660*	7.64	13.6	......	V Lac	224455	4.98	9.01	F2-G5
W Sgr	*175829*	7 60	5.8	F2-G6	CN Car	*101257*	4.91	11.4	......
RS Ori	061614	7.56	9.4	F2-G0	SX Car	*104257*	4.86	9 7	F5-K0
WX Lac	220953	7.52	14.5	......	VZ Cyg	214742	4.86	9.71	F5-G5
R Mus	*123668*	7.51	7.7	F8-G5	S Cru	*124857*	4 69	7.91	F5-G8
η Aql	194700	7.18	4 98	G0-G5	WW Car	*104758*	4.68	10.4	......
UY CrA	*185337*	7.13	15.2	......	DY Car	*104859*	4.67	11.0	......
U Aql	*192407*	7 02	7.58	G0-G6	T Vel	*083447*	4.64	9.01	G0-G5
X Sgr	*174127*	7.01	5.6	F5-G9	UX Per	020657b	4.6	11.5	......
XX Vel	*103255*	7.0	11.4	......	FN Car	*105759*	4.59	11.10	......
AR Sco	*161522*	6.90	14.2	......	XY Cas	004459	4 50	11 0	F8-K
AY Sgr	*181718*	6.74	11.86	......	T Vul	204727	4.44	6.8	F5-G1
U Sgr	*182619*	6.74	8.0	F8-K0	V Vel	*091955*	4.37	8.70	F8-G5
T Cru	*121561*	6.73	8.17	G0-G5	Y Lac	220550	4.32	9.7	......
V Car	*082659*	6.70	8.77	G2-K0	UU Cas	234560	4 31	10 50	......
CS Car	*103057*	6 66	13 4	......	SX Per	041041	4 29	11 70	......
XX Sgr	*181816*	6.43	9 97	F8-K0	CY Car	*105360*	4 25	10 3	......
RR Lac	223755	6 41	9.96	F5-K0	EW Car	*101760*	4.24	14 9	......
S TrA	*155263*	6 32	7.86	F5-K0	X Sct	*182513*	4.20	10.4	F5-K2
X Vul	195326	6.32	9.75	G0-K5	SY Cas	000957	4 07	9 6	F8-G5
RS Cas	233261	6.30	11.4	......	BF Oph	*165926*	4.07	8.4	F8-K2
AB Vel	*101655*	6.25	15 2	......	ST Tau	053913	4 04	9 5	F5-G5
X Cru	*124058*	6.21	8.75	G0-K0	BB Cen	*114862*	4 00	9 8	......
VV Cas	014459	6.21	11.66	G	α UMi	012288	3.97	3.29	F5
RV Sco	*165133*	6.06	8.0	F5	Y Aur	052142	3.86	10 6	F8-G5
VW Cas	005961	5.99	11.2	......	SU Cyg	194029	3.85	7.0	F3-G1
R Cru	*121861*	5.82	8.34	F8-K0	XX Vir	*141105*	3.83	11.8	......
Y Sgr	*181518*	5.77	6.7	F5-G5	RT Aur	062230	3.73	6.27	F1-G5
FH Car	*104460*	5.66	14 2	......	SS Sct	*183807*	3.68	9 07	F8p
XZ Oph	*164829*	5.55	12.5	......	UX Car	*102557*	3.68	9 77	F2-K0
UY Car	*102861*	5.54	9.3	......	Y Car	*102957*	3.64	8.7	F5
VY Per	022058	5.53	12.13	......	R TrA	*150166*	3.39	7.85	G0v
RZ Gem	055622	5.53	10.33	F8-G5	UZ Cen	*113662*	3 33	9 2	G5
V Cen	*142556*	5.49	8.1	F5-G7	AZ Cen	*112060*	3.21	9.8	......
X Lac	224555	5.44	9.33	G1-G5	ST Cen	*110551*	3.15	10.4	......
SW Cas	230258	5.44	10 33	G0-K0	SZ Tau	043118	3.15	7.55	F4-G2
δ Cep	222557	5 37	4.93	F4-G6	ET Car	*100861*	2.91	14.7	......
UY Per	022758	5 36	12.3	......	EY Car	*103860*	2.88	10 6	......
UW Car	*102359*	5 35	10.5	......	U TrA	*155862*	2.57	8 78	F2-G5
TT Her†	164717	5.33	10.13	......	TU Cas	002050	2.14	8.54	F8.5
AY Cen	*112060*	5 30	10.8	......	SU Cas	024368	1 95	6 81	F2-F9
CQ Car	*102759*	5.3	13.6	......	SW Tau	041903	1.58	10.2	A7-F2

† This star has recently been shown by Kordylewski to be an eclipsing star.

NAME INDEX

SUBJECT INDEX

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